"I Only Have One Black Student… With a Sample Size of One, It's Hard to Say": A Critical Analysis of Epistemology in Mathematicians' Racial Sensemaking

R. Taylor McNeill
Michigan State University

Mathematics departments nationwide are tasked with addressing racial inequities, including students' racialized experiences in calculus courses. Faculty must interpret racial information critically to advance equity through change initiatives. However, mathematicians often hold colorblind views of instruction that are reinforced by mathematics epistemological values, such as neutrality and objectivity. It is, thus, unclear whether mathematics epistemology inhibits the advancement of racial equity within mathematics departments. This analysis reports on racial sensemaking among 22 mathematicians at a research-intensive university amidst a reform effort to improve Black and Latin students' calculus experiences. Guided by theories of epistemic culture, I compare mathematicians' mathematical interpretations of racial phenomena with those proposed by critical race theory. Findings exhibit how mathematicians conflated contextualizing racial information with stereotyping, and focused on quantities without recognition of students' lived experiences or the social justice implications of their arguments. Implications are provided for using epistemology as a lever for change.*

Keywords: epistemology, mathematics faculty, critical race theory

Study Purpose & Background

Postsecondary mathematics is rife with racial inequity. For example, at the entry level, racialized pressure and isolation in undergraduate calculus can demand additional labor from Black and Latin*[1] students to persist in science and mathematics fields (Battey et al., 2022). At the faculty level, racial dynamics in mentorship and collaboration can contribute to differences in early-career support and faculty development (McNeill & Jefferson, 2024). While these racial issues are, arguably, produced by mathematicians, mathematicians are also tasked with remediating such inequities. For example, research aiming to advance calculus instruction recommends that mathematics departments collect and interpret local data to guide improvements (e.g., Zazkis & Nuñez, 2015). Research focused on practitioner inquiry into higher education inequities emphasizes that faculty must approach such work with equity-mindedness to resist colorblind or deficit interpretations of racial phenomena (Dowd & Bensimon, 2015). However, faculty in mathematics often hold colorblind interpretations of instruction (McNeill et al., 2022; King et al., 2023). It is unclear, then, whether mathematics faculty are prepared to critically interpret racial data when these same racial phenomena may escape their awareness in their teaching and other professional work.

How one curates and interprets information to generate knowledge is a question of epistemology. Disciplinary epistemologies can influence how faculty consider racial issues in their professional work. For example, in a study exploring equity in doctoral admissions, Posselt (2015) found that disciplinary theories, epistemologies, and methodologies shaped committees' perceptions of applicants' qualifications. For example, in the highly quantitative field of

[1] The term, Latin*, is inclusive of all genders (Salinas, 2020). Our use of Latin* avoids the patriarchal use of "Latino" to describe people of Latin American descent.

economics, "economists not only looked for quantitative expertise; they also relied primarily on quantitative measures to gauge quantitative expertise" (p. 818), while philosophers were "deeply uncomfortable with the false precision" (p. 821) of quantitative rankings. These different epistemological orientations played a role in committees' reliance on racialized metrics for evaluating applicants. Similar evidence of interplay between disciplinary epistemology and racial equity has also been found in mathematics departments. For example, mathematics departments can employ seemingly-neutral quantitative metrics for faculty evaluation that invisibly reward traditionally white achievements while neglecting Black women faculty's less traditional scholarly work (McNeill & Jefferson, 2024). Similarly, mathematicians' views of mathematics as fixed and pre-existing parallel common perceptions of students' mathematics ability as fixed and innate, reinforcing inequitable practices like tracking (Burton, 2004).

In the social sciences, the epistemologies promoted by critical race theory (e.g., Delgado & Stefancic, 2023) have become an accepted standard for antiracist thinking. However, mathematical thinking holds a privileged position in society (Gutiérrez, 2013) and many mathematicians believe that mathematical thinking is the purest form of reason (Burton, 2004). For this reason, developing mathematicians' capacity for addressing racial inequity may require unsettling deeply entrenched ways of thinking that are believed to be superior.

