Exploring Discrepancies in Mathematicians' Use of Programming in Teaching and Research

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We present preliminary findings from an ongoing investigation into the discrepancies between how university-level mathematicians incorporate programming into their research versus their teaching. Motivated by earlier studies that highlight a gap in the use of programming between research and teaching, we explore this tension through interviews with mathematicians at a university recognized for supporting programming in science and mathematics teaching. Specifically, we draw on Engeström's Cultural Historical Activity Theory to analyze the interview with one mathematician who illustrates this discrepancy. Our preliminary findings suggest connections to more deeply held epistemic beliefs about which tools and purposes are considered legitimate in mathematical practice.

Keywords: Mathematicians, Programming, Mathematical Practices, Activity Theory

Background

In this paper, we present preliminary findings from an investigation into the underlying causes of the discrepancies in how university-level mathematicians use programming (writing and executing computer code) in their research versus their teaching. A motivation for this project is a survey of 302 Canadian mathematicians by Buteau et al. (2014), where 43% reported using programming in their research compared to 18% in their teaching. In a follow-up interview study, Broley et al. (2018) found institutional factors commonly cited as barriers to using programming in teaching (e.g., students do not know how to program). However, other research (e.g., Iannone and Simpson, 2022) indicates that mathematics students generally have access to courses in programming, suggesting the significance of factors beyond institutional issues.

In an interview study with six mathematicians, Lockwood et al. (2019) found that the mathematicians mainly used programming in their research to gain intuition into problems or to provide numerical results where analytic methods fell short. This suggests that the discrepancy between the use of programming in teaching and research could be linked to a previously established tension between the high-value mathematicians place on intuition in research and the comparatively lower emphasis on intuition in teaching (e.g., Burton, 2004; Hersh, 1991).

To explore this question, we conducted interviews with 15 mathematicians at a site with long-standing and recognized institutional support for the use of programming in teaching. While it was immediately clear that the interviews confirmed the existence of a discrepancy between the use of programming in teaching and research, we aimed to probe deeper for underlying causes by drawing on Engeström's Cultural Historical Activity Theory (CHAT). Here, we present an analysis conducted on one of the interviews that exemplified this discrepancy.

Specifically, we explore the following questions:

- What features do we see in the data when using CHAT as an analytic lens to compare how a mathematician describes their teaching and research activity systems as they relate to the use of programming?
- Is additional data needed to further support these findings?
- Should we modify or further refine our operationalization of CHAT?

Theoretical perspective

Engeström's Cultural Historical Activity Theory (CHAT) builds on the activity theory of Vygotsky and Leon'tev (Engeström, 2015). Given the scope of this paper, we focus on operationalizing Engeström's extended activity triangle (Figure 1).

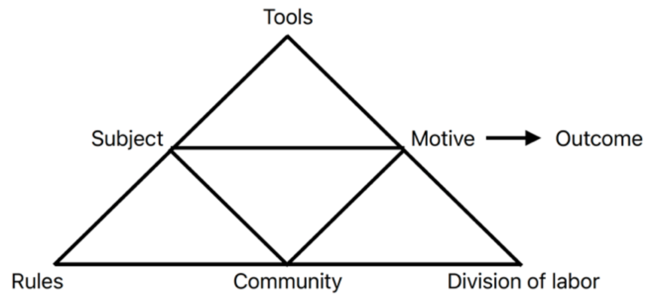

Figure 1. The extended activity triangle (Engeström, 2015).

Engeström's activity triangle expands Vygotsky's mediating triangle by emphasizing the collective aspects of activity (rules, community, division of labor). Activities are defined as enduring, motive-driven, situated, and collective human endeavors, such as mathematical teaching and research. To provide an analytic lens, we follow Mwanza and Engeström (2005) in operationalizing Engeström's triangle as a checklist of questions to ask the data (see Table 1).

Table 1. Operationalization of the nodes of the activity triangle.

Node	Example of questions
Subject	What role does the subject identify with as part of the activity?
Motive	Why is the subject pursuing the activity?
Tools	With what means does the subject pursue the activity?
Rules	What rules and norms regulate the subject's pursuit of the activity?
Community	Who does the subject see as supporting their pursuit of the activity?
Division of Labor	What power structures affects who does what in the activity?
Outcome	What is the 'output' of the activity as perceived by the subject?

Following Leon'tev, Engeström views the motive as the activity's reason for existence and a force that drives all other components. Our analysis, therefore, begins by focusing on the motive, using the other nodes to add layers of detail. The aim is to identify certain cultural tensions, called dialectic contradictions, that serve as historical driving forces, such as the tension between applied and pure mathematics. While we see the nodes as acting like the diagnostic tools of a physician, dialectic contradictions are not issues to be 'cured' but mechanisms to be understood and leveraged for change and innovation within the activity.

