Inquiring About What? Student Questions in Inquiry-Oriented Abstract Algebra

Corinne Mitchell
Virginia Tech

Michael D. Hicks
Virginia Tech

Rachel Rupnow
Northern Illinois University

Lindy Hearne
Virginia Tech

While extensive research has been conducted on curriculum development, teaching strategies, and student thinking in inquiry-oriented instruction, there is little focus on the nature of students' inquiry as it relates to question-posing; we know almost nothing about how and why students ask questions in these courses. Since inquiry-oriented mathematics teaching aims to shift mathematical authority to be shared between teacher and students, we wonder about the role of student inquiry in this shift. We present preliminary findings from an inductive analysis of students' questions and the mathematical authority relations that arise during classroom interactions where students ask questions in an inquiry-oriented undergraduate abstract algebra class.

Keywords: Questions, abstract algebra, authority, assessment

The "inquiry" in inquiry-oriented instruction (IOI) has a two-fold conceptualization in research: teachers inquire into students thinking while students inquire into advanced mathematics (Kuster et al., 2017). While extensive research has been conducted on curriculum development, teaching strategies, and student thinking in IOI, there is little focus on the nature of students' inquiry itself; we know almost nothing about how and why students ask questions in these courses. If one of the goals of inquiry-oriented mathematics teaching is to shift mathematical authority to be shared between teacher and students, what is the role of student inquiry in this shift? Further, if asking questions is a predominant mode of engagement for students across undergraduate mathematics courses, what is the mathematical nature of this activity? We present preliminary findings about students' questions and the mathematical authority relations that arise during classroom interactions involving student questions in undergraduate abstract algebra.

Questioning in the mathematics classroom has primarily been explored at the K-12 level, with just one notable study concerning undergraduate mathematics. Paoletti et al. (2018) explored the nature of 11 lecturers' questioning in advanced mathematics courses, finding that students were most often asked to recall facts or produce the next step in a proof, but questions requiring other kinds of mathematical activity were infrequently asked. These results were consistent with the findings of K-12 literature on teacher questions (Kawanaka & Stigler, 1999). K-12 research has also explored how teacher questions can be posed to impact student thinking (Martino & Maher, 1999; Van Zee & Minstrell, 1997), which questions promote students' mathematical development (Kawanaka & Stigler, 1999; Melhuish et al., 2024), and how pre-service teachers develop questioning practices (Gaspard & Gainsburg, 2020; Moyer & Milewicz, 2002; Nicol, 1998). The how and why of student questioning in math class is scant in both K-12 and undergraduate mathematics classrooms. Outside of the mathematics education literature, Van Zee et al. (2001) found that students posed questions in science class when they were invited to do so, during conversations in familiar contexts, or in discourse environments that prioritized understanding each other's thinking (p. 182-183). While Van Zee et al. (2001)

focused on the conditions that may lead to students' questions, our analysis explores the role of students' questions in class discourse.

Theoretical Framing

On the surface, the authority relation when students ask questions in math class seems obvious: students may simply be deferring to the mathematical expertise of the teacher to provide an answer. However, the framework described in this section proposes a finer grain for parsing this relation to describe the modes of authority in which students are engaged during questioning activities. Authority is often conceptualized in terms of perceived status or positionality of actors within a situation, both colloquially and in the literature (e.g., Gerson & Bateman, 2010; Otten et al., 2017). In contrast, we emphasize the actions of the actors in the situation, thus deemphasizing traditional authority roles. Like Hicks et al., (2023) we see mathematical authority as "rooted in the activities within a classroom that contribute directly to the mathematics, as well as who gets to partake in those activities" (p. 826). Bishop et al. (2022) and Hicks et al. (2024) identified the following modes through which teachers or students can take up mathematical authority. The primary contributors of the main mathematical idea under consideration are in authority for authorship. Those who are publicly communicating or extending ideas through writing or speaking are in authority for animation. Finally, participants are in authority for assessment when they are evaluating contributions made in service of an authored idea.

In this report, we frame the activity of posing questions as an authoritative action in order to investigate the authority relations that arise during such interactions. Further, we explore the conditions under which the activity of questioning transitions from being an indicator of student authority for animation, to being an indicator of student authority for authorship, or even assessment. In viewing questioning as a mathematical activity through which members of a classroom community negotiate authority, we expand both the notion of mathematical authority-in-activity (see Bishop et al., 2022) and our scientific community's understanding of questioning in the advanced mathematics classroom.

Methods

This study arose as part of an ongoing project exploring the activity of authoritative assessment in a semester-long inquiry-oriented abstract algebra class, using video data collected by the fourth author. While developing this theoretical construct, we identified several instances of student questioning which caused debate as to whether the utterance made by the student was authoring or assessing an idea.

