Complexity, Signaling, and Goal-Orienting Feedback in Specialized Interactive Digital Tools

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The implications for students and teachers of educational digital tool design have thus far been under-examined. We find it important to problematize and bring attention to relationships between the explicit or hidden choices of digital tool designers and implications for the possible learning taking place with the tool. Exploring one aspect of these relationships, we examined extant digital tools within the undergraduate mathematics curricula. Grounding our analysis in Piaget's genetic epistemology, we took on the perspective of the epistemic user and identified features of extant digital tools that hold implications for the learner-user experience. With this we generate initial categories that contribute to a user-oriented phase of a framework for digital tool design and discuss implications for further research.

Keywords: digital tools, digital task design, undergraduate student learning

Digital tools can be leveraged in a wide variety of contexts ranging from asynchronous learning environments, in-class digitally supported activities used by individuals or groups, discussion-orienting didactic objects (Thompson, 2002), or tools to be used throughout a curriculum. Designers of educational digital tools engage in unique task design, at times facing different constraints than those of traditional in-class or at-home task-design. For instance, digital tools are necessarily contextualized within software constraints, and designers have varying levels of control over user-interface and visual design features that can enhance or constrain the user experience. These decisions, made implicitly or explicitly by the designer, all have the potential to orient the user towards learning goals set by the designer or to potentially obscure the underlying intended mathematical content.

Not yet directly addressed within the literature on digital tools is the relationship between designer choices and the experiences of users engaging educational tools. The broad aim of our research agenda is to highlight this relationship and infer implications for users. Pursuant of this goal, we analyzed a variety of extant digital tools (e.g. "micro-worlds", online interactive textbooks, educational games, computational tools, etc.) to infer features of the tools that might constrain or afford users' experiences for learning. We address the following research question: *What design features of specialized interactive digital tools might enhance or constrain the learning of users engaging with the tools?* We provide initial explanatory and operationalizable constructs in the form of digital tool features that demonstrate the relationship between design choices and the user experience.

Literature and Theory

This report is part of a larger project exploring educational *digital tool* design. By *digital tool*, we mean an instantiation of a digital technology created within a specific software for the purpose of student use in learning (Margolis et al., 2025). We aim to provide a framework for designers of undergraduate mathematical digital tools, enabling them to approach tool design with attention tool features that might constrain or afford users' progress towards established learning goals. We think of this design process as unfolding in two phases: 1) a macro-orienting phase in which the designer situates possible digital tools within *design space*, and 2) a micro-orienting phase in which the designer makes tool-level design choices, continually de-centering to consider the implications of the choices for users (Margolis et al., 2025).

This conceptualization of the design process draws from Yerushalmy's (2005) notion of a *design space* for interactive digital mathematics textbooks, which she broke down according to "forms of interaction", "interdependency of diagrams and text", and "forms of represented data" (Yerushalmy, 2005). We adopt our own notion of a design space to all digital tools and think of the space as variations in the spectra of *specialized to generalized*, *interactive to inert*, and *dynamic to static* (Margolis et al., 2025). For the purpose of this report, we analyze extant tools from undergraduate mathematics curricula that are specialized (e.g. supporting a specific learning trajectory) and interactive (e.g. users manipulate controls and receive feedback). Our analysis attends to features of tool-level design choices that impact the experience of the epistemic user.

Situating Within the Literature

Thus far, the literature on digital tool design includes well-investigated but siloed regions of design space, such as micro-worlds (Kaput, 1992), didactic objects (Thompson, 2002), videos, or projects like TELMA (Bottino & Kynigos, 2009) or ReMath (Artigue & Mariotti, 2014). When not siloed, much of the literature also takes broad framings of the digital tool use that foreground the interplay between the student, teacher, tool, and situated social or pedagogical environment (Leung & Bolite-Frant, 2015). While these silos are of value, to our knowledge links have not yet been made between design decisions, the user experience, and the learning taking place.

In a recent review of the literature on digital tool design, Leung and Bolite-Frant (2015) described a still extant gap between our knowledge of "...how to use a tool and how the tool usage leads to mathematics concept development." (p. 207). They cautioned that without more careful attention to tool use and students' cognition, "…the use of tools may result in epistemological and didactical obstacles for students, depriving them from reaching the desired or intended mathematical knowledge." (p. 207). They concluded that no theory yet exists that productively encompasses "tool-based task design", which is a gap that we intend to remedy.

