"When I Write in English, It Doesn't Touch My Heart": Bilingual Graduate Students' Language Switching in Mathematical Understanding

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Bilingual students are an integral part of the mathematical graduate education system, so understanding their perspective on learning mathematics is important to ensure they are provided proper support. While many students learn undergraduate mathematics in English, some graduate students' first exposure happens at the upper undergraduate or graduate level, introducing additional challenges. The purpose of this study is to discover reasons for bilingual graduate students' language switching when understanding and experiencing mathematics. All four bilingual participants were in a math PhD program and acquired their undergraduate degrees outside of the United States in their respective languages. The results revealed that students switch languages not only between reading textbooks or during lectures, but also when working on solving mathematical problems and internalizing difficult concepts.

Keywords: code switching, graduate students, bilingualism

Bilingualism is a complex concept that does not have a universal definition. Although a consistent component of the term is the use of two or more languages, there are slight differences in definitions seen in the literature. Grojean (1997) defined bilingualism as "the ability to produce meaningful utterances in two (or more) languages" (p. 164) and bilinguals as "people who use two (or more) languages (or dialects) in their everyday lives" (p. 164). On the other hand, Planas and Setati (2009) added a focus on comprehension of language describing a bilingual as "someone who has acquired or learned to speak and understand the world by means of two languages, though the contents of her/his understanding may not be the same depending on the language that s/he uses" (p. 38). This highlights a potential discrepancy between using or speaking a language and the application of that language in different settings.

The science and engineering fields, many of whom utilize mathematics, report an increase in foreign-born students. In 2017, almost 60% of the workforce in computer science and mathematics were foreign-born (National Science Board, n.d.). Because of the almost universal use of English in higher level mathematics, many of these students need to shift their mathematical thinking from their native language to find success in advanced education programs. This exploratory study seeks to understand the language shifts of students with a bachelor's degree in mathematics achieved predominantly in their native language. Therefore, our research question is: For those students, what are the reasons behind switching languages when seeking help in understanding mathematical concepts?

Literature Review

Analogous to the myriads of definitions of bilingual people, language switching, or code switching, has been defined in numerous ways (Green & Wei, 2014; Hofweber et al., 2020; Moschkovich, 2007). Zeller (2020) claimed that code switching "ranges from inserting single words into the main language of discourse to alternating languages for larger segments of a discourse and can occur at various syntactic positions and with various word classes" (p.1). Alternatively, Bosma and Blom (2019) adopted a definition of code switching as an "ability of a bilingual to alternate effortlessly between two languages" (p. 1432). Both definitions have one

common element – code switching is a dynamic process of integrating one language into another and should not be regarded as a lack of language proficiency in any of the languages, but rather as a sign of bilingual competence (Bosma & Blom, 2019). When considering code switching, the asymmetry between language switching sometimes corresponds to the notion of minority-majority language pair, where the majority language is used by most inhabitants of the country (Bosma & Blom, 2019).

Despite the existent notion and practice of language switching, the approach in classrooms mathematics bilingual students follow may be more intuitive. In a report by Clarkson (2005), eighty Vietnamese students were studied while solving mathematical problems in English and 51% described switching to Vietnamese for at least one problem, which supports the use of code-switching as one of the strategies for problem solving. Bernardo (1999) discovered that the success of Filipino-English students in solving word problems in Filipino was higher than in English due to a better comprehension of the stated problem. According to Bernardo and Calleja (2005), during their study of eighty-five Filipino fifth graders, when mathematical problems were presented in English, students reverted to their native language for several reasons. One reason was the social aspect of the classroom which allowed students to seek help from other classmates in Filipino (Bernardo & Calleja, 2005). Another circumstance of language switching occurred when students perceived a problem to be too hard, whether it was a written word problem in English or what may be regarded as straight processing of algorithms or counting. Students were less likely to produce solutions when the problems were presented in their non-dominant language (English) and were more successful in applying appropriate arithmetic procedures to problems written in their dominant language (Filipino).

The research shows that bilingual students tend to use more advanced strategies for solving complicated mathematical problems in their native language (Domínguez, 2011). According to Domínguez (2011), "if bilinguals are strategic language users, they may also be strategic problem solvers, especially if learning to problem solve is viewed as a communication-based experience" (p. 309). While many bilingual students use their native language outside of school, different language of instruction might provide additional cognitive costs which could result in lower accuracy and processing speed (Saalbach et al., 2013). Domínguez argued that integrating students' everyday experiences into mathematical problems could blend two languages together, which was an approach that has been posited for equitable mathematics education (Adiredja et al., 2020). Participants tended to have more discussions, arguments, and risk taking along with mutual learning, which appeared during solving problems in Spanish, their native language, while reverting to traditional approaches when working on problems in English.