In line with the AAA framework, we analyzed segments, stretches of classroom activity possessing consistent mathematical focus and participatory structure. We focused specifically on segments in which students' activity included posing questions to develop inductively a theory of questioning as an authoritative activity. We identified a question as any utterance that requested information of some kind. To determine the influence of students' questioning on the overall authority relations within a segment, we attended to not only the mathematical content of their questioning but also the source of that content. We asked, "Whose idea is the subject of the question? What mathematical contribution does it make? Does the contribution become the focus of classroom discourse?" Using the method of constant comparison (Corbin & Strauss, 2015) to develop and refine our definitions, we first made the distinction between questioning activity that contributes new mathematics to the discourse and that which does not. For instance, we differentiated questions regarding a previously spoken idea, uttered by the teacher or another

classmate from those about a student's own ideas. This distinction between questions that clarify and those that contribute does not value one type over the other, but rather foregrounds the public mathematical activity involved so that the influence of these interactions on the classroom authority relation can begin to be understood. Within these two types, distinctions arose in the data that warranted further development into two subcategories for each type, dependent on the scope of the mathematical activity involved. We develop our classification below.

Findings

In this section, we present preliminary findings in the form of four types of questioning that arose throughout the semester, describing how students' questioning garnered them authority for authorship, animation, and assessment. Figure 1 summarizes our classification.

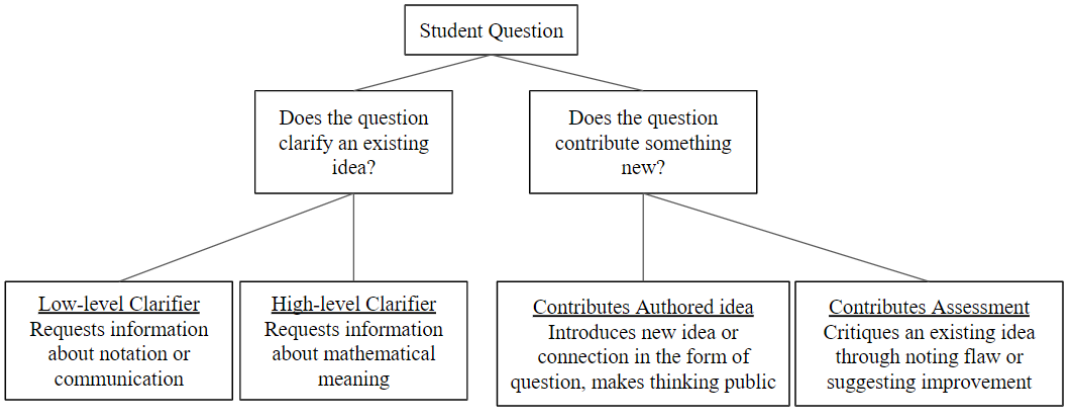

Figure 1. Classification of Student Questions as authoritative activities.

Clarifying Questions

Students regularly asked questions which aimed to clarify an element of the classroom discourse. We saw a distinction between questions that clarified notation and questions that clarified mathematical content. In the first type, which we designated low-level clarifiers, the question asked serves to clarify the visual and audible communication of mathematics, rather than the creation of it. These questions attend to notation or vocabulary within the class discourse and generally are not mathematically substantial. For example, when the teacher had finished presenting a proof, a student asked, "Is that the letter U or V used to represent that set?" Questions like this are focused on an aspect of communication rather than the mathematical content itself. While we see it as likely that this student was engaging in meaningful sense-making of the teacher's proof during this interaction, such activity was not publicly available and so did not contribute to the authority relation of the classroom beyond simple animation, i.e., the student's utterance garnered authority to animate an existing idea through speaking.

On the other hand, we also saw students ask clarifying questions that attended to the meaning of the mathematical content at hand. These questions, which we called high-level clarifiers, made students' reasoning about the mathematics public, usually by alluding to a new idea or connection. For example, while the class was investigating why a non-trivial homomorphism from the group of rotations of the square into the integers (Z) could not be constructed, the following interaction occurred:

Ella: Could you do equivalency classes instead?

Teacher: The problem is that I'm mapping to Z.

Ella: Not Z mod 4?

Teacher: Yeah, right? So I can't map to equivalence classes because the elements here aren't equivalence classes.

Ella's question suggests attention to the operation-preserving property of a group homomorphism and an attempt to rectify the issue. While the utterance was posed as a clarifying question about whether the codomain can be changed to preserve the operation between groups, the question also served as a vehicle for an idea. We saw activities like this as being more mathematically substantial than a low-level clarifier because they involve public student reasoning in ways that questions aimed to clarify simple communication often do not. However, while high-level clarifiers contained new ideas or perspectives, these did not become the focal idea of the conversation in a way consistent with authorship authority. Therefore, for both types of clarifying questions, students had authority for animating the mathematics through speaking.