Other authors have problematized the interplay between pedagogical intention and design. Bottino & Kynigos (2009) consider the pedagogical intentions of digital tools as inseparable from the tools itself. Our work aligns with this general conception of digital tool design, and we intend to generate features of design informed by foregrounding the user experiences and the possible hindering or supporting of users' achievement of the designers' pedagogical goals.

Theoretical Framing

For this paper, we foreground the experience of the epistemic user of digital tools. The user engages with the digital tool to complete an activity or to learn a mathematical idea within an undergraduate mathematics setting. For space, we will consider only individual student-users, leaving other scenarios such as groups of student-users or teacher-users for future work.

With this in mind, we take a radical constructivist framing (Glasersfeld, 1995) of the user, who learns through interaction with the digital tool. The epistemic user takes actions within the digital tool, receives and interprets feedback from the digital tool, and imputes and develops mathematical meanings from their tool engagement via assimilations to and accommodations of schemes of meaning (Piaget, 1971) for which the inscriptions or visualizations within the digital tool are figurative material for their re-presentations. Within this epistemological framing of users' experiences with these tools, in analysis we consider the epistemic student's digital and mental actions using the tool and inferences possible from assimilations, abstractions and generalizations (ibid) resulting from their interaction with the digital tool.

Similarly, while our analysis primarily considers the user perspective, we find it also important to note that extant digital tools for mathematical learning were necessarily created in the context of the (however broad or specific) pedagogical goals of the designer. We consider that when designing a digital tool to meet some learning objective, the designer is making explicit or implicit conjectures about how students' interactions are going to support particular ways of reasoning. While we can reasonably establish the pedagogical underpinnings of some digital tools designed with respect to curricula that well-define hypothetical learning trajectories and targeted ways of thinking, such as quantitative and co-variational reasoning (Thompson & Carlson, 2017), we foreground our analysis on the features of the tools made for such curricula.

Methodology

The primary goal of this initial analysis was to examine extant digital tools from the perspective of a user. Because of the magnitude of digital tools publicly available, we sought digital tools that would likely generate theoretical user-oriented insights, rather than attempting to survey or descriptively analyze the breadth of tools available. As such, we drew from our own diverse libraries of tools, tools that we were aware of from active curricula or research projects, and tools submitted via survey. All tools discussed are masked, and in our results we generically describe features of the tools and give general diagrams of the tools' layouts and controls. This masking is to analytically describe and explain features of tools without being read as passing undue judgment on any one digital tool or a collection of digital tools.

Analysis

Our analysis attends centrally to observable features of extant digital tools as they might matter for the actions, feedback, and potential meaning-making of the epistemic student. This involves critically examining tools by taking on the perspective of a user. Our approach is analogous to prior work taking radical constructivist framings of exam items from the perspective of "users" (i.e. the students taking the exams) (Tallman, Reed, Oehrtman, & Carlson, 2021; Reed, Tallman, Oehrtman, & Carlson, 2021). Analyzing exam items for the purpose of identifying features by which one might assess mathematical understanding, Tallman and colleagues considered the mental activity necessary for the epistemic test-taker to successfully answer the exam item. They identified reasonable requirements on mental activity and thusly inferring features of items that reasonably assessed particular mathematical meanings. In doing so, the authors foregrounded: 1) the problem contexts, 2) the relative complexity of the various mental actions and inferences necessary to successfully complete the prompt, 3) the mathematical modeling required to establish quantities relevant to the problem, 4) the necessity of the student to make strategic decisions at the onset of solving the problem, 5) and other necessary mental actions taken by an epistemic user (Tallman et al., 2021; Reed et al., 2021).

We take a similar approach, in that we consider the epistemic user of digital tools, analyze the tools with respect to the actions and inferences from feedback reasonably necessary for use of the digital tool to achieve an objective, or for mathematical meaning making. Such analyses, independent of knowledge of the designer intention, are important, as how users might perceive or engage with the tools might drastically differ from how designers intend tools to be used.

