Argument-Mirroring Proof-Texts: A Tool for Leveraging Students' Incoherent Thinking to Effect Greater Coherence

Steven Ruiz
Arizona State University

Keywords: calculus, chain rule, coherence, clinical interview, teaching experiment

This report details a teaching experiment (Steffe & Thompson, 2000) which consisted of clinical interviews (Clement, 2000) and exploratory teaching interviews (Sellers, 2020) designed to perturb selected students' incoherent thinking. In short, this study addressed the following research questions:

1) In what ways are students' understandings of the chain rule coherent with one another?
2) How do students respond to proof-texts which mirror their own incoherent thinking?

Theoretical Perspective

Glasersfeld (1988) describes schemes as quanta of knowledge held by an individual. Per Thompson (2008), incoherence is a lack of consistency between schemes. I used Toulmin's (1958) method to identify within students' arguments the claim – an assertion to be proven, data – evidence used to support the claim, and warrant – why the data is relevant to the claim.

Data Collection and Analysis

Ten students from a public research university in the American southwest participated in the initial clinical interviews, designed to examine the subjects' cognition regarding the chain rule of differentiation and its fundamental ideas e.g., continuity, differentiability, composition of functions, subjects were each presented with three functions and asked where each function was defined, continuous, and differentiable. The student was encouraged to think aloud and make annotations as appropriate.

Analysis focused on partitioning each student's transcript into arguments and identifying relevant claims, data, and warrants. These facets of a student's argument were then compared to one another and where appropriate (in my estimation), categorized as incoherent in one of four ways, including *incoherent claims* – two or more claims result in an impossibility and *justifications not broadly applicable* – the warrant from one argument would cause the claim from another argument to be altered. From each source of incoherence, *argument-mirroring proof-texts*, conditional statements and justifications built from the participant's reasoning were generated and presented to them to examine how they responded to their incoherent thinking and whether they take measures to resolve it.

Results and Discussion

The bulk of this report focuses on the four subjects who participated in both the clinical interviews and the argument-mirroring proof-text phase of the study and the effectiveness of the argument-mirroring proof-texts in particular. The latter method showed great effectiveness in a pilot study (Ruiz, in press). Furthermore, argument-mirroring proof-texts are highly individualizable, have the potential to be applied to a wide variety of contexts inside and outside of mathematics, and as such, the method represents a novel contribution to the field.

References

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In R. A. Lesh & A. E. Kelly (Eds.), Handbook of research design in mathematics and science education (pp. 547–589). Lawrence Erlbaum.

Glasersfeld, E. (1988). Cognition, construction of knowledge, and teaching. In *Constructivism in Science Education* (pp. 11–30). Springer.

Ruiz, S. (in press). Argument-mirroring proofs: A methodological approach for helping students recognize incoherence in their thinking. *Proceedings of the 46th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Cleveland, OH.

Sellers, M. E. (2020). Students' Quantifications, Interpretations, and Negations of Complex Mathematical Statements from Calculus (Doctoral dissertation, Arizona State University).

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), Research design in mathematics and science education (pp. 267307). Hillsdale, NJ: Erlbaum.

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In Proceedings of the annual meeting of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 31-49). Morelia: PME.

Toulmin, S. E. (1958). The philosophy of science (Vol. 14). Genesis Publishing Pvt Ltd.