Examining Connections Between Students' Mindsets and Beliefs About Mathematics

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Students' beliefs influence the ways in which they learn mathematics. These beliefs can be categorized in different ways and related to growth or fixed mindsets towards mathematics. In our work, we seek to better understand connections between students' beliefs and mindsets about learning mathematics. We present case studies of two undergraduate students who were enrolled in a first-year college algebra course who demonstrated a shift in mathematical mindset. Our findings highlight the complexity of students' beliefs and how students can hold both growth and fixed mindsets towards mathematics. We explore how mindsets can be tied to specific categories of beliefs and how those beliefs can influence a student's overall perspective about learning mathematics. We conclude that mindset interventions and future research should consider how to leverage these mixed mindsets to help students foster more growth-oriented beliefs about mathematics.

Keywords: Beliefs, Mindset, First-Year Mathematics, College Algebra

Students' beliefs influence the ways in which they learn mathematics. These beliefs are situated at the intersection of the cognitive and affective domains and have been categorized in various ways. For example, McLeod (1992) emphasized the affective aspect of belief systems and distinguished between beliefs about mathematics, beliefs about the self, beliefs about mathematics teaching, and beliefs about the social context of mathematics education. Similarly, Kloosterman (1996) organized beliefs into two primary categories: beliefs about mathematics as a discipline and beliefs about learning mathematics which also includes beliefs about oneself as a learner of mathematics and beliefs about the role of the teacher as subcategories. Building on past research, Op't Eynde and colleagues (2002) used models of different student beliefs to develop a theoretical framework of beliefs related to mathematics learning and problem solving. More recent work relating to affect and beliefs in mathematics has focused on mindsets (Dweck, 2016; Boaler 2016) and how these can influence the ways in which students engage in learning mathematics. In our work, we seek to better understand connections between students' beliefs and mindsets. We present case studies of two undergraduate students who were enrolled in a first-year college algebra course. These cases highlight how mindsets can be tied to specific mathematical beliefs and how those beliefs can influence a student's overall perspective about learning mathematics.

Theoretical Framing

Our research draws on the concepts of mindsets and mathematical beliefs. In her seminal book, Dweck (2016) described two types of mindsets that affect the way students view their learning abilities. Students with a growth mindset believe that their mathematical abilities can be improved with practice, while students with a fixed mindset believe that they only have a certain amount of intelligence or mathematical ability (Dweck, 2016). Boaler (2016) applied Dweck's mindset theory to the field of mathematics, describing how growth and fixed mindsets can affect students' views of themselves as mathematical learners. In her work, Boaler framed having a mathematical mindset as "knowing that math is a subject of growth and that [a student's] role is to learn and think about new ideas" (Boaler, 2019, p. 29). Students' mathematical mindsets are tied to their beliefs about themselves, the nature of mathematics, and their role in relation to it (Boaler, 2019).

In our study, we draw connections between mathematical mindsets and beliefs using Op't Eynde et al.'s (2002) beliefs framework. We analyze how students' expressed mindsets may be linked to their mathematical beliefs. Op't Eynde et al.'s (2002) framework, outlined below in Table 1, describes how students' beliefs about mathematics fall into three categories: beliefs about the discipline of mathematics and mathematics education, beliefs about themselves, and beliefs about the social context of learning. Our findings suggest that students can have a certain mindset linked to beliefs in one category, but a different mindset about beliefs in another category. In addition, we postulate how mindsets and beliefs in one category may influence those in another category. This research gives practitioners insights into undergraduate students' mathematical mindsets and how they might change over a semester. Our work also provides a basis for researchers to continue investigating connections between mathematical mindsets and beliefs.

Table 1. A Framework of Students' Mathematics-Related Beliefs (Op't Eynde et al., 2002)

Category	Belief	Example
Beliefs about mathematics education	Beliefs about mathematics as a subject	Mathematics is a collection of facts and procedures to be memorized.
	Beliefs about mathematical learning and problem solving	There is only one correct way to solve a mathematics problem.
	Beliefs about mathematics teaching in general	A good teacher will show me how to solve a problem before I am expected to do it myself.
Beliefs about the self	Self-efficacy beliefs	I am confident that I can understand the mathematics material in this course.
	Control beliefs	I will be able to learn the material if I put in enough time and effort.
	Task-value beliefs	It is important for me to learn the material in this course.
	Goal-orientation beliefs	Fully understanding the mathematics material will give me great satisfaction.
Beliefs about the social context	Beliefs about social norms in their own class	If I speak up during a class discussion, my classmates and teacher will consider my ideas and value my contribution.
	Beliefs about socio-mathematical norms in their own class	I must be able to explain my thinking to present a good mathematical solution to my classmates and teacher.

