Students' Perceptions of Differences Between Physics and Mathematics Classes

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Student learning of physics is built on a foundation of mathematical knowledge. Accordingly, physics education research has devoted much study to students' use of specific mathematical content in specific physics contexts. More generally, though, students also experience various "cultural" differences between the (mathematics) courses where they learn this content and the (physics) courses where they apply it. Researchers have discussed these cultural differences, but to our knowledge they have not investigated whether students are aware of them, or students' beliefs about how these differences may affect their learning. We used a survey questionnaire and individual interviews to study this in a group of math-physics double majors (or majors in one with minors in the other). These students are indeed aware of many such cultural differences, cite many classroom examples, and have strategies to cope with them.

Keywords: physics education, mathematics, disciplinary cultures

Introduction

The physics education literature has extensively studied students' use of mathematics in their physics coursework (Caballero et al., 2015). Often this focuses on specific mathematical techniques in specific physics contexts, such as students' use of vector calculus in electromagnetism (Bollen et al., 2015). However, some authors have also pointed out more general "cultural" differences between mathematics and physics which likely affect students' ability to make use of mathematical knowledge in a physics context (Redish and Kuo, 2015). We interpret this idea broadly to include, for example, the standards of proof or derivation of mathematical facts, the ways that mathematical notation is used, how approximations are used and justified, and the use of potentially ill-defined concepts such as the Dirac delta function.

An illustration of such a cultural difference, in the use of function notation, is provided by "Corinne's shibboleth", attributed to Corinne Manogue (Dray and Manogue, 2002). Mathematicians and physicists are claimed to give different answers to this question: If $F(x, y) = x^2 + y^2$, then what is $F(r, \theta)$? Mathematicians should treat the function arguments as dummy variables and answer $r^2 + \theta^2$, while physicists typically assume that F represents a physical quantity that can be expressed in either rectangular or polar coordinates, and answer r^2.

Our interest is in whether students themselves are aware of such cultural differences and how they may impact their learning, and whether they have coping strategies to deal with them as they apply their mathematical knowledge in physics. We studied these questions through a survey of double majors in mathematics and physics at our institution, plus interviews with a subset of the participants.

Purpose and Theoretical Background

The purpose of this exploratory study was to learn whether undergraduate students are aware of the cultural differences in the treatment and use of mathematical content between their mathematics and physics courses, how they see these differences as affecting their learning, and what coping strategies they may employ. Our understanding of cultural differences here is consistent with that of Rabin et al. (2021), which explored obstacles to interdisciplinary communication between STEM fields arising from different practices around notation, terminology, modeling, and rates of change. On the other hand, it is distinct from the notion of disciplinary cultures in Reinholz et al. (2019) which refers to the administrative organization of the disciplines within the university setting. We were particularly inspired by the work of Redish and Kuo (2015), who point out that "Math in science (and particularly math in physics) is not the same as doing math. It has a different purpose – representing meaning about physical systems rather than expressing abstract relationships" (p. 563). We interpret the construct of cultural differences very broadly, and indeed our students discussed a wide range of topics ranging from specific mathematical content to the teaching styles and assignments in their mathematics and physics courses. One could frame our questions in terms of transfer of learning from mathematics to physics, but we think the issues are broader than that, and not unidirectional.

The authors are two physicists and one mathematician having a postgraduate physics background. In formulating our survey and interview questions, we drew upon our own experience and the literature cited above.

Our research questions were:

1. Are undergraduate double majors aware of cultural differences between mathematics and physics, and differences in the presentation and usage of mathematical content between their physics and mathematics courses?
2. What do they cite as major differences in "common" mathematical content?
3. How do they perceive these differences as affecting their learning?
4. What coping strategies do they employ?
5. What specific classroom experiences do they cite?

Participants and Methods

We designed our study to include all undergraduate double majors in mathematics and physics at our institution. The rationale was that such students would have taken enough high-level courses in both disciplines to have experienced a range of cultural differences, content areas, and teaching styles. These would include proof-based mathematics courses beyond the basic calculus sequence. There were fewer double majors than we had hoped, so we decided to add students majoring in either discipline and minoring in the other. By email we invited each of them (36 in all) to complete a survey questionnaire in exchange for a gift card, obtaining 20 completed surveys. Following an initial analysis of these, we invited a subset of students to be interviewed in person, with another gift card as compensation. There were 11 completed interviews in all, which were audio recorded and transcribed.

The surveys asked students' opinions about specific mathematical topics that we suspected might be problematic, such as Fourier series/transforms, infinitesimals, and linear algebra; differences in notation between students' physics and mathematics courses; and differences in the standards of proof or justification between the two fields. We asked whether they have strategies for coping with differences in presentation of "the same" topic in their physics versus mathematics courses, and whether they have advice for mathematics or physics instructors about presenting such cross-disciplinary content. We also included the "Corinne's shibboleth" task since we have not seen published data on it in the literature.

