Examining Series Convergence via Script Writing: The Case of Jordan

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Undergraduate students in the United States typically encounter the topic of infinite series in second-semester calculus courses (Kung & Speer, 2013). Students study techniques for determining convergence of infinite series which also rely upon leveraging prior knowledge about limits and convergence for both functions and sequences. As such, McDonald et al. (2000) assert the necessity for students to have "a rich sequence schema … to fully understand series as the limit of a sequence of partial sums" (p. 98). Similarly, Oehrtman et al. (2014) found students engaging in guided reinvention for sequence convergence capable of giving a definition for series convergence that aligned with its concept definition. Other research on student understanding of series convergence focuses on understandings of series and meanings for series-related concepts including ways students conceptualize and think about series convergence (e.g., Eckman & Roh, 2022; Martínez-Planell et al., 2012). In addition, Eckman and Roh (2024a, 2024b) and Larsen et al. (2024) explore how students interpret summation notation or develop notation describing partial sums of a series while Alcock and Simpson (2004) and Martin (2013) examine the type of imagery students rely upon when examining convergence of power and Taylor series. Researchers acknowledge that students face difficulties learning infinite series and that meaningful learning activities should facilitate the development of deeper understandings of series and series convergence (e.g., Lindaman & Gay, 2012; Martínez-Planell et al., 2012).

This exploratory case study aims to examine second-semester calculus students' understanding of series convergence and identify ways that script writing elicits aspects of their understanding. Script writing (Zazkis & Zazkis, 2014), used primarily with prospective teachers in mathematics courses for engaging in an imagined role-play through the process of writing a script, can also be used to explore or assess mathematical understandings (Zazkis, 2018). We examine student understanding through the theoretical lens of concept image and definition (Tall & Vinner, 1981). This poster presents findings related to the following research questions: (1) What aspects of student understanding of convergence of infinite series arise when working a scripting task versus a non-scripting-based task? and (2) In what ways do scripting task responses convey a student's concept image and definition regarding series convergence?

Jordan (pseudonym) participated in two interview sessions spaced two weeks apart, taking place in weeks 11 and 13 of a 15-week semester. Jordan completed the course midterm exam on series two weeks prior to the first session. The first session included a 75-minute task-based interview that involved general questions and tasks on sequences and series along with a 30-minute scripting training activity. The second session involved Jordan creating a script on series convergence within 45 minutes followed by a 45-minute interview focused on Jordan's script.

During the first task-based interview, Jordan confounds the sequence of partial sums of a series with the terms of the series. Jordan also claims that a series converges given that the terms of the series converge to zero. In the task-based script writing interview, Jordan's partial understandings persisted, but their script exposes these partial understandings in a more robust manner and provides distinct opportunities to convey their understandings of the concept definition. This exploratory work provides some evidence for using script writing as a viable alternative assessment of student understanding of series convergence.

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