The present analysis uses the context of a calculus program's equity-oriented reform effort to examine mathematics epistemological values (MEVs) in mathematicians' racial sensemaking. To inform mathematics faculty development initiatives that seek to advance racial equity, the present analysis explores the following questions. What role do MEVs play in mathematicians' sensemaking about racial (in)equity in undergraduate calculus? How do these mathematical interpretations of race compare to those established through critical race epistemologies?

Epistemic Cultures

Knorr Cetina's theory of epistemic cultures (1999) guided the analysis. Epistemic cultures are "cultures of knowledge societies" (p. 241). More specifically, epistemic cultures arise when an "internally referential" (p. 2) community generates and codifies knowledge based on a largely shared set of epistemological values. Disciplinary communities, such as mathematicians or critical race theorists, are examples of such communities as they each hold values and practices related to the creation of knowledge that are largely distinct from everyday thinking. I hold these two epistemic cultures as the subject of examination and summarize their epistemologies.

Mathematics Epistemological Values

Although there is no universal definition of mathematics, mathematics can be characterized as relating either to proof or to application (Hacking, 2014). Mathematicians generate knowledge from the epistemological practice of proving claims via arguments based in formal logic and reliant on axioms. Although applied mathematics is intertwined with proof-based mathematics, applied mathematics is often distinguished by its use of computations which "proceed in a mechanical way by set rules" (Hacking, 2014, p. 145).

The notion that mathematical arguments constitute "proof" highlights common perceptions of mathematical claims as unequivocally, universally true or false (Burton, 2004; Hacking, 2014). Indeed, when studying how mathematicians generate knowledge, Burton (2004) found that 90% of mathematicians held an objectivist, positivist view of mathematics as existing independent of human thought. Beyond this, mathematicians tended to view mathematics as "the highest form of rationality" (p. 6), and as governing the nature of reality, even beyond the scientific. However, as with all human endeavors, the mechanisms for building mathematical

knowledge are socially constructed (Knorr Cetina, 1999). For example, there are ambiguities related to what constitutes formal logic and what constitutes an axiom (Dutilh Novaes, 2011; Shapiro, 2004). Furthermore, despite framings of mathematics as objective, aesthetics, affect and intuition play centrally in mathematicians' knowledge generation (Burton 2004).

Critical Race Theory & QuantCrit as Epistemological Values

CRT originated in legal studies and theorizes how race and racism operate in U.S. society. Characterizations of CRT vary, but share a core set of beliefs including: (1) CRT recognizes that racism is endemic to American life; (2) CRT expresses skepticism toward dominant legal claims of neutrality, objectivity, colorblindness and meritocracy; (3) CRT challenges ahistoricism… [and] presumes that racism has contributed to all contemporary manifestations of group advantage and disadvantage along racial lines; (4) CRT insists on recognition of the experiential knowledge of people of color; (5) CRT is interdisciplinary; and (6) CRT works towards the end of eliminating racial oppression as part of a broader goal of ending all forms of oppression (Mastuda et al., 1993, p. 6). CRT can also be seen as a set of epistemological values describing ways that knowledge about race and racism is generated. For example, CRT suggests that the lived experiences of students of color are valuable sources of knowledge about race and racism, while ahistorical, decontextualized comparisons of mathematics achievement among white students and students of color are less robust sources of knowledge.

CRT has primarily been engaged in qualitative contexts and some scholars "remain skeptical about the possibility that numbers can ever fully capture the 'material impact' of intersectional racism" (Gillborn et al., 2018, p. 169). However, given the power and utility of numerical information in American society, a quantitative extension of CRT, known as QuantCrit, has been developed that incorporates similar principles. In addition to CRT principles of advancing social justice and understanding racism as ordinary in American society, QuantCrit takes as given that: (1) Numbers are not neutral; (2) Racial categories are socially constructed; and (3) Data cannot speak for itself and must be contextualized by the experiential knowledge of people of color.

Methods

Research Context and Participants

This study took place at Wesselman University, a large, elite, predominantly white, research-intensive university in the southern U.S. During the 2022-2023 academic year, the mathematics department had approximately 30 tenure-track faculty, 20 postdoctoral faculty, 10 senior lecturers, and 25 doctoral students. The demographics of the entire teaching staff were approximately 70% white and 85% male. During the same year, the domestic undergraduate population was roughly 40% white, 20% Asian American, 10% Black, 10% Latin*, 5% multiracial, and 5% unknown race.