After engaging in individual analyses, the team met to discuss the salient features of tools for supporting or constraining meaning-making. We report on some of the features that emerged from our analysis and give examples that problematize the need for a greater dialogue around the interplay between design choices, learning goals, and the user experience.

Results

We will elaborate the emergent features of two primary types of tools that fall within the interactive and specialized regions of design space. In this analysis, we highlight the features of *complexity, signaling,* and *goal-orienting feedback*.

Dynamic Interactive and Specialized Tools

We first discuss digital tools of interest that are also dynamic (e.g. “micro-worlds” (Kaput, 1992)). This particular design subspace tends to contain tools that display at least one or more non-symbolic representations of mathematical quantities and give the user some means of dynamically altering the state of the representations as well as any linked textual or symbolic information related to the mathematical representation. We attend to features that help delineate similar tools with respect to the student experience and discuss then ways that these features extend less dynamic, but still interactive and specialized tools.

The first collection of tools come from curricula that intentionally leverage dynamic tools to promote quantitative and covariational reasoning in pre-calculus and calculus. We examined tools promoting quantitative meanings for Riemann Sums and definite integration by appeal to applied contexts. As masked stand-ins for the common design elements of these tools, we will use hypothetical digital tools that we refer to as the *Hydrostatic Force Tools*.

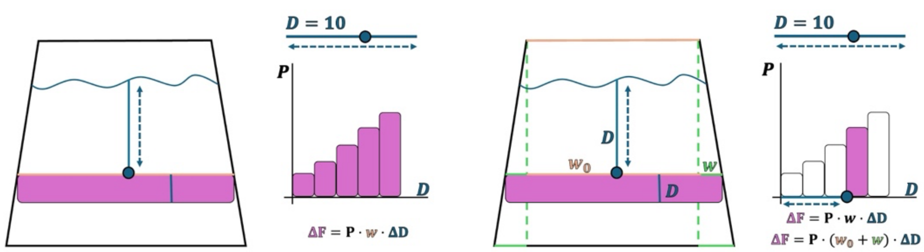

Figure 1. Variations on Hydrostatic Force Tools

These masked tools use hydrostatic force on a dam as the central context for definite integration. Figure 1 depicts variation in the typical layout of such tools and gives representations of the common design choices for control elements, feedback elements, and stylistic elements of the *Hydrostatic Force Tools*. As depicted Figure 1, common to these tools were contextual representations of the quantitative relationships relevant to defining a definite integral (seen on the left), a separate but linked representation (typically graphical) on the right, symbolic or numerical representations of the quantitative relationships at selected states, and one or more control elements (sliders) that the user could manipulate. The masked task for these tools asked students to come up with a Riemann Sum or definite integral that would calculate the hydrostatic force on the depicted dam. Some tools would be implemented as didactic objects, others as formative asynchronous quizzes, and others as supports for in-class group work.

Two important features that emerged from analysis of these tools are *complexity* and *signaling*. *Complexity* relates tool controls to the types and levels of feedback given in response to control change. *Signaling* refers to stylization of the perceptual features of the digital tool with the potential to highlight information or make connections between representations. Many Hydrostatic Force Tools employed color to signal links between multiple in-tool representations of quantities. In Figures 1a and 1b, each representation for depth (e.g. graphical, contextual, symbolic, and slider) is colored blue. Similarly, force representations are colored pink, and width representations are colored orange (and green). Moreso in Figure 1b, contextual and graphical representations of the local force quantities are signaled when the slider varies. As the local representation of force moves throughout the dam, the rectangle at the associated depth is singularly colored pink. Conceptualized as dynamic supports for students' quantitative and covariational reasoning, signaling was a key design feature of these tools that served the purpose of not only emphasizing the key quantities in the modeling process, but also linking the representations of the quantities within the tool amidst variation of the independent quantity.

Complexity was also a distinguishing feature of these tools. Figure 2a depicts one tool in our data with higher complexity, in a way that potentially hindered present signaling. There were multiple sliders in the tool, each controlling possible variation in the setup of a Riemann Sum such as interval size, sample point location, interval location, and interval number. Many of these control elements influence each other, including the interval number and interval size being interrelated, and the sample location impacting all approximation representations at once. With this higher complexity, particularly the interrelatedness of the control variables, the user must attend to numerous feedback elements at once when altering the value of any number of sliders.