Interviews were semi-structured, following a general script but allowing for detours to explore student comments in more depth. In addition to asking students to expand on their survey answers, we asked what they thought the role of mathematics in physics was; how mathematics and physics classes "feel" different; and whether seeing the same topic presented in both a mathematics and a physics class had facilitated or impeded their learning of it. We tried to keep our questions unbiased, not showing favoritism to either mathematics or physics, and open-ended to allow students to bring up topics we might not have expected.

After an initial analysis of the surveys, we selected students to interview whose comments we wanted to explore in more depth. All surveys and interview transcripts were read multiple times by all authors, and discussed as a group. Surveys and interviews were analyzed thematically (Terry et al., 2017), coding ideas that multiple students brought up and consolidating them into general themes. Our results below also include student comments that were less frequent but worth exploring in future research.

Results

Theme 1: The Nature of Mathematics and Physics

Our students perceive mathematics to be deductive, standardized, and highly structured, whereas physics to them is more inductive, flexible, and eclectic. This applies to the nature of argumentation or proof in each field, the presentation of material in lectures, and the homework and exam problems posed. They find mathematical proofs highly structured, with logical steps spelled out in detail and named theorems cited as justifications. Derivations in physics look more like calculations, relying on known techniques from algebra or calculus without explicit justifications. When words appear in a physics derivation, they may actually interrupt the flow of the reasoning to introduce a physical assumption or approximation. Students feel that mathematical proofs contain all the reasoning needed to understand them, whereas they may have to stay after class to ask their physics instructor to explain the steps of a derivation if they want to fully understand it.

Topics presented in mathematics courses are fully developed from definitions and axioms, whereas mathematical methods used in physics may be introduced without complete justification and simply illustrated through specific examples. Students understand and accept that mathematics is used as a tool in physics, and this tool need not be fully justified from first principles.

> My math classes have been, they have, like a familiar structure … they have some motivation, they have some definitions, a theorem, and a proof … Whereas in physics I feel like it's kind of been more all over the place. Like the transitions between when somebody's giving like a heuristic argument or an actual proof of what they're trying to show, it's a bit blurry. [Interview, double major]

Students find homework and exam problems in mathematics to be quite standardized and predictable in terms of what a correct solution should contain, even across different courses. In contrast, they see wide variation in expectations across physics courses, and are unsure what methods or approximations are allowed in homework solutions:

> I can get full credit on the assignment even though I don't know why the thing I just applied works in this instance…you can BS the physics assignment and still get 100 on it, even though you don't understand why the stuff you're writing works…in a math homework assignment, if you BS the homework assignment you're not getting 100 on it…because you're going to be missing some crucial steps in your proof, and you know

you're missing those crucial steps, and when a proof is complete you feel it. [Interview, physics major]

Theme 2: Physics "Tricks"

Students referred to some uses of mathematics in physics as "tricks," which includes two distinct categories. Some tricks, such as interchanging the order of derivatives and integrals, could be mathematically justified but this justification may be omitted in class. Others are physically motivated approximations. Both were sources of confusion for students.

We were surprised that physical approximations were considered problematic by many students. Often these rely on Taylor series expansions, a topic that many students felt was insufficiently emphasized in their calculus courses. But even the small-angle approximation for a pendulum raised questions. Why not give an exact solution instead? Won't the error accumulate over time as the pendulum swings? Why not give a rigorous error bound, as would be the case in a mathematics class? Why not retain additional terms in the Taylor expansion? We suspect that students' prior mathematics classes have contributed to a general preference for exact solutions and an aversion to discarding information by approximating.

> I've taken real analysis, when you do real analysis and you're giving the approximation functions for equations in that class like the Taylor expansions, they of course give you the error and they prove why the error works and they are like, this is how we minimize the error to make sure that there is as little error as possible. We weren't given anything like that [in the physics course]… I personally find it a little bit confusing to have that stuff and then not have any explanation for why these work the way they do and like what the margin of error is and how we can minimize that margin of error, especially since it feels like it would be important. [Interview, mathematics major]

Theme 3: Preparatory Mathematics Courses

In general, students felt that it was most helpful to learn specific concepts first in mathematics courses and then to see them applied in physics. The sequencing of their mathematics and physics courses did not always allow this, however. Vector calculus in mathematics, followed by its use in electromagnetism, was cited as a successful example of this synergy. Those students who took complex variables in math prior to quantum mechanics were also pleased with the foundation they had gained. In contrast, students felt that their introductory linear algebra course did not provide enough background for quantum mechanics, and that their physics instructors were not always aware of this. Our basic linear algebra course typically gets to eigenvalues at the very end, and does not cover Hermitian operators or complex vector spaces. Also, eigenvalues in quantum mechanics are often not calculated by the standard characteristic polynomial method taught in linear algebra. Most students had not taken a second linear algebra course, which does cover spectral theory in more depth.