The findings from the Dominguez study were further supported by Ní Ríordáin and Flanagan (2020). Two undergraduate Irish-English students, whose dominant language was Irish, chose to interact in English, as it was the dominant language of instruction in their university. This reflects the impact of the social context on students' choice of language. That social context was also apparent in the study of Catalan-Spanish speaking students (Planas and Setati, 2009), where students used Catalan in a socialization setting while briefly addressing the whole group and Spanish when acting like mathematical learners in their small groups (Planas & Setati, 2009). Other studies (e.g., Planas and Civil, 2013) showed that students' overall demeanor changed to more relaxed when using their home language in mathematical explanations. In contrast, when using their non-native language, students' presentation style changed to less natural and non-engaging.

According to Ní Ríordáin and Flanagan (2020), alternate use of two languages is not only a demonstration of proficiency of the languages but of the social circumstances of the languages in use. The dependence of bilingualism's expression on the immediate surrounding makes this process a highly socially contingent phenomenon. In academic settings, the shift to a common, or a higher status, language may occur more often as it pertains to students' perception of access to social goods, while lower status language may bear the perception of being related to segregation and marginalization (Ní Ríordáin & Flanagan, 2020). It is crucial to note that in these settings, the social process is integrated in social interaction where issues of identity and power are structural (Ní Ríordáin & Flanagan, 2020), so the freedom of expressing oneself is often constrained by a social aspect or the surroundings.

The reviewed literature demonstrates the occurrence and reasons for language switching in mathematics, and it describes reasons and effects of language switching. However, to our knowledge, examining language switching among mathematics graduate students has not been investigated. With that, we hope our findings will contribute to the evolving literature and bring awareness to the experiences and learning approaches of bilingual graduate students in mathematics.

Methods

The aim of this exploratory study is to acquire insights on reasons behind bilingual graduate students' language switching when working on mathematical problems. To achieve that, four graduate students were recruited from a central, high-research, U.S. university's doctoral program. All participants were approached in person and a set of qualifying pre-screening questions was asked. Among ten reached graduate students, four agreed to participate in this study that earned their bachelor's degrees outside of the United States (Table 1).

Table 1: Participants and Backgrounds

Participants	Bachelor's Degree	Master's Degree
Bach	Vietnam	U.S.
Ergun	Turkey	Turkey
Maria	Colombia	U.S.
Miguel	Mexico	Mexico

The semi-structured interviews were video- and audio-recorded in a seminar room ranging between thirty and sixty minutes. Open-ended interview questions were developed with a purpose to learn each student's educational background along with their mathematics-related studying and learning practices. For example, participants were asked, "Can you describe your transition to all-English lectures?" or "Did you ever use online or offline resources in your native language to understand some mathematical concepts better?". Additional follow up and clarifying questions were posed.

Once data was collected, the interviews were transcribed. Since the study is exploratory and no known coding scheme was used, all transcripts were coded using a hypothesis coding approach (Saldaña, 2013). The hypotheses were "developed from a theory/prediction about what will be found in the data before they have collected or analyzed" (Saldaña, 2013, p. 147) and additional codes were identified and classified as "other". The coding was done individually by interviewers, compared, and finalized based on mutual agreement: "[they] made no attempt to

develop a numerical reliability rating, because our goal was consensus, with each point of difference being debated and clarified until the group agreed on appropriate usage of the set of codes" (Harry et al., 2005, p. 6).

Results

During the interview analysis, and after the original hypotheses, the research team discovered three main categories of language switching: external factors (during lectures and reading textbooks); internal processes (processing mathematical information and solving problems); and social influences (driven by social or environmental factors).

External Factors

As all four participants earned their undergraduate degrees outside of the United States, their initial learning of mathematics primarily happened in their native language until higher mathematical content was introduced. This equipped them with the ability to understand mathematics in multiple languages, widening their pool of resources. For example, Ergun's current interest is p-adic groups, so he discovered a Turkish professor's work on that topic in a Turkish mathematical digital magazine, explaining the concept clearly. Miguel has searched for recent Spanish dissertations written on topics of interest to receive the most recent information available in his own language. Although helpful, online sources are generally not frequently used by the participants because of the lack of these resources online. Maria stated that finding advanced math-related resources online in Spanish was not easy compared to those in English and Ergun stopped consulting Turkish resources altogether due to their scarce numbers. Similarly, Bach found it disadvantageous to consider online sources in Vietnamese because questions he might seek answers for are already stated in English:

> And I am Googling something like that... Becauseif you look at the words in the textbook, you will type that in English, right? You don't type it in Vietnamese. So, you type that and it will show many English textbooks so that's... Obviously, I have to look at English only.