Methodology

We conducted semi-structured interviews with 15 mathematicians at the department of mathematics of a major northern European research university, who were sampled purposefully to represent a wide variety of mathematical research orientations (pure, applied, and computational). Each interview lasted 1.5 – 2 hours and addressed programming in their research and teaching, as well as institutional issues. All interviews were audiotaped and transcribed. To ensure a high level of communication, the interviews were conducted by the first author, who has a background as a research mathematician. Additional data, such as CVs, examples of publications, and webpages of taught courses, were consulted to support the analysis.

In this paper, we analyze the interview of an applied computational mathematician, given the pseudonym Emil. He was selected because he uses programming extensively in research but much less in teaching. Using our operationalization of CHAT over repeated readings of the full interview transcript, we generated descriptions of his teaching and research practices. We then compared these descriptions, looking for tensions indicating dialectic contradictions that relate to the discrepancy in the use of programming exemplified by Emil. These findings were discussed among the authors and compared with prior analyses of the complete data set.

To protect Emil's anonymity, we have altered quotations to obscure specific details about his research focus and the courses he has taught.

Findings

To save space, we combine our interview analysis with a description of Emil's teaching and research activity systems. The main points are summarized in Figure 2.

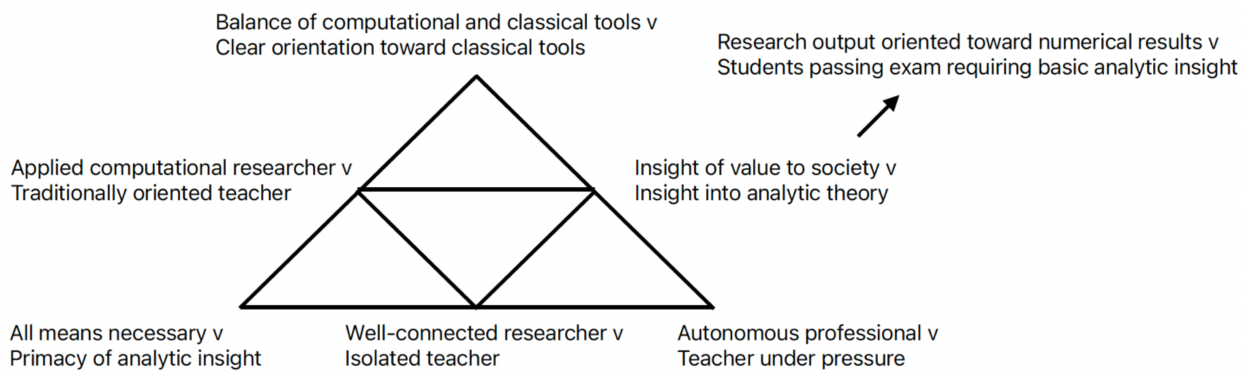

Figure 2. The main features of Emil's research (top) and teaching (bottom) activity systems.

Description and comparison of Emil's two activity systems

Subject. As a researcher, Emil explicitly identified as an applied computational mathematician. As a teacher, his preference to avoid programming in introductory courses suggests a more classical focus where students are to work analytically with pen and paper.

Motive. Regarding research, Emil stated that "the end product is that I want a deep understanding of how a physical phenomenon behaves, based on an approach that integrates an analytic and numerical approach with the use of real-world data." In discussing his teaching, Emil gave additional details, saying that, ideally, he would want students to have the same combination of abilities so that they can "piece them together into an end product that is useful for the industry and society." This motivation was tempered, however, by his low expectations of the capabilities of first-year students: "I have to say that the challenge for students to get through the first-year course that I teach, it is not their ability to use a computer, on the contrary, it comes down to their ability to understand how basic mathematical objects behave."

Tools. As a researcher, Emil said, "If I did not have computers and programming as tools, then I would not be able to do anything." He explained that analytic approaches to applied mathematics ran out of steam 50 years ago, and since then, progress has been entirely reliant on computers and programming. As we noted in the motive node, Emil was skeptical of whether it was possible to leverage programming in the same way in teaching first-year students. The following quote indicates that not being able to tap into this synergy bothers him:

> I think that being able to use a computer to represent the mathematics you do is very advantageous. If you can do it in concert with your ability to represent the same things using your head, paper, and pencil, I think you will get further. […] I wish our students would reach such an advanced level that we could begin to talk about synergies. However, [that is] aiming far higher than what we are able to get in [my course].

Rules. In his research, Emil views computers as indispensable for doing research in applied mathematics. However, when talking about first-year students, he stresses the primacy of analytic mathematics over the use of programming. That is, he seems to insist that any use of programming in mathematics must be guided by analytic intuition or insight - it does not work the other way around. He explains this as follows in his own words:

> … programs are, to a larger degree, acting like black boxes. And there is no reason to believe that these researchers have written their own programs either. They just sit and use it as a black box that someone else has programmed. Then, I get worried because I feel that they miss some of the deeper analytic insight that I am used to having. So, personally, I prefer that you not just show the result of the numerical simulation but also have some deeper analytic understanding. And maybe, I think it is, there is something to implementing a simple model rather than just implementing the full equation governing the physical system. Because then you can focus on what a simple model... how it behaves, as I think, deep down, can give you more insight.

