Comparing the Strategies of Two Precalculus Students Responding to a Novel Mathematics Task

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This study examined the problem-solving abilities of two male Pre-Calculus students at a Southwestern University as they completed three novel mathematics tasks. The tasks prompted students to explain and represent how two quantities in a problem context vary together. My analysis revealed that students' success in constructing meaningful formulas and graphs depended on the students' first conceptualizing the quantities in the problem context and visualizing how they are related and vary together.

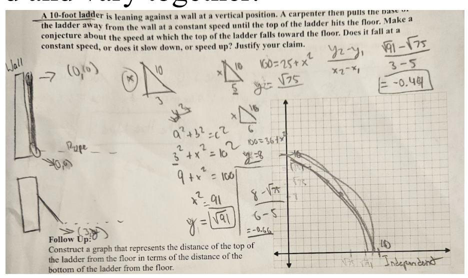

Figure 1. A picture of the task and Student A's work

Student A's mental image (Carlson, O'Bryan, 2022) of the quantitative structure of the ladder problem is illustrated (Figure 1) in the student's drawing. To address the prompt (Figure 1), he sketched the ladder's initial position (top left) and modeled its motion as the bottom of the ladder was pulled away from the wall. He constructed a sequence of triangles to represent snapshots of the ladder falling, ultimately allowing him to conceptualize and compute how much the top of the ladder dropped with each fixed change in the base's movement. His use of covariational reasoning led to him correctly recognizing that the top of the ladder would speed up as the base was moved further from the wall.

Student B immediately made a drawing of the context and concluded that he could use the triangle he constructed and the Pythagorean theorem to write the equation of a circle.

Student B: … the rates of change aren't constant because [omitted] it's a circle… so the ladder is going to start slow, speed up, and then slow down again.

Student B demonstrated fluency in using geometric relationships when he represented the Pythagorean theorem as an equation of a circle and then solved for one variable. When asked to restate how the ladder would fall, he referenced the curvature of the circle on different intervals and claimed the rate of the top of the circle is the inverse rate of the bottom right-hand part. He appeared to be recalling facts he had learned rather than conceptualizing how the quantities in the problem context were related. While his approach of comparing rates of change across intervals was notable, his reliance on abstract geometric reasoning prevented him from verifying whether his answer aligned with the real-world scenario.

While there's no single correct approach to solving a novel problem, constructing a mental image of how quantities are related before modeling them mathematically helps students understand how the quantities vary together in relation to a graph's concavity. The poster will focus on this task so viewers can engage with the task and consider excerpts from each student.

References

Carlson, M.P., O'Bryan, A., Rocha, A. (2022). Instructional Conventions for Conceptualizing, Graphing and Symbolizing Quantitative Relationships. In: Karagöz Akar, G., Zembat, İ.Ö., Arslan, S., Thompson, P.W. (eds) Quantitative Reasoning in Mathematics and Science Education. Mathematics Education in the Digital Era, vol 21. Springer, Cham. https://doi.org/10.1007/978-3-031-14553-7_9. (pp. 221–259). Springer, Cham.