While most online resources were deemed insufficient, in some situations, participants were able to find complement material to available resources including books and hand-written notes in their native language. For example, Miguel kept Spanish copies of "Topological Groups" and "Introduction to Topological Transformation Groups" as well as Spivak's Calculus, and Maria continuously referenced a Spanish copy of Munkres' "Topology" textbook. Similarly, personally written notes in participants' native language were regarded as a valuable resource. Both Maria and Miguel tend to consult their own previously taken notes in Spanish due to familiarity with notations or simply because the content of those notes was once challenging. Interestingly, Maria reports that the frequency with which she refers to her old notes changed as she learned more mathematics in English throughout her academic career, identifying her knowledge replacement in a new language.

The externally influenced language shifts during lectures were beyond the control of our participants, but they seem to have had a positive impact. During Bach's senior year of undergraduate study, mathematical lectures were held primarily in English, which provided early exposure to mathematics in English. Likewise, Miguel's early introduction to mathematics in English happened during conferences:

> Some people came to the National University of Mexico, they did their lecture, and it's, in some sense, I believe a really safe environment because it's not like they're going to question you if you understood the material or not. So, that helped a lot, I think.

The notion of gradually easing into learning mathematics in English appears to have a positive effect on Bach and Miguel's perception of language switching in a math setting. Beyond lectures, textbooks written in English were often used during undergraduate education. Bach shared that the number of comparable mathematical textbooks written in Vietnamese was scarce, so while some texts were recommended by his professors, he often searched for relevant English literature on his own. The practice of favoring English texts also influenced Maria's undergraduate experience, where the textbooks were in English or Portuguese, but the proofs were still expected to be written in Spanish, even if she referenced the non-Spanish textbook.

Internal Processes

The translation is not foreign for mathematical graduate students and, rather, at some stages of their academic careers, is integral. Translation can be considered as an internal factor of language switching controlled by the student compared to external factors which are outside of participants' control. Bach shares his communication experience while using internal translation for double switching between receiving information in English, generating a response in Vietnamese, and translating it back to English: "So, before we actually speak that one, we think about it in Vietnamese and we translate words by word, word by word," which is not applicable solely to verbal communication, but to written, too. Maria shared that, when solving math problems stated in English, she began by "thinking in Spanish and then trying to write it down in English when I first came here," but then by "one semester after my first semester I started to think more in English". For Ergun, reading math problems is not the challenge, "the difficulty starts from generating arguments. And so, to attack the problems... my brain automatically goes to Turkish." This presents a vivid example of an internal language switch, where participants unintentionally default to their native language for response creation or problem solving due to sufficient mathematical knowledge acquired in their native language.

The notion of mental internalization and translation we label as "inner voice", which entailed mental work participants conducted while learning or solving mathematics. For instance, as Ergun worked on a math problem stated in English, he wrote questions and keywords to himself in Turkish:

> Yeah, I have some English statement, mathematical, maybe proposition. So, now, I'm working on some bilinear forms. So, I kind of check myself and I see that these questions… I'm asking lots of questions to myself, and writing lots of keywords, and it's kind of a puzzle and makes sense after some point if I work on it in one hour or two hours. But if I get some new good proposition, write in English. But then, still the teaching part is always Turkish to internalize, to understand some deep, deepness... To have some deepness still. My voice, inner voice is Turkish.

There is a strong connection of Ergun's mathematical work to his language, as he shared:

> So, some writings are Turkish, and some write things are English, and it's a mixture… When I write in English, it doesn't touch my heart. So, if I write in Turkish, I feel much more at home and just... I get more motivation... The pop-up questions are much more fruitful for me. Sometimes English is mechanical. I don't know. Maybe it's a linguistic issue. I mean, the words always trigger something, and they are connected to each other.

> And, in my native language, it's connecting very quickly in my mind, in brain, but English words is much more slow works for me.

It is prominent that Ergun's linguistic attachment to his native language plays a significant role in solving mathematical problems. While participants' current lectures are purely in English, there are times, like with Ergun, their notetaking is still in their native language. For example, Maria finds herself jotting down quick notes in Spanish to provide herself a gist of presented material. The language switch during note taking emphasizes the connection of our participants to their native language, as it is an important aspect of their identities.

