At the Crossroads of Inquiry and Equity: Synergies and Tensions in Instructor Decision-Making

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Although some contemporary definitions of inquiry learning in college mathematics include instructor attention to equity as a core precept, evidence suggests the practice of inquiry-based instruction in proof-based undergraduate mathematics courses entails many decisions whose implications for inquiry and implications for equity and access may not be mutually reinforcing. We present the results of our analysis of over 40 design and facilitation decisions made by the second author as he taught an undergraduate-level number theory course for high school students in a summer residential mathematics program. In our analysis we identify some categories of instructional decisions that have both inquiry and equity implications. We also discuss in detail a scenario in which the instructor was able to choose a course of action that simultaneously supported inquiry and equity goals of the class, and one in which the goals of inquiry and equity appeared to be in tension.

Keywords: Inquiry, Equity, Instructional Practices, Proof-Based Mathematics

In 2019, Laursen and Rasmussen defined a set of four “pillars” for *inquiry-based mathematics education* (IBME). Of these, the fourth pillar called for instructors to “foster equity in their design and facilitation choices” (p. 138). However, Laursen and Rasmussen acknowledged that it was not yet known how the other pillars of inquiry – student engagement with rich mathematical tasks, collaborative discovery and discussion of mathematical ideas, instructor inquiry into student thinking – might best be brought into alignment with equity goals in college mathematics teaching. Indeed, several contemporaneous studies (Andrews-Larson et al., 2018; Johnson et al., 2020) have suggested, at a minimum, that an inquiry-based instructional approach is not a guarantor of equitable outcomes in conceptual learning or in public ownership of mathematical ideas. This raises a broad question: what are the mechanisms by which inquiry-based instruction supports or hinders equitable outcomes in student learning and mathematics identity development? In this report, we broach a narrower question: how does the relationship between inquiry goals and equity goals appear through the eyes of an instructor with an explicit commitment to both?

We argue that this question has significance for addressing Laursen and Rasmussen’s (2019) call for equity research in IBME. Research has begun to explore theoretical perspectives that, if translated into practice and college mathematics instructor professional development, can help instructors set the stage for students’ mathematical inquiry and take an asset-based approach in interpreting and responding to student thinking (Adiredja & Zandieh, 2020; Ellis, 2022; Louie et al., 2021). However, teaching in ways that center student inquiry while attending to access and opportunity for all students still presents some technical challenges for instructors. For example, stereotypes about traditionally marginalized groups may emerge in group discussion in ways in which they might not occur in whole-class discussion (Ernest et al., 2019). The skills that might help an instructor address such threats to the status and participation of minoritized students (Cohen & Lotan, 2014) are not the same as the skills required for delivering smooth, technically competent lecture-based instruction. Additionally, the process of orchestrating discussion of students’ mathematical thinking, and weaving this thinking into a shared narrative that addresses the mathematical goals of a lesson, raises questions that have both inquiry and equity implications: Whose thinking gets shared, and in what order? How to address thinking that contains reasoning errors, or that is logically sound but not communicated with the targeted level of formality or generality? How can we help students make sense of and appreciate their peers' thinking without immediately shifting to an evaluative frame of mind?

In this paper, we report on a retrospective analysis of the second author's teaching of an advanced undergraduate-level number theory course in a summer mathematics program. In this analysis we seek to address the research question: *In what ways do goals of student mathematical inquiry and equitable learning intersect in an instructor's design and facilitation decisions when teaching a proof-based undergraduate-level mathematics course?* We anticipate that findings from this analysis may inform the design of future studies of how instructors simultaneously address goals of inquiry and equity in proof-based mathematics courses and suggest possible ways for professional development to prepare instructors to manage classroom situations and decision points that have implications for both inquiry and equity.

Inquiry and Equity in The Context of Undergraduate Mathematics

To discuss synergies and tensions between inquiry-promoting teaching practices and equity-promoting teaching practices, we need to establish what we mean by both. The following sections review existing literature regarding inquiry and equity as constructs and conclude with definitions of each which we use to classify the teaching moments analyzed later in this paper.

Inquiry

There are two predominant characterizations of inquiry-centered instruction in the contemporary literature in the U.S., Inquiry Oriented Instruction (IOI) and Inquiry Based Learning (IBL). The two share many characteristics and philosophies (Kuster et al., 2018), but differ in some of their nuance and origins. Both offer valuable contributions to our conceptualization of inquiry. The tradition of IOI is deeply rooted in Realistic Mathematics Education (RME) (Freudenthal, 2005). Lockwood and Purdy elaborate on their use of RME, "Ultimately, a goal of leveraging guided reinvention (and RME more generally) is to infuse mathematics with meaning for students" (Lockwood & Purdy, 2019, p.227) illustrating that creating powerful and meaningful mathematical experiences is central to IOI.