Many physics departments provide a "mathematical methods" course that can function as a bridge between mathematics and physics. Mathematical topics are covered in more depth than in other physics courses, but still with physical context provided for them. Along with more applied courses in the mathematics department, students did find these useful for connecting physics and mathematics. We view such courses as "trading zones," a term from anthropology that has been adapted to describe interdisciplinary settings where mathematicians and physicists, for example, might interact using a common language (Galison, 2010). Such courses have considerable potential to help students coordinate their mathematical knowledge with its physical applications.

Other Topics, Not Part of Major Themes

Infinitesimals

We anticipated that the common use of infinitesimal amounts of charge, displacement, or work in physics might conflict with students' prior experience in calculus, where limits are emphasized and infinitesimal or differential quantities may be considered meaningless on their own. Students did express some confusion about the ways in which differentials were *manipulated* in physics, which they included in their lists of "tricks," but were generally not bothered by their supposed infinitesimal nature. Some recognized infinitesimals as an alternative way of expressing the concept of limit, which could be translated into that language if required.

> I know that if I cancel them [infinitesimals] then I would be able to get an answer pretty easily, but there's a reason that I was stuck on this problem for an hour and it wasn't that I didn't see that I could cancel them, it was that I kept looking at it and saying, well why can I do that? [Interview, physics major]

Fourier Transforms

Many students cited Fourier transforms (or series) as a particularly challenging mathematical technique they encountered in physics. This seemed to be because they encountered them early, in a lower-division optics course, without prior exposure in any mathematics course. They felt that the technique was presented simply as a tool without a clear mathematical derivation. Also, although a Fourier transform has the form of an improper integral, it is rarely evaluated from the definition of improper integrals provided in calculus. Rather, "tricks" such as completing the square in the exponent are used. Some students were unfamiliar with complex numbers and confused by the switch from sines and cosines to complex exponentials. In quantum mechanics the Fourier transform converts between the position and momentum representations, but students did not always see how this interpretation was justified. Its use in quantum mechanics is also linked to the Dirac delta function, which has problematic aspects of its own for students. A mathematics major commented that since quantum mechanics violates our physical intuition, it makes sense that its mathematical tools (for example, the delta function) would also violate our mathematical intuition!

> I personally felt like there was a lot of like tricks that are used in the Fourier transform that are just kind of well-known and you just have to look up … I didn't know they were tricks, so I would like try to actually solve them. And it would take me so long, and I would like get together with friends, try to like, solve these problems … And then we just look it up, and then it's actually quite easy and it's just like a well-known fact, or whatever. [Interview, physics major]

Corinne's Shibboleth

We asked students this question in the "bare" form cited in the Introduction, without placing it in a physical context as did Dray and Manogue (2002), thinking that this might eliminate possible bias toward the "physics" response. However, we did add this follow-up question: could you imagine a physicist or mathematician giving a different answer than you did? We found that *every one* of our students gave the physics answer r^2 (or, in two cases, simply r – probably an algebraic error rather than a distinct interpretation), indicating that they interpret the variables meaningfully as rectangular and polar coordinates. Only one (a double major) answered the follow-up by saying that the mathematics answer was possible, and that the physics answer is technically an abuse of notation. We are not surprised that undergraduate students would interpret the variables concretely as coordinates. However, even mathematics majors did not suggest the standard mathematical function syntax when prompted for an alternative answer.

Notation

The only specific example of conflicting notation (really a convention) between mathematics and physics classes that students cited was the reversal of the symbols θ and φ for the three-dimensional spherical coordinate angles between the mathematics and physics literature. They got used to it, however. Other conventions that were mentioned were the use of complex exponentials versus sines and cosines as well as the placement of factors of 2π in Fourier analysis.

Discussion

We found that our students are indeed aware of many cultural differences between physics and mathematics, and how these impact their learning. Sometimes they found it beneficial to have both a mathematical and a physical perspective on common topics, sometimes lack of coordination between courses in the two disciplines impeded their synthesis of these topics. Students' strategies for coping with these differences included consulting online resources, asking professors for clarification, and mentally separating their "physics understanding" from their "math understanding."

Our study is subject to several limitations. Although we contacted every student at our institution who was a double major in mathematics and physics, or majoring in one and minoring in the other, our sample was still small: 20 surveys and 11 interviews. The students we interviewed all had senior standing at the time, and therefore might have taken many of the same classes with the same instructors. Some of our findings may reflect characteristics of those instructors rather than the course content or culture as such. We are also on the quarter system and our curricula may differ from semester-based institutions. We did not observe systematic differences in the responses of our subpopulations of double majors, mathematics majors, and physics majors, but our small sample size would not support such claims in any case. It would be desirable to conduct a similar study at multiple institutions, with more students, allowing for some quantitative analysis of the results.

Recognizing the small sample size, we cautiously make some pedagogical suggestions. Greater awareness of the cultural differences identified here would be beneficial to both physics and mathematics instructors. Both should be familiar with what is taught in the other discipline and the relative sequencing of prerequisite courses. Students themselves suggested that physics instructors make supplementary mathematical resources available for topics that are taught rapidly, or not in depth, in class. Physics instructors might pay more explicit attention to motivating and justifying approximations and other "tricks".

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