In 2020, university administration identified a need to improve the calculus program, particularly to ameliorate racial disparities in calculus grades and attrition from STEM majors. A research team began a collaboration with the Wesselman mathematics department in Spring 2021 to support this reform effort. Our partnership took three phases: (1) Documenting Black and Latin* students' experiences in calculus; (2) Exploring instructor and stakeholder sensemaking about racial equity, and (3) Facilitating professional development with calculus staff. The present analysis is based on data from the study's second phase.

Prior to the beginning of the Fall 2022 semester, instructors teaching calculus I courses were invited to participate in the study at an in-person meeting of calculus staff, and again through

email. Faculty who were stakeholders in the calculus reform effort, such as the department chair, the coordinator for calculus courses, the director of undergraduate studies, and members of the "rethinking calculus" committee, were also invited to participate through email. All who expressed interest participated. In this report, I describe participants in aggregate to preserve anonymity given the unique departmental context. The 22 participants consisted of 11 calculus instructors (five lecturers and six doctoral students) and 13 stakeholders (seven tenure-track faculty members, five lecturers, and a postdoctoral scholar), with two participants serving in both instructor and stakeholder roles. Participants were majority white and men.

Researcher Positionality

In alignment with CRT's rejection of objectivity, I do not detach myself from the present analysis, but instead harness my personal experiences at the juncture of mathematics and social science contexts as a source for knowledge production. I am a white transgender person that holds doctoral training and faculty experience in both mathematics and education fields. While I perceive myself as belonging to the epistemic culture of critical race theorists, my past and present experiences immersed in the culture of mathematics have allowed me to engage either set of epistemic machinery with relative ease. I leveraged this ability while coding to critically evaluate the epistemological underpinnings of participants' reasoning.

I spent three and a half years in partnership with Wesselman mathematics, serving as a researcher, a calculus instructor, and an instructor educator. I frequently experienced the epistemological conflicts that this analysis formally characterizes first-hand. This first-hand experience, combined with my prolonged collaboration with Wesselman mathematics, contribute to the study's trustworthiness.

Data Collection and Analysis

Data consists of interviews which were semi-structured, audio recorded, and transcribed verbatim. Interviews averaged one hour in length, with stakeholders participating in one interview and instructors participating in four. Interview questions solicited participants' perspectives on the role of race in calculus courses and related changes to the calculus program. As examples, stakeholders were asked, "How do you think proposed changes to calculus courses would impact Black and Latin* students' experiences?" Instructors were asked, "How do you see race playing a role in students' experiences or opportunities for success in your course?"

The theory of epistemic cultures guided a data analysis that followed an open, axial, and selective coding scheme (Strauss & Corbin, 1998). Two sets of open codes were used to flag data for implicit epistemological values. One set of open codes captured participants' use or rejection of MEVs. This included instances where participants discussed numbers, used mathematical language (e.g., "sample size," "it follows that"), or otherwise employed mathematical reasoning (e.g., testing a generalization by looking for counter examples), as well as instances where participants framed mathematical thinking as irrelevant or unhelpful. These codes helped to understand how participants did and did not use MEVs in their racial sensemaking, addressing the first research question. Another set of open codes flagged epistemological orientations to race, including excerpts that agreed or contrasted with the CRT and QuantCrit.

Axial coding found connections between the two sets of open codes, addressing the second research question comparing mathematical and CRT interpretations of racial phenomena. Specifically, axial codes identified categories at the intersection of these open codes, where included excerpts engaged or rejected MEVs in similar ways, and shared a common relationship

with CRT epistemologies. Lastly, selective coding extracted thematic relationships between MEVs and CRT epistemological values.

Findings

Although data contained instances in which participants rejected MEVs or engaged MEVs in ways that aligned with CRT epistemologies, participants most often engaged MEVs uncritically in ways that contrasted with CRT epistemologies. In what follows I demonstrate three themes that arose: (1) Using quantitative arguments to naturalize racial inequality; (2) Focusing on representation over experiences; and (3) Conflating contextualization with stereotyping.