On the other hand, IBL was borne out of work done by practitioners rather than from educational theory (Laursen & Rasmussen, 2019). Hence, IBL courses center student ideas as the main content of the course. In an IBL class, "Following a carefully designed sequence of tasks rather than a textbook, students construct, analyze, and critique mathematical arguments. Their ideas and explanations define and drive progress through the curriculum." (Laursen et al., 2014, p. 407). Both this aspect of IBL and IOI's emphasis on constructing mathematical meaning were central to both the large-scale design and day-to-day pedagogy of the number theory class.

Laursen and Rasmussen characterize IBME, a term meant to encompass both IOI and IBL, as a branch of active learning, where students do things, think about what they are doing, and talk to others about what they are thinking and doing (Laursen & Rasmussen, 2019). This take on inquiry positions students as practitioners of mathematics, something that Artigue and Blomhøj also do in their description of inquiry-based pedagogy. Artigue and Blomhøj say that inquiry-based mathematics education (IBME) "aims to offer students the opportunity to experience how mathematical knowledge can meaningfully develop." (Artigue & Blomhøj, 2013), which, like IOI, places meaningful mathematics at the center of inquiry.

Artigue and Blomhøj claim that Inquiry Based Education (IBE) historically aims to support "emancipation and democracy" (p.808) and use their analysis of IBME to set forth "ten concerns", the last of which is "the critical and democratic dimensions of IBME" (p.809). We thus imagine that these authors might align with Laursen and Rasmussen's equity pillar for inquiry, though possibly for different reasons. Laursen and Rasmussen support their claim with seven "Equitable Teaching Practices" identified by Boaler in 2006, which they recognize as overlapping with the other three pillars. Hence, they suggest that equitable instructional practices are an essential fourth pillar of IBME. On the other hand, Artigue and Blomhøj examine the origins of inquiry, specifically the research and philosophies of Johnathan Dewey, who was a proponent of education as a tool to support democracy by producing students with both technical knowledge, but everyday skills (Dewey, 1916). To better illustrate this as an aspect of equity, we now turn to a brief review of literature in this area.

Equity

Gutiérrez (2011) characterized equity in mathematics as having two axes: the dominant and the critical. She describes dominant mathematics as, "the components students will need to be able to show mastery of in the discipline as it is currently defined and to participate economically in society." (Gutiérrez, 2011). Meanwhile, the critical axis "builds upon the idea that mathematics is a human practice that reflects the agenda, priorities and framings that participants bring to it." (p. 21). She associates the dominant axis with Access and Achievement, and the critical axis with Identity and Power, and argues that students ought to gain both dominant and critical mathematics identities. In the conceptualization of equity we adopt for the purposes of our analysis, we focus on the identity dimension of the critical axis.

Rubel (2017) identifies specific equity-directed instructional practices, two which she claims fall on the dominant axis and two on the critical. These teaching practices are: teaching for understanding, connecting mathematics content to students' experiences, fostering multidimensional participation, and connecting mathematics content to students experiences. Rubel uses these instructional practices to study teacher professional growth, and proposes that, "Indeed the practices on the dominant axis are practices that lend themselves to daily instruction, whereas the critical practices arguably could be seen as practices that teachers engage with less regularly" (p. 91) Therefore, when we later identify teaching practices and classify the extent to which they promote equitable learning experiences, we keep in mind that Access and Achievement may be more common than Identity and Power.

Our Definitions

We define inquiry-promoting teaching practices to be those which make the primary mathematical ideas of a class legitimately stem from the flow of students' thinking, thus making the content and progression of the course meaningful for the students. We define equity-promoting teaching practices to be those which give all students the opportunity to see themselves as central participants within the community and legitimate doers of mathematics.

Context And Method of Inquiry

The following analysis is based on teaching decisions made during a course on number theory taught at an informal summer mathematics program for high school students. In the program, students learn to prove foundational theorems in elementary number theory; the course is approximately equivalent to a semester-long advanced undergraduate course. The class was comprised of 36 students, all rising tenth or eleventh graders. The class met for an hour and fifteen minutes on Monday through Friday for six weeks. In addition to the class meetings, students also attended daily hour-long problem sessions with eight to twelve students facilitated by undergraduate student mentors. The problem sessions reviewed major proofs from class and addressed students' questions about the material. The students also participated in nightly four-hour study sessions in groups of four, each led by a mentor. During these sessions, students worked on problem sets designed to foreshadow ideas in future lessons and build intuition behind major concepts in the course.