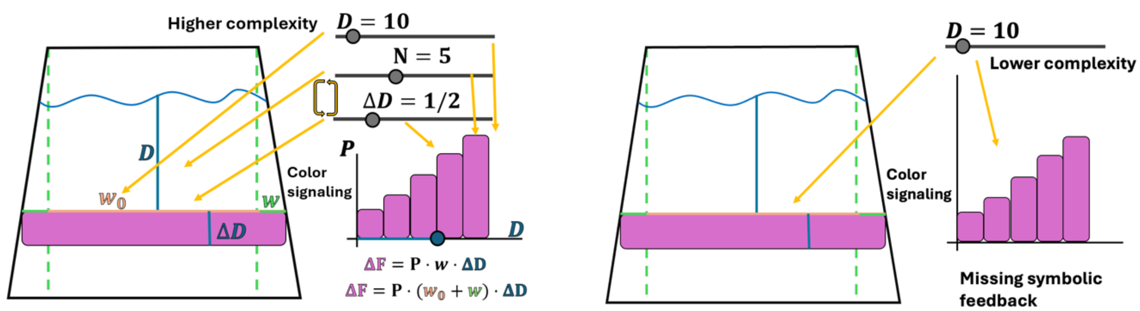

Figure 2. Tools with higher and lower complexity

This feedback variation was across representations and across other control elements. This adds mental actions necessary for a user to infer the important quantitative relationships in the accumulation process, potentially distracting from color-based signaling taking place within the tool. Contrastingly, Figure 1b has lower complexity, hides certain feedback elements, and contains similar intentional signaling. Each slider directly alters only one aspect of each representation, increasing the signaling potential to a user intentionally varying the control elements. Moreover, the symbolic feedback is hidden, reducing the amount of information that the user might observe at any one time, and increasing the weight of the color-based signaling.

River Swimmer Tools

We now broaden the design subspace by including digital tools that share features with Hydrostatic Force Tools but are more static. Such tools are largely text-based and highly scaffolded, centering on singular vector or trigonometric problems. Our masked hypothetical tool is called the *River Swimmer Tool*, involving velocity vectors and correction trajectory angles. The following directions and problem statement are placed at the top of the webpage: ***Directions*** *Use the interactive application below to visualize the problem and guide you through the process of solving. You may choose your own path to the solution.* ***Problem:*** *A swimmer wants to cross a river that is 120 meters wide. The river's current flows at a speed of 1.5 m/s parallel to the riverbanks. The swimmer can swim at a speed of 2 m/s in still water. The swimmer wishes to* *reach the point directly across the river from the starting point. What angle relative to the riverbank should the swimmer aim for in order to reach the point directly across?*

This is followed only by a single statement and dropdown menu:

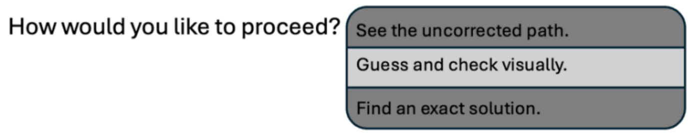

Figure 3. Branch Choices for the River Swimmer Tool

If the first option is selected, the user is presented with a simple dynamic diagram with three points: one for the starting location, another for the intended destination, and a third representing the path of the swimmer. Pressing the "GO" button in the diagram causes the swimmer point to move diagonally away from the starting point, downstream of the intended destination. Choosing the second option provides a similar interactive element.

The third option in the dropdown menu provides a sequence of steps, containing boxes in which users provides numerical data (pertaining to angles, directions, and coordinates) and the boxes give immediate feedback based on the entry. More dropdown menus then provide further branching opportunities. Each option in Figure 3 creates one of three main branches in which the user progressively acts, receives feedback, and is provided more branch options. For example, on one branch the user can choose between solving the problem via a vector table or solving via drawing a triangle, with either choice creating a new sub-branch of activities. Certain paths also contain minimalistic diagrams, such as an unlabeled triangle for the triangle solution path.

This digital tool leverages a branching mechanism, guarded by dropdown menus, as an effective means of reducing complexity. The user not only interacts with one branch at one time, but also only one activity within the branch at any one time. The more static design of the tool provides the user with clear singular feedback from any one action taken within the tool. Even more unique to the design of this tool, the hiding of further steps along any branch contrasts similarly set up tools in which information is not hidden, but the action of scrolling (rather than option selection) is the mechanism for progress through a task.