Research Question

The purpose of our research is to better understand how students' mathematical beliefs and mindsets are related. In this paper, we address the research question: *What types of mathematical* *beliefs are expressed by students who exhibit changes in mathematical mindset in a first-year mathematics course?*

Methods

The research presented in this paper comes from a larger study of students' perceptions of a first-year mathematics course. Data for this study comes from two written mathography reflections that were assigned to 611 students enrolled in 15 sections of a corequisite college algebra course at a large, Midwestern, metropolitan university during Fall 2023. This course involved active learning strategies and standards-based grading; it was designed to help students foster a more positive relationship with mathematics. Mathographies were collected at the beginning and end of the semester. These reflection assignments were based on past work by Drake (2006) and had students respond to prompts such as "What do you think it takes for a student to be good at math?" Out of the 611 students, 445 students submitted a reflection at the beginning of the semester. Of these 445 reflections, 55 were selected for further analysis using results from a Qualtrics survey on mathematical mindset and self-efficacy that was also administered at the beginning of the semester.

Beginning mathographies were analyzed during the Fall 2023 semester using MAXQDA 2022 (VERBI Software, 2021). These reflections were coded using a set of primary codes including Growth Mindset, Mixed Mindset, and Fixed Mindset. We coded excerpts that included both fixed and growth mindset characteristics as a "Mixed Mindset." For example, "I believe math is a strong suit for some but not for others but if you really took the time to practice and pay attention you can be successful." At least two authors independently coded the mathographies and met to reconcile differences until agreement was reached. At the end of the semester, ending mathographies were collected from the same cohort. Of the 55 students whose beginning mathographies were coded and analyzed, 51 students submitted an ending mathography. These ending mathographies were tagged using the same codebook and coding process as the beginning mathographies, with at least two authors meeting to reconcile differences to ensure consistency.

To understand how the mathematical mindsets of students expressed at the beginning of a semester compared to mathematical mindsets shown at the end of a semester, each of the 51 student's beginning and ending mathographies were analyzed side by side, specifically examining any potential changes in mathematical mindset codes. From this secondary analysis, we selected six students who exhibited a noticeable shift in their expressed mindset for further analysis. We then analyzed each student individually and created a detailed description of their expressed mindsets and mathematical beliefs as categorized by the Op't Eynde et al.'s (2002) framework from both their beginning and ending mathographies. Our goal was to compare the different types of beliefs relating to changes in mindset from the beginning to the end of the semester. In this paper, we use a case study approach (Stake, 2003) to better understand the mathematical beliefs and mindsets of two students who exhibited a change in mindset from the beginning to the end of the semester. To protect the privacy of the students, both names are pseudonyms.

Findings

In studying the mathematical beliefs expressed by students who had a change in mathematical mindset during a first-year mathematics course, we looked closely at individual stories. We found that students hold different beliefs about math, which potentially influence their mindsets. Some students stuck to their initial mathematical mindset that they expressed at the beginning of the course, while others seemed to exhibit a change in mathematical mindset by the end of the semester. These case studies help us understand what types of beliefs are expressed by students, and why some students might have a change in beliefs and mathematical mindset.

Case 1: Emily

Emily's case provides an example of a student who initially demonstrated a fixed mindset towards mathematics but ended the semester with more of a growth mindset. In her intro mathography, Emily expressed a fixed mindset relating to her beliefs about mathematics as a subject stating, "I think students who are good at math honestly were able to grasp the concepts at a young age like in elementary school," and "I believe math is a strong suit for some but not for others." This view of mathematics as an innate ability corresponds to having more of a fixed mindset about mathematics as a subject. However, Emily went on to add "but if you really took the time to practice and pay attention you can be successful," which reflects more of a growth mindset relating to control beliefs about the self. Thus, in her initial mathography, Emily demonstrated both a growth and a fixed mathematical mindset relating to two different categories, beliefs about mathematics education and beliefs about the self, under Op't Eynde et al.'s framework.

In addition to discussing beliefs about the self and mathematics education, Emily also discussed past experiences with mathematics teachers relating to beliefs about the social context in her initial mathography. She explained that "when asking for help [my high school teacher] would make me feel not smart with the way she responded to me." She also described negative experiences with assessment: "when taking a test and not understanding I would breakdown and cry and have to step out of the classroom. I felt like the odd one out like everyone else understood but I did not." Thus, the social context in Emily's past mathematical experiences potentially contributed to her statements reflecting a fixed mindset, shaping her belief that mathematics ability is innate, with some inherently or 'naturally' strong in it while others, like herself, struggled due to negative interactions with teachers and comparing her abilities to her classmates.