Community. In his research, Emil collaborated with fellow researchers, contacts in the industry, and MSc and PhD level students. This is corroborated by a look at his publication record and personal webpage. As a teacher, the level of collaboration with colleagues seems less pronounced (which is also a recurring theme in the other interviews).

Division of labor. Emil expressed significant autonomy concerning his teaching and research. However, there are some discrepancies. Although he interacts with a network of collaborators in his research, he adopts a more independent stance in his teaching. In his research, Emil expressed the absolute necessity of using computers. In his teaching, he expressed the difficulty of using programming based on his impression of his students' analytic proficiency level. He also pointed to continuous reorganizations, both within the university and externally, as taking so much energy that he did not feel the strength to look at the 'big picture.'

Outcome. In his research, the outcome is research papers of interest to the industry, with numerical simulations at the forefront. In his teaching of first-year students, the outcome is that mathematics students pass exams that test what Emil considers basic analytic skills.

Discrepancy between activity systems

In the above analysis, we see that Emil, as a researcher, desires insight into physical phenomena based on a combined analytic and numerical approach that allows him to work with real-world data. He does mathematics to contribute to our understanding of the real world and he does it using programming, as it would be impossible otherwise.

While Emil wants students to have 'the whole package,' he is concerned that their lack of analytical mathematical ability makes it difficult for them to pass courses in their first year. Therefore, he teaches classical analytical mathematical content using pen and paper, without involving computers, except for using ready-made tools for visualizations. He believes this approach gives students the necessary insight required to work numerically.

We interpret this as a discrepancy related to tensions in the epistemic dimensions of the motive of the activity and by what means the activity is pursued. We represent this in the 'epistemic matrix' shown in Figure 3. We note that while Emil would like students to use 'the whole package,' he seems to feel pressured by circumstances outside his control (such as his perception of students' low analytic proficiency) to adopt another, more classical, stance concerning his teaching.

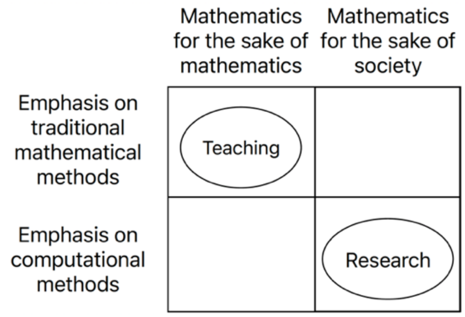

Figure 3. An 'epistemic matrix' representing the discrepancy between Emil's research and teaching activities.

Discussion

Using CHAT as an analytic lens, we are naturally led to focus on Emil's motives for research and teaching, highlighting discrepancies between the 'reasons why' of the nodes. The most conspicuous discrepancy concerns the use of programming. While Emil views his mathematics as something that should serve society, with programming as a necessary tool to handle real-world data, he believes that any use of numerical methods must be guided by analytical ability. For Emil, students must first learn basic analytic skills before being tasked to program anything. Thus, he emphasizes traditional methods in teaching, not because he devalues programming, but because he sees analytic ability as a prerequisite for insightful use of programming. This observation resonates with and connects the findings of Buteau et al. (2014), Broley et al. (2018), and Lockwood et al. (2019), which we discussed in the background section. We propose the epistemic matrix as a tool to explore the gap between the use of programming in mathematics research and teaching. Our findings suggest that this gap stems from a tension between Emil's belief in the primacy of analytic ability and his perception of first-year students' skills.

A first natural question is whether the dimensions of the epistemic matrix are strongly correlated. This does not seem to be the case, as preliminary analyses of other interviews suggest that there are mathematicians corresponding to other areas of the matrix. A second question is whether the dimensions of the epistemic matrix represent dialectic contradictions (i.e., historical driving forces of the activities). To understand this, analyses of additional interviews, a closer look at institutional factors, and a review of related results in the philosophy and history of mathematics are necessary.

While one might argue that using CHAT to analyze a single individual is a contradiction in terms, as it is a socio-cultural theory meant to analyze *collective* activities, we maintain that if the goal of the analysis is to uncover epistemic dimensions, then the relevant collective component is chiefly how the subject *perceives* the collective dimension, which may be different to how peers report the collective dimension.

We conclude with potential questions for the audience:

1) Does our use of CHAT seem useful and appropriate as we have operationalized it?
2) Do you see the epistemic matrix as having a potential to inform mathematics education?
3) Are there other frameworks than CHAT that are better suited for our analysis?

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