Using Quantitative Arguments to Naturalize Racial Inequality

Participants engaged epistemologies of mathematics in their interpretations of large-scale information such as racial disparities in students' grades across disciplines. It was common for participants to question the methods by which large-scale statistics were calculated, as well as their interpretation. Through this, participants framed racial inequity as minimal and exempted themselves from responsibility for remediating racial inequity. For example, some participants denied that racial disparities in grades were cause for concern in mathematics, arguing that grade inflation in other departments produced compression that simply made racial disparities appear less pronounced: "Once you normalize students' GPAs by GPA average in departments, it's not clear how much the problem is centered in math." Similarly, another participant argued,

> This [racial] disparity [in grades]… the statistics were not done right because you have to divide by the variation, the sigma... mathematics classes could have a huge variation, while, let's say English can have a very small variation. If you do that, then everybody is actually falling correctly more or less.

The use of the word "correctly" is notable as it naturalizes the unequal distribution of grades among racial groups. However, its juxtaposition with statistical computations makes this subjective naturalization of racialized grades appear objective. Other participants similarly used quantities to naturalize racial disparities.

> I don't know what the expectation is from the university for the calculus classes… Less people who are Black, Latinos, to fail?... All of them, they don't want to fail?... Let's say you have all of your Black and white students get a similar SAT score, [but] Black students [are] doing worse in calculus. Then that, to me, that's a great problem. But if students with around the same SAT score stay around the same, more or less, then to me that's a lesser problem.

Expecting students with similar standardized test scores to have similar levels of achievement in calculus is a mathematical framing that may seem neutral because it suggests that Wesselman mathematics courses do not exacerbate racial disparities. This frames the perpetuation of preexisting racialized disparities in mathematics achievement as natural.

These interpretations contrast with the epistemological values of CRT and QuantCrit. The idea that any mathematical framing of racial issues is "correct" relies on notions that mathematics is objective. This directly opposes QuantCrit's position that numbers are not neutral and contrasts with CRT's rejection of objectivity. Furthermore, these examples argue against remediating racial inequities in students' grades, conflicting with CRT's social justice aims.

Focusing on Representation Over Experiences

Participants' focus on racial representation to the exclusion of acknowledging students' racialized experiences also illustrated the influence of MEVs. For example, when I shared preliminary findings with the department showcasing narratives from Black and Latin* students,

I heard that faculty had "disagreements with your presentation" because "the data was not quantitative." Instead, participants often focused on percentages and probabilities, neglecting the human experiences that accompany these figures. For example, when interpreting a Black student's narrative of racial hypervisibility when participating in a class where white students dominated discussion, one participant reasoned,

> If there is one dominant student in the class, race aside, your odds of being the dominant student in the class are one in however big the class is. And if you are already in a minority of the people in that class, your odds of being the dominant... it's like these differences are exacerbated by the very fact that they're already in a minority.

The participant uses the mathematical meaning of "minority" (a number that is less than half of the total) and frames racial inequity as a byproduct of the class's racial representation rather than resulting from issues of power and privilege. Participants focused on probabilities and representation even when describing their goals for advancing racial equity. When responding to the same prompt about racialized participation dynamics, another shared,

> I would guess that I'm fitting into fairly standard trends in terms of participation. I think I can get my minority students to speak up a little bit… Yesterday, I think I had two Black students speak up, actually, when I asked questions. So, I'm getting some in there. But... even accounting for probabilities, I would say my white students are much more willing to speak up, even compared to Asian students.

The participant's reflection implicitly frames Black students speaking in class as the goal and does not consider the quality of their learning experiences. Their remark that two Black students spoke, combined with their consideration of probabilities, frames racial equity as numerical.

Both CRT and QuantCrit emphasize the importance of lived experience as a source of knowledge. The QuantCrit value of contextualizing numerical information within experiential knowledge directly contrasts with participants' inattention to students' affective experiences.