At the end of the semester, Emily's reflections had transformed. She acknowledged her capability in math, attributing her progress to consistent study and support from her instructors and colleagues in class. Emily wrote, "I learned that I am capable of doing math and doing a good job as long as I keep studying and making time for math outside of school." She highlighted the positive impact of a supportive learning environment and her beliefs about the social norms in the class, noting that her interactions with teaching assistants were judgment-free and beneficial. She reflected that, "On some problems I just did not know what to do and when I would get help, I felt no pressure whatsoever when getting it incorrect." Emily added that,

> I really enjoyed how much assistance we got during class and the involvement with other students really helped me out, because when I would find myself confused my group would always make sure to ask if we all understood, it was like a team effort.

In her ending mathography, Emily's beliefs about the socio-mathematical norms were different than the ones she expressed in her initial mathography when she felt like "the odd one out" and "not smart" because of the social context. In her initial mathography, Emily demonstrated both fixed and growth mindsets in the category of beliefs about the self. In terms of control beliefs, she expressed more of a growth mindset, using the third person language of

"...you can be successful". However, Emily demonstrated a fixed mindset pertaining to her self-efficacy belief shaped by her previous class experiences. In her ending mathography, the social context in the class seemed to influence her self-efficacy beliefs positively as she used first person language to state, "I learned that I am capable of doing math." Thus, Emily's mathematical mindset changed from a fixed to a growth mindset in relation to her self-efficacy beliefs while also internalizing a growth mindset about her control beliefs. Emily's case exemplifies the types of mathematical beliefs expressed by students who experience a change in mathematical mindset during their first-year mathematics course. This finding underscores the role of the mathematics teaching and learning environment in influencing students' mathematical beliefs and mindset development.

Case 2: Ryan

Ryan's experience in mathematics shows a transformation from a mixed mathematical mindset at the beginning of the semester to a growth-oriented mindset at the end of the semester. In his intro mathography, he reflected, "In general, my experience with math has been pretty poor. My intelligence favors the English/Reading side of education by a vast margin." Ryan's mathematics experiences prior to college seemed to have created a self-efficacy belief that he was intelligent in subjects other than mathematics, corresponding to a fixed mathematical mindset. However, in addition to stating his limitations with mathematics as a subject, he discussed how with hard work, one can be successful at learning mathematics. He wrote,

> I used to think that some are born with the ability to do math, and some aren't. Although I feel that still applies to a certain extent, it's not that black and white. It's true that for some people, it just clicks and for others it doesn't. However, I have learned that working hard and applying yourself is all it takes ... I know it's a bit cliché saying, "all it takes is just hard work."

Here, Ryan shared a belief about mathematics education learning and problem solving – that some people are "born with the ability to do math" and "for some people, it just clicks." Yet he also expressed a control belief about the self that one can be successful in mathematics through hard work. Thus, we can see in this example how someone can hold a fixed mindset relating to beliefs about mathematics education (learning and problem solving) while demonstrating a growth mindset about control beliefs relating to the self.

In his intro mathography, Ryan compared his current classroom experience to past ones. He stated, "I really like how this class emphasizes group work […] also, the attitude in the classroom is very open and non-judgmental. I feel like in my previous math experiences, as horrible as it sounds, I would feel dumb for getting help or the teacher would try not to roll their eyes when I asked a question like they were thinking, 'Again? Seriously?'" This quote shows how social contexts have the potential to shape beliefs, although those beliefs may be difficult to identify. Ryan's comparison of classroom norms may relate to a belief that the role of the teacher should be to support and help students, rather than to judge their mathematical worth. This belief may also affect mindsets and beliefs in other categories such as beliefs about the self and mathematics education. Students who have these types of negative experiences in their educational past (e.g. mathematics teachers who make them feel "dumb") are more likely to view mathematical ability as innate and hold fixed mathematical mindsets as the classroom social norms contribute to defining students who are or can be good at mathematics.

In his ending mathography, Ryan expressed more of a growth-oriented mindset stating that anyone could succeed in math with enough effort. He reflected,

> I do believe that any student can succeed in math. My 16-year-old self wouldn't believe this, but all it takes is effort. It may seem like you're predestined to suck at math, but that isn't the case. It'll just take you a little more work and effort. If you really want to succeed, you will.