The class was taught by the second author and observed by the first. During the six weeks, both authors kept journals detailing events in class and instructional decisions made by the second author including small, in-the-moment interactions as well as larger pre-planned curricular design decisions. After the completion of the program, we used our journals to compile a list of over 40 instructional and design decisions. Separately, we utilized open coding to identify themes across the decisions then compared our codes. Ultimately, we identified six categories of decisions. We discussed possible implications of each decision for inquiry and for equity, as defined in the preceding section. We selected several decisions that we identified as having both inquiry and equity implications and performed an in-depth retrospective analysis of each. We present two of these cases below.

Analysis

In this section we provide an in-depth look at our thinking on two specific cases that illustrate distinct ways in which inquiry and equity goals can interact. We also present the six categories identified from our list of instructional decisions.

Illustrative Case 1: Building an Activity From Student Work

In the second week of the number theory course, students proved the theorem that if a divides bc and a and b are relatively prime, then a divides c. Students were asked to complete this proof in small groups during their nightly study session. The following day after class, one group presented their proof to the second author. Their approach was one that to our knowledge no students in the program had devised previously; however, there was a gap in their argument. Recognizing this gap, but seeing a way for the group to complete the proof, the second author decided that the afternoon's problem session should focus the attention of each group on completing the proof. That afternoon, mentors presented the incomplete proof and gave credit to the original authors, highlighting their creativity. The students were not told what the gap in the argument was, rather they were asked to identify a gap and attempt to fix it.

As a result of this decision, the proof became a central idea in the students' mathematical work that day. The instructor later attested to having chosen this course of action because it offered the group that authored the proof greater ownership of the class's mathematical ideas – a cornerstone of inquiry. Furthermore, the four students who originally brought forth the proof had not yet received public recognition of their ideas in class, particularly the female students who had had relatively few opportunities to share ideas publicly in class. Making the group's inventive proof an object of inquiry for the class highlighted these four students' ingenuity and the value of their ideas, consistent with our conceptualization of equity.

The instructor could have addressed these students' contribution in different ways. For example, he could have chosen to present an activity in problem session in which part of a more typical proof was given, and students were asked to critique or complete it. In this case, the students would still have had the opportunity to engage with a proof, look for gaps, and attempt to complete the argument. However, this would have focused more of the students' work on an artifact of mathematical thinking external to the class, rather than their classmates' own thinking, and would not have done as much to highlight the mathematical work of students who had not received much prior recognition in class. In this sense the instructor had a choice available to him – using the group's proof – that advanced both his inquiry goals and his equity goals simultaneously. Thus we see this decision as an example of synergy between the two constructs, with the whole greater than the sum of the parts: greater inquiry-related goals were achieved by attending to equity, and vice versa.

Illustrative Case 2: Selecting a Class Activity on Quadratic Reciprocity

The number theory course culminated with an exploration of Gauss' law of quadratic reciprocity, which establishes a surprising relationship between the question of whether p is a perfect square modulo q and that of whether q is a perfect square modulo p, where p and q are distinct odd primes. The second author knew that the proof of this theorem is notoriously difficult for students to understand, so he wanted to provide multiple opportunities for students to engage with the ideas of the proof and try proving some of the foundational lemmas before seeing a full proof of the theorem in class.

This created a quandary for the instructor, especially given that this exploration took place during the last two days of the course: any class activity would need to run efficiently to allow sufficient time for the instructor to complete the proof of the theorem during class. He considered two possible in-class activities: either (1) have students get into groups and write up proofs of some of the lemmas that had been assigned for the previous nights' study sessions, or (2) have students do an exploratory activity in which they divide multiples of p by q, record quotients and remainders, and notice a relationship between the quotients and the remainders that serves as a foundation for one of the lemmas. Of these two options – we do not claim that there might not have been other possible options available – the instructor considered (1) more supportive of the inquiry goals of the course, because this activity would give more students the opportunity to anticipate more of the reasoning of the proof of quadratic reciprocity. On the other hand, he considered (2) more supportive of equity goals: he knew that only a few students had worked in prior study sessions on the lemmas he would assign in class, and he observed that many students were struggling with the level of abstraction of the parts of the argument the class had developed so far. In option (2) he saw an opportunity to roll some of this abstraction back and have students engage in a data collection activity (performing calculations for different pairs of primes p and q) that would generate strong evidence of a relationship that would later prove instrumental in the proof. While the reasoning available to students in this activity would not be the same quality as in option (1), the instructor knew that option (2) would be accessible to most students in class and would provide more concrete reference points for the reasoning in the proof.