Contributing Questions

In some cases, students posed questions that indicated an existing idea or belief and requested the teacher's judgement to verify. When this happened, we saw the student's question as contributing an authored idea. To exemplify this, we present the following exchange where Levi and Cammie ask about another student's solution on the board.

Levi: Does it hold that like, we don't have to check any elements in H? So if you checked the coset {a, b, d} to make sure that the left and right are equal, we don't have to do that because H is a group so any left- or right-product of d and H is gonna be in there?

Teacher: Yeah, if H is a subgroup, then I know it's closed, so I don't have to check any of the things in there when I'm checking my left- and right-cosets. Yeah?

Cammie: So for cH you could also use eH or fH?

Teacher: Yes, I could use any name I wanted for this.

In the course of asking his question, Levi introduces the idea that the closure property of the subgroup H simplifies the process of constructing cosets and explains his reasoning. The novelty of Levi's idea together with the public justification for the thinking within the question earned credit for authorship because the question was substantive enough to become the focal mathematical idea of the segment.

Although Cammie's question displays some reasoning about the choice of coset representatives, we see this question as a high-level clarifier rather than a contributing question because she appeals to an idea authored in a previous discussion by the teacher. Observe that Levi's question authored mathematics while Cammie's question animated existing mathematics through speaking. We see similarity between Cammie's question in this excerpt and Ella's high-level clarifier in the last section because neither student elaborates on nor justifies their thinking, though both clearly demonstrate sense-making of the mathematics at hand.

Another kind of contribution that arose through students' questioning activity came in the form of authoritative assessment. Questions that *contributed an assessment* typically contained a critique of an authored idea through noting a flaw or suggesting an improvement. In the following example, a student's question about a proof the teacher has presented does both.

Justin: Why do we need to do eH? Why can't we just do H times gH?

Teacher: I could've done H●gH.

Justin: Right.

Teacher: Yeah. Because H is the identity coset, it's the same name as eH. And I didn't use that e for anything. I was trying to highlight for you guys that this was the identity.

Consistent with our framing of assessment, we noted that the teacher's response to Justin's question defended her choice of notation when she said, "I was trying to highlight for you guys that this was the identity," indicating that the question was playing the role of a public critique within the discussion.

Implicit within Justin's question is an attention to the technique of proof-writing, with some implicit regard to whether it is more appropriate to express the idea of multiplying by the identity coset using fewer symbols. Viewing proof-writing as a cultural adaptation to the discourse of mathematicians provides a potential reason for why Justin would phrase this critique as a question; perhaps he experiences some uncertainty about whether mathematicians would find his expression more appropriate. As the teacher points out, his technically-more-concise statement has the potential to obscure for someone reading the proof that the coset used is in fact the identity. This discussion, though subtle, arises from Justin's critique and contributes something of mathematical importance to the discourse, so we see his question as a way of taking up the authority to assess the teacher's mathematics.

Discussion

In our preliminary work, we have seen how student questioning activity may move beyond authority for animation to become authority for authoring or assessing mathematics. Rather than treating all questions as students appealing to the authority of the teacher for authorship or assessment, we attend to the public nature of students' questions and the mathematical activity involved in formulating and communicating them to parse ways that students' questioning creates an authority relation. In viewing questioning as an authoritative mathematical activity, we expand the scope of possible outcomes in interactions when students question and explore the boundaries between animation and authorship.

We distinguished Ella's high-level clarifying question from Levi's contributing question in our analysis because Levi's question became the mathematical focus of the segment while Ella's did not, although both involved the students bringing their own mathematical ideas into the class discourse. In this example, the difference between a high-level clarifier and a contributing question lies in two places: the mathematical scope of the segment and the teacher's response to the question, how it was taken up or developed. Levi's question occurred in a segment where the classroom activity was narrowly focused on students asking questions after the presentation of the teacher's proof, so the introduction and justification of a new idea garnered Levi authority for authorship. In contrast, Ella's question occurred during a rich discussion of a particular example, so her idea had less influence in the context of the broader conversation. Similarly, the teacher's respective responses to Ella and Levi influenced the way the authority relation in each segment was created. The teacher validated Levi's thinking and formalized the language used, indicating that Levi was leveraging the properties of a subgroup rather than a group. In response to Ella, the teacher pointed out a flaw in Ella's suggestion. In both cases, the teacher took up authority to assess the students' ideas. We wonder whether this is consistently the case: does the teacher's (or other students') response determine the authoritative nature of students' questions?

Questioning is not the only activity during which the authority relation can shift from animating to authoring or assessing mathematics. While we looked at questioning in this preliminary analysis, we wonder about the circumstances required for such a shift when students are engaged in other classroom mathematical activities like validating proofs, writing mathematics publicly on the board, or working together in small groups. Knowing where this boundary lies in these varying contexts may further illuminate how authority relations are formed across activities.

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