Preliminary Analysis of Undergraduate Student Problem Posing in a Scaffolded Calculus Intervention

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Problem posing, that is, the reformatting of existing problems and the generation of new ones, has been promoted for many years by the National Council of Teachers of Mathematics (NCTM, 2000, 1991) as a valuable practice for deepening students' *mathematical thinking* and habits of mind. Problem posing has been associated with problem-solving ability (English, 1998; Silver & Cai, 1996, 1993) and mathematical creativity (Van Harpen & Sriraman, 2013; Yuan & Sriraman, 2011; Leung & Silver 1997). As a practice, problem posing has also been shown to develop student conceptual understanding (Kapur, 2015; Cai et al., 2013). Problem-posing tasks are not only more cognitively demanding than problem-solving tasks. Due to their low-floor nature, they are also more accessible to students of different mathematical backgrounds (Cai & Hwang, 2021), including undergraduate students (Silber & Cai, 2021).

Single-Variable Calculus serves as a gateway course for undergraduates of various majors and mathematical backgrounds. Yet, such 100-level courses may not provide many opportunities for deep mathematical thinking. NCTM (2000) policy has called for student use of multiple *mathematical representations* for promoting mathematical thinking. Such representations are important for developing and reflecting understandings of mathematical ideas (Huntley et al., 2000; Lloyd & Wilson, 1998). In this study, I explored what kinds of problem scenarios (i.e., analogous, somewhat original, or original) undergraduate Calculus students created during a scaffolded, representation-based intervention to analyze students' depth of mathematical thinking.

During two semesters, I, the principal investigator, led a nine-week intervention in four undergraduate Calculus courses in a small, mid-Atlantic college. Over the nine weeks, all students received instruction on average rate of change, limits, continuity, the derivative, and applications. Students were paired randomly by an outside researcher. As a preliminary analysis, problem scenarios created by six dyads with similar Calculus and problem-posing backgrounds were analyzed. For Phase 1 (five weeks), students were told to create problem scenarios for approximately 20 minutes per session based on the mathematical content taught that week. For Phase 2 (four weeks), students were instructed on the concept of problem posing, that is, the difference between analogous problems, "what if?" problems, and "original" problems (e.g., inversions), along with the incorporation of one, then two mathematical representations. Then, students created problem scenarios for approximately 20 minutes per session.

During Phase 1, almost 90% of the created scenarios were analogous to those received during instruction (not original), no matter the students' experiences with Calculus (if any) or with problem posing prior to taking the course. However, during Phase 2, most scenarios were somewhat original or original in nature. Such scenarios in Phase 2 were mostly more mathematically challenging than those in Phase 1. However, they varied in mathematical reasoning, understanding required by the solver, and in complexity. Phase 2 appeared to foster deeper mathematical thinking. Yet, as this did not occur for all student groups, more research is needed to explore ways to use the activity to foster greater originality.

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