This statement corresponds to a control belief about the self where Ryan used third person language to describe what it takes to be successful at mathematics. In addition, Ryan also used first-person language to discuss another control belief specifically relating to himself, which potentially was influenced by being in a positive and non-judgmental environment. He stated,

> When it comes to new aspects, I've learned about myself that relates to Math, one of them would be just show up. As long as you just go (to the class, in this instance), then 75% of the work is done.

Thus, Ryan's experience in a positive and non-judgmental classroom environment promoted a control belief in the value of participation and effort. He realized that engaging in a supportive class setting can significantly contribute to a student's success and that he could be successful at mathematics by "just showing up."

Ryan's case highlights how an encouraging learning atmosphere can help promote growth-oriented control beliefs about the self. Having held a mixed mathematical mindset at the beginning of the semester, Ryan's development towards having a growth mindset was facilitated by his beliefs about the social norms in the class and the support from his instructors and peers.

Discussion

The mathematical beliefs of students are complex and often show characteristics of both fixed and growth mindsets. This phenomenon, which we call having a "mixed mindset," is evident in the expressed beliefs of our participants. For example, Emily expressed a fixed mindset belief about mathematics as a subject, i.e. "I believe math is a strong suit for some but not for others" while maintaining a growth mindset belief about their mathematics learning and problem solving, i.e. "but if you really took the time to practice and pay attention you can be successful." This complexity exists between the general categories of the beliefs framework (beliefs about mathematics education, the self, and the social context) and also between the subcategories of the main mathematical beliefs (e.g. different beliefs about self-efficacy and control beliefs in Ryan's case). This finding suggests that if students express different mathematical mindsets and beliefs about mathematics education, the self, and the social context, then a one-size-fits-all approach to mindset interventions may not be sufficient to address the full spectrum of students' beliefs.

Our findings also show that the social context plays a crucial role in shaping students' mathematical mindset, although these influences are often more nuanced and harder to categorize as strictly fixed or growth oriented. In our analysis, it was more straightforward to classify beliefs about mathematics education and the self as either fixed or growth. However, it became more difficult to apply the same classification to beliefs about the social context because oftentimes this relied on making inferences from students' mathematical experiences. For example, Ryan's experience of his teacher making him "feel dumb" potentially relates to a belief that the teacher's role is to support and assist students, which could be classified as a growth mindset. We were able to infer this based on Ryan's other statements about the social context of his current mathematical classroom. However, without this context, a statement similar to Ryan's could be taken as a belief that a teacher's role is to pass judgment on students, which may suggest more of a fixed mindset. Thus, while it was sometimes difficult to categorize beliefs about the social context as relating to fixed or growth mindsets, it was possible in some cases by looking at the broader set of student experiences.

Finally, beliefs about the social norms in the class can influence a student's beliefs about the self and mathematics education. A supportive and encouraging classroom atmosphere can foster a growth mindset by helping students feel capable of improvement. The social context defines expectations of what it means to be a good student, and these expectations are assimilated by students as a personal belief about the self, which influences the student's mathematical mindset. Ryan's experience illustrates this point. Similarly, Emily's story tells how positive social norms in the class can nurture growth mindsets in beliefs about mathematics education. Mathematics-related beliefs in one category can influence and even change beliefs in another. For instance, a student's belief about the self can impact their attitudes toward mathematics education and whether they hold fixed or growth mindsets along with these beliefs.

Our findings raise important questions about different types of mathematical mindset related beliefs and the role of the social context in influencing these beliefs. As students begin to adopt different mathematical beliefs through shifts in social norms in their class, they may revise their beliefs about what it takes to succeed in mathematics, hopefully pushing them towards developing more growth-oriented mindsets. Interventions should address specific areas of fixed mathematics-related beliefs while building upon existing growth-oriented beliefs. In our data, we found that students appreciated the supportive classroom environment and multiple opportunities given to understand the material and demonstrate their knowledge. Interventions instructors incorporated in the course included activities related to productive struggle and metacognition (McGuire, 2015). In addition, the course curriculum emphasized understanding mathematical structures rather than only developing procedural knowledge. These types of interventions and teaching practices can help shift student beliefs and mindsets. It is important to acknowledge that even though a student may display a fixed mindset related to a certain type of mathematical belief, they may still hold a growth mindset related to other belief categories, what we have termed in this paper as a "mixed mindset." Future research could explore how educators can leverage this concept of mixed mindsets to foster more growth-oriented beliefs in students, particularly in areas where fixed mindsets persist.

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