Supporting Mathematical Instruction Around Definitions Through Values-Centered Collaboration

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Extensive research shows students do not learn as much from undergraduate mathematics courses as we would like (e.g., Asiala et al., 1997; Borji & Martínez-Planell, 2020; Siyepu, 2013). Despite research demonstrating the efficacy of non-lecture-based approaches (e.g., Inquiry Oriented Abstract Algebra, Larsen, 2013; Larsen & Lockwood, 2013; Learning Abstract Algebra with ISETL; Leron & Dubinsky, 1995), most mathematics instructors lecture in lower-division courses like Calculus (Kirin et al., 2017) and upper-division courses like Abstract Algebra (Johnson et al., 2018). Dawkins and Weber (2023) suggest mathematicians choose to lecture because it meets their goals, including content coverage, preparing future mathematicians, precision and language, and efficiency; thus, they call for improving instruction by enhancing lectures in ways that feel comfortable to instructors. Our project supports this novel approach to improving undergraduate instruction by centering collaboration between mathematics education researchers and mathematics instructors. We focus on teaching the norms and values of definitions, as definitions are fundamental across different undergraduate courses and have been identified as an area where students and instructors can struggle to communicate (e.g., Martín-Molina et al., 2020).

This project integrates focus groups, class observations, surveys, and interviews with instructors and students to develop a toolkit of instructional innovations for effectively communicating mathematical norms and values around definitions in both upper-division (Abstract Algebra and Real Analysis) and lower-division (Calculus I and II) lectures. This poster presents preliminary observations in the first round of focus groups and surveys with 10 mathematics faculty from two universities. The aim is for the instructors to develop their own minimally invasive activity centered on a definition that communicates a mathematical norm or value (Rupnow & Randazzo, 2023).

Instructors reported challenges with limited time for activities in lecture, highly coordinated courses limiting freedom of instruction, as well as more content-specific challenges. For instance, some instructors explained how it was difficult to plan an activity that shared norms and values of mathematical definitions in such a way that complemented their learning goals. For example, instructors reported difficulty in communicating precision without becoming over-encumbered in fine details (e.g., tensor products). Other instructors expressed difficulty in growing students' intuition about certain definitions (e.g., Laplace Transform) or translating between students' informal and technical understandings (e.g., the informal notion of closeness with the technical nature of ϵ and δ in convergence proofs). As the project continues, the project team is continuing to collaborate with the mathematics faculty to address these challenges (e.g., by understanding which norms and values might afford which learning goals) while supporting their instruction to communicate norms and values in ways that are comfortable and non-intrusive for both the instructors and their students.

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