We view this as an instance in which the instructor faced a decision in which he had to choose whether to prioritize maximizing opportunities for student inquiry or prioritize maximizing students' access to the proof he would ultimately present at the end of the course. This is not to say that inquiry and equity are inherently in tension with each other: this binary choice was forced by a number of factors, including the limited time remaining, missed opportunities to better engage all students in thinking about the lemmas during preceding study sessions, and the instructor's sense that completing the proof of the theorem was a non-negotiable requirement of any course of action. However, subject to local constraints, the instructor was forced to choose, in a certain sense, between inquiry goals and equity goals.

Categories of Decisions

Based on our compiled list of instructional decisions, we identified six categories of decisions that may have bearing on the inquiry culture of a class, equitable learning opportunities, or both: *defining class culture*, *exposition of content*, *facilitation structures*, *tasks and activities*, *eliciting and selecting student thinking*, and *responding to student thinking*. Given that our list is based on observations of one instructor and filtered in part by that instructor's perspective on his own practice, we do not anticipate that this list is complete. However, each instructional decision we identified fits well into at least one of these categories; some decisions fit into multiple categories. We speculate there may be categories of decisions that are overall more likely to create synergy for inquiry and equity, while other categories might generate more instances of tension, or a need to prioritize, between the two.

Discussion and Implications

We wish to acknowledge some limitations of this analysis. By taking the second author's teaching as an object of inquiry, we are able to account for insights, values, and obligations behind instructor decisions in ways that can be difficult to match in studies in which the instructors are third parties. However, the second author's perspectives on inquiry and equity are likely not perfectly representative of those of the inquiry teaching community or the wider community of undergraduate mathematics instructors. To build upon our preliminary results, a study with more instructors, in which researchers regularly check in with instructors for insight about key decisions while instructors' memories of those decisions are still fresh, would likely generate a more representative body of evidence of how instructors weigh inquiry and equity considerations in teaching.

Additionally, we acknowledge that what we identify as a tension between inquiry and equity may not be a tension if viewed from a different perspective, or with a more comprehensive set of design and facilitation options. We have both found anecdotally that as we gain more experience in inquiry-based teaching, we gain tools that help us renegotiate false choices (*e.g.*, between inquiry and content coverage) and false dichotomies. Therefore, we do not present any of the analysis in this paper as proof of any inherent incompatibility between inquiry and equity as guiding perspectives for college mathematics instruction. Rather, we present the experience of the second author as evidence that college mathematics instructors, even if experienced with both inquiry and equity, might at times perceive a tension between the two. If such a tension is perceived, then it is reasonable to infer that it could at times influence decision-making.

Based on our analysis, we see that instructional decisions arise in which the decision maker must simultaneously consider equity and inquiry related instructional goals. Our first case exhibits one way in which inquiry and equity goals synergize to create a learning experience for students in which they engage in legitimate inquiry and in which students are positioned as valued and capable practitioners of mathematics. Our second case exhibits one way in which the decision to prioritize a more equitable learning experience perhaps decreased some students' opportunity for deeper inquiry, placing the two goals in tension with each other.

The primary contribution of our analysis is considerations for future research and practice in inquiry teaching in proof-based courses. Our list of categories of decisions that impact inquiry and equity, while possibly incomplete, suggests some areas in which researchers might ask instructors using inquiry about design and facilitation decisions. If a further investigation revealed that some categories of decisions regularly generate perceived tension between inquiry goals and equity goals, professional development and professional learning communities could equip instructors to navigate these tensions in ways that preserve the best of both constructs. For example, the [blinded] project (supported by [blinded award]) is helping college mathematics instructors address a common tension in inquiry-oriented proof-based courses: the fact that some students enter these courses with extensive background in mathematical proof, and as a result have disproportionate opportunities to drive the work of the class. One way the project is helping ease this tension is by developing group-worthy activities that offer all students high-quality, structured engagement with proof in forms other than proof construction. By pursuing such opportunities to broaden the range of activity and facilitation options available to instructors, the undergraduate mathematics education community can continue to work toward the fulfillment of the vision articulated by Laursen and Rasmussen (2019): a journey of mathematical inquiry that leaves no student behind.

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