The River Swimmer Tool made less use of color-based signaling, instead signaling in ways different from the Hydrostatic Force Tools. As much of the tool was text-based, textual stylization such as italicization, bolding, and fully capitalized words were implemented to emphasize tool instructions, to call attention to situational considerations such as units, or to draw attention to optional tool features such as drop-down hint boxes.

One feature of this digital tool not present in the Hydrostatic Force Tools was explicit feedback provided in response to user actions that informed the user whether the input or action advanced them towards (or away from) completion of a goal. For instance, users received "Red X" (incorrect) or "Green Check" (correct) feedback for angle measure, magnitude, and vector coordinate inputs. Further, textual feedback given at the end of certain branches similarly oriented users to other branches that might yield more accurate results. Much feedback of this type had the function of explicitly guiding users to a final answer. Each step, including the optional step of guessing and checking, was accompanied by written and/or symbolic affirmations (e.g. "Yes! We want to use sine for the vertical component.") or formative guidance (e.g. "Consider how a negative angle would affect the path of the swimmer in this problem."). We call this feature of the digital tool *goal-orienting feedback*. Key to this feature is an explicit tool function of providing actionable feedback, indicating that the user is closer to or farther from achieving a goal. *Goal-orienting feedback* was less present in Hydrostatic Force Tools, though it was identified in some. We interpret that rather than directly in the tool itself, any goal-orienting feedback intended for the Hydrostatic Force Tools would be given by an instructor.

Discussion and Conclusion

In this report we examined extant digital tools, attending to design features that hold implications for the user experience in the context of learning. We now discuss implications of the features for user learning and conclude with future directions for this avenue of research.

Complexity, signaling, and goal-orienting feedback all interact with each other and contribute to the barriers or supports for user learning. Use of a digital tool with high complexity of the kind seen in Figure 2a would require the user to strategically vary individual controls attending to interrelationships between the controls inferred from systematic variation, to then infer the particular quantity represented by individual controls, to then infer the quantitative operations that construct hydrostatic force. This complexity might obscure extant signaling, and in the absence of explicit goal-orienting feedback within the tool users might need to spend a substantial amount of time engaging with the tool itself before being enabled to make the necessary abstractions to the underlying mathematics. This is not to say that increasing the need for user observation and inference is a constraint for every task. In fact, there are plenty of curricular contexts in which a higher need for inference is an intentional design element.

In contrast, River Swimmer Tools were heavily scaffolded, and complexity was lessened by restricting viewable and actionable tool elements, ensuring that users were only tasked with one small component of the larger problem solving at any one time. High volume of goal-orienting feedback densely distributed across the tool provided users with more explicit indications that their actions brought them closer to or farther from the task goals. This requires less inference on part of the student, and more heavily guides users towards goal achievement, such that the task can be entirely achieved within the tool. This lessening of user inference is not suitable for all tasks, however, and in particular removes much of the need for guidance from an instructor.

As such, the three features identified from our results allow for analysis of digital tools of varying types such that one might build initial hypothetical models of interactions taken by the epistemic student, as well as the constraining or affording of learning by use of the tool. Moreover, these features also enable us to distinguish (and perhaps eventually establish) design choices for supporting students in ways unique to the location of tools within design space and unique to the designer’s goals. While our features were applied across disparate tool types, we were also able to speak into the tool-specific design elements in relation to the features. In other words, reducing complexity in a static environment might be different from complexity in a dynamic environment, and tools residing in similar regions of design space might have unique design components that contribute to a feature such as complexity.

Such constructs can be incorporated into a process of design-based research on digital tools, wherein theoretical analyses of newly-built tools might provide initial models for the learning taking place by use of the tool. Such models can then be refined through observation of actual user engagement with the tool. Considering categories such as these in the process of tool design can be an important perpetual act of de-centering by designers. Further work from which more similar features might emerge will contribute to an eventual process for iterative design of digital tools that is informed by the thinking of users, thus enhancing their impact. Future work will also include interviews with designers and interview observation of students’ use of these tools in clinical learning environments.

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