Conflating Contextualization with Stereotyping

Participants were extremely hesitant to provide racial interpretations of their instructional observations. Although this was not explicitly mathematical, participants' engagement of mathematical language such as "sample size" and "conjecture" suggested that their sensemaking was influenced by MEVs. For example, when asked their perception of whether race played a role in a successful white student's performance, one participant shared, "Everything I'm about to say is conjecture… but I think she's Russian. Russia has a strong mathematical culture… maybe there's something there." The term "conjecture," in the context of mathematics, describes an act of generalizing from one or more cases to make a universal claim. The participant's hesitation to provide a racial interpretation, combined with their use of "conjecture," implicitly suggests that they equate interpreting racial observations with generalizing or stereotyping. This is also evidenced in the following excerpt, in which a participant was asked to describe their racial observations related to participation in their classroom. The participant, unprompted, voiced skepticism about the validity of a racial generalization one might draw from their observation, using MEVs as justification: "I only have one Black student. He doesn't participate a ton… With a sample size of one, it's hard to say. Is that all of the Black students, or that this one guy is kind of quiet?" The participant's consideration of the "sample size" suggests that knowledge of race should rely on statistically significant trends. Given that generalizing is a common and valued mathematical practice while placing information in a sociohistorical context is relatively less common, it is possible that MEVs shaped participants' misunderstanding of tasks which asked them to contextualize their instructional observations.

CRT and QuantCrit presume that racism is ordinary in American society and plays a role in producing present-day inequities that fall along racial lines. This includes inequities in prior educational opportunities or access to classroom participation. Furthermore, CRT's refusal of colorblindness encourages consideration of racial elements of events that may be seemingly neutral at face value. Hence, a CRT perspective would presume that race plays a role in students' calculus experiences while holding an antiessentialist assumption that race influences students' learning in different ways.

Discussion & Implications

K-12 mathematics education scholars have long critiqued school mathematics as grounded in white, middle-class values that perpetuate racial oppression (e.g., Ladson Billings, 1997). Postsecondary mathematics has been historically shaped by the same global forces of white supremacy and antiblackness (McNeill & Jefferson, 2024; Bishop, 1990; Hottinger, 2016). However, interrogation of the mathematics discipline at this level has been much more limited. Some scholarship has illustrated how dominant epistemological values in mathematics can reinforce the oppression experienced by students of color (Adiredja & Zandieh, 2020; Street 2024). The present analysis extends these findings, demonstrating that dominant epistemological values may reinforce oppression, not just when students' engage with mathematics, but in any context where mathematical thinking and racial issues collide. For example, Adiredja and Zandeih (2020) describe how linear algebra examples produced by women of color and based on their everyday experiences could readily be dismissed as "idiosyncratic" (p. 254) in the dominant mathematical view. The present analysis illustrates that mathematicians may readily dismiss the racial experiences of individuals as idiosyncratic more generally, potentially because dominant epistemologies of mathematics assign little value to individual cases.

Implications for Developing Mathematicians' Critical Consciousness

Faculty development programs focused on issues of equity are becoming more commonplace (Castillo-Montoya et al., 2023). However, equity-focused development programs for mathematics faculty demonstrate mixed efficacy (e.g., Ching, 2018). The present findings demonstrate that mathematicians' sensemaking about race exhibits mathematical features. This suggests that developing racial consciousness may require mathematics faculty to critically re-evaluate the epistemic machinery they employ to consider racial issues. However, given the privilege that mathematical thinking receives in society, it may be unlikely that mathematics faculty doubt their epistemological approaches. Those engaged in developing mathematics faculty racial consciousness should consider making critical epistemological approaches explicit for faculty and addressing epistemological conflicts that faculty raise.

Implications for Research: Conceptualizing a Critical Mathematics Epistemology

The present analysis demonstrates ways that mathematical and antiracist thinking conflicted. However, this is not to say that mathematical thinking is inherently oppressive. A body of research in K-12 education contexts has worked towards reconceptualizing mathematics as critical and liberatory (e.g., Gutiérrez, 2018; Gutstein, 2012; Martin, 2019). Despite the fact that people of color practice mathematics at all levels, critical postsecondary mathematics is undertheorized. Critical race epistemologies suggest that the lived experiences of mathematicians of color may be a valuable resource for such knowledge generation. Future research should explore how mathematicians of color engage mathematics in critical sensemaking to support an epistemic culture change in mathematics.

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