The thought process around problem solving in different languages is unique and intriguing. As it can be observed, Ergun integrates Turkish and English as his problem-solving strategy where the translation process is no longer obvious but, rather, fluid and continuous. Ergun has a well-established practice that intertwines the language of the problem and language of mental processing. However, Maria shared a different experience: "when I translated from Spanish, there is where I have those problems. Worse. They are, like, worse if I translate. No, I need to think in English and try to write it down in English." This indicates that language switching when solving problems for some students is not a helpful strategy.

While language can be tied to previous experience, it can also serve as a helpful resource. Maria described a time during her topology class where a notion of universal set was introduced. At first, she encountered difficulties with conceptualizing the definition, so the professor provided an alternative name – ambient set. Since Maria took topology class in Colombia, she immediately recalled the familiar Spanish word "ambiente" which triggered previously acquired knowledge about ambient sets. Similarly, while working with his advisor, Miguel encountered concepts stated in Spanish and struggled to understand their differentiation. When he attended to his professor, he was given English alternative definitions of these concepts that immediately distinguished them from one another.

Social Influences

Another type of language switching identified seemed to be dictated by social norms and a strive to learn English. For example, Maria shared her early attempts to write proofs in English, saying: "When I was finishing my undergrad, I was... trying to write down the statements of the homework and try to write down the proofs in English… it's easier in English to find a master's, with a scholarship of course." The described strive to improve proof-writing in English was driven by a notion of socially accepted high status of the English language in a mathematical setting. Similarly, Ergun stated that, "Mostly I gave up to look up Turkish resource maybe at the beginning, after one year in my master's program, because I was feeling that I need to learn to read English, just need to improve myself more." The idea of integration is perceived by participants to relate to improvement of their English skills, which is what Bach shared about:

> Later I figured out that this is not convenient to just use Vietnamese. So, in general, I want to improve my English, so I stop thinking as Vietnamese. So, then I stopped using... Googling or finding this term in Vietnamese… I will say I preferred English. Yeah, because now I am in a completely 100% English program, so I have to do more with English. So, instead of using these sources in Vietnamese, I just start reading English.

The data shows that our participants view English as either higher status language or majority language (Bosma & Blom, 2019), which could skew their perception of their native language's importance or relevance.

Discussion

Our goal in this paper was to learn why bilingual graduate students shift between languages when working through mathematical problems to further understand how language and problem solving intertwines as well as to highlight potential hurdles bilinguals might face during their graduate careers. Another objective of this study was to determine what impact transitioning from (mostly) non-English lectures to purely English mathematical courses had on our participants.

In the light of our research question, we conducted four semi-structured interviews with mathematics graduate students who earned their bachelor's and, sometimes, master's degrees outside of the United States with the primary language of instruction being not English. During their bachelor's or master's experience, all participants consulted English advanced mathematics textbooks based on professor's reference and for the expansion of available information and resources. We also found that our participants' personal notes taken in their native language served as a valuable resource as they contained familiar mathematical notation and content viewed as challenging in the past. Even though online resources were not indicated to be in the highest demand, two graduate students used queries in their native language to widen the pool of information or simply because they prefer a particular non-English author. We also discovered that internal language switching while solving math problems was not uncommon among our participants, as some switch to their native language when producing mathematical arguments which enhances their understanding and success in proving problems.

To our knowledge, there are no studies done with bilingual graduate students pursuing a PhD in mathematics on language (code) switching, so we strive to contribute our results to the currently growing literature. Almost all, if not every, department of mathematics have graduate students with similar backgrounds as our participants, and while carrying multiple languages for these students can be a tool for studying mathematics, lack of certain vocabulary in their second language can impose difficulties during their studies. We believe it is important to be aware of additional support graduate students might need to succeed in graduate programs, even if it is a simple recognition and understanding of potential language barriers.

While participants were representing four different continents and three different languages, our results were limited to a small number of perspectives. A broader study to include experiences of those from other countries and continents would gain insight into the global needs of different students.

Implications and Future Research

The results of our findings reveal a potential need to improve integration of bilingual students into English-led courses. This may lead to professors and educators being more mindful of a language gap in not only mathematical domain, but also in everyday language. As the results showed, some students found success in intertwining both their native language and English, while others were convinced switching to English was the best choice for them. Instructors should encourage graduate students to practice different problem-solving techniques which could involve language switching. Future work in this area should consider increasing participants' pool to diversify participants' experience, which would enhance the knowledge and awareness of various graduate students' practices and difficulties and could more broadly be done for other domains of graduate programs. Also, interviewing participants in their native language could reveal more insights to the researcher while circumventing a language barrier.

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