

Preservice Teachers and the Triangular Area Puzzle: A Look at Common Errors

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Research has demonstrated that enhancing teacher candidates' comprehension of area concepts largely emphasizes the deepening of content knowledge in geometry through the application of geometric principles to real-world problems (Litoldo & Amaral-Schio, 2021). Building on prior studies (McNeal et al., 2018; McNeal et al., 2023), this study examines the common errors identified in elementary preservice teachers' (PSTs) written responses to triangular area questions, linking these errors to PSTs' conceptual and procedural understanding.

The conceptual framework for this study is based on Hiebert and Carpenter's (1992) model, which distinguishes between two types of knowledge: procedural and conceptual understanding. Procedural knowledge encompasses the application of rules, algorithms, and symbols, whereas conceptual knowledge involves grasping the relationships among concepts and the principles underlying the procedures. This framework guided the analysis in the present study, enabling an examination of the relationship between the common errors made by preservice teachers (PSTs) in their written submissions and their conceptual and procedural understanding of the triangular area.

We collected 85 elementary preservice teachers' written work, including homework, midterm, and final assessment questions related to the calculation of triangular area. A total of 200 written submissions were gathered from four cohorts of a Geometry and Measurement content course for elementary preservice teachers over three consecutive academic years. Employing content analysis methodology (Elo et al., 2014), we first analyzed the content of the students' written submissions to identify common errors. We then examined the relationship between these errors and the preservice teachers' conceptual and procedural knowledge, in accordance with our conceptual framework. After 5 individual coding sessions and 6 group checking-in meetings, we finalized developing 6 codes of common errors in preservice teachers' written work including *using of area units, identifying height or/and base of a triangle, using of area formula, connecting the area formula with the adding and moving principle, connecting triangular area with different shapes' areas, and calculation mistake*.

Analysis revealed that among 6 identified error codes, the highest percentage of errors pertained to the use of area units. For instance, preservice teachers (PSTs) frequently confused the terms "square unit" and "unit square," and struggled to differentiate between them using examples. The second-highest percentage of errors involved difficulties in drawing or identifying the height of triangles, particularly in the case of obtuse triangles. These errors often occurred when PSTs attempted to draw the height for calculating the triangular area, struggled to extend the base and determine the appropriate point to start drawing the height, or create a non-perpendicular segment and treating it as the height. The use of incorrect units and the misidentification or incorrect drawing of the height are linked to a conceptual misunderstanding of the properties of triangles. These errors significantly contributed to the frequency of mistakes in area calculations. Although most PSTs correctly stated and applied the area formula in their written submissions, the incorrect use of units and/or height often led to incorrect answers.

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