The Impact of Questions on Students' Independent Reading of Mathematics

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This study contributes to research on the impact of questions on students' reading of mathematics and informs design considerations for questions included in workbooks. We used eye-tracking complemented by open-text questions. Participants were engineering and science students from the first to the third year of their studies. We considered proceduralness, conceptualness, and difficulty as dimensions of the workbook questions. We found that what we call a 'conceptual question' triggered more attention to the explanation than the example part of the workbook text, but so did what we call the 'procedural question'. In addition, relative question difficulty may have influenced our results. An implication of our findings is the need to investigate the interrelations of the conceptualness and proceduralness of mathematical questions while also considering the role of questions' relative difficulty.

Keywords: Reading Attention, Questions, Conceptual Knowledge, Eye-Tracking

Questions are used for both assessment and as learning tools, with one aim being to help students attend to important content and prepare for exams (Quiroz et al., 2022) . Educators often develop mathematical questions integrated within worksheets or workbooks, which can be studied independently, with expectations that tertiary students should develop as autonomous learners. However, students have been observed engaging only minimally with the content of mathematical texts (St Clair-Thompson, Graham, & Marsham, 2018). Therefore, research is needed that investigates the impact of questions on independent reading in undergraduate mathematics, specifically regarding questions' effectiveness in directing attention to particular parts of a mathematical text.

In this paper we report outcomes from a study that aimed to provide evidence of variations in where students direct their attention while reading mathematics with the goal of answering questions. Our specific focus is on comparing the impact of questions targeting explanations versus those targeting examples. Findings may enhance our understanding of how different question types influence students' reading of mathematics.

Literature Review

We provide a theoretical framing for the study by synthesizing the educational psychology literature on relevance manipulation and the mathematics education literature on conceptual and procedural knowledge. We also draw on the educational assessment literature to consider the relative difficulty of mathematics questions.

Relevance Manipulation

The directive to read with a specific purpose acts as a form of relevance manipulation (McCrudden & Schraw, 2007). Relevance is the degree to which a text segment is germane to a reader's goals. Authors assign importance to text segments according to the degree to which they are essential to understanding the ideas intended to be communicated in the text (McCrudden & Schraw, 2007). However, readers often prioritize relevance over importance (McCrudden & Schraw, 2007; Pichert & Anderson, 1977). Even when in-text cues indicate importance, readers are more likely to focus on and recognize the significance of text segments that align with their perceived relevance (Sperber, 1995).

Relevance has been associated with optimal cognitive efficiency in processing the information presented in the text (Sperber, 1995). Moreover, questions can be designed to evoke particular kinds of reflection. For example, the Cognitive Reflection Test (CRT) (Frederick, 2005) comprises three questions that activate what are called System 1 cognitive processes, which are fast and generate intuitive answers with little conscious deliberation. To answer the questions correctly, these processes must be overridden by what are called System 2 cognitive processes, which are slower and more reflective. Accordingly, we are interested in how questions can be utilized as an integral part of mathematical texts to facilitate reading through relevance manipulation and to instigate cognitive reflection.

Mathematics Questions

We consider three dimensions of questions from the assessment literature and the mathematics education literature.

Difficulty. Some questions are more difficult to answer than others due to various factors including requisite background knowledge, length or complexity of the question, whether it is open or closed, use of context, and so on (Holmes et al., 2017). In the assessment literature, difficulty, and its converse facility, are typically measured in terms of the proportion of accurate responses by a sample of students (Bond & Fox, 2007). Questions of different difficulty have been found to impact student reading comprehension differently in other subjects (Cerdán et al., 2009), and this might be expected to be the case for mathematics too (e.g. Engelbrecht , Bergsten & Kågesten, 2009).

Conceptualness. Some questions, often used in research studies or to generate discussion in lectures, are intended to assess conceptual knowledge. Well-known examples include carefully designed and validated multiple-choice items such as those in Epstein's (2007) Calculus Concept Inventory. Conceptual knowledge has been defined in terms of understanding a domain's definitions, representations, rules and categories, as well as comprehending underlying principles and the connections between them (Crooks & Alibali, 2014). We might expect what we call here *conceptual questions* to direct students' attention to explanatory parts of a text.

Proceduralness. Routine questions, such as those that appear in common mathematics examinations and tests, are often considered to be procedural (Sangwin & Jones, 2017). Such questions tend to be closed and require students to apply one or more steps to produce a number or transformed expression or equation. We might expect what we call here *procedural questions* to direct students' attention to worked examples within a text.

The Study

We conducted a study designed to identify variations in students' attention while reading mathematics with the goal of answering questions, specifically comparing the impact of a question focusing on the explanatory part versus one targeting the example part of a text. In particular, we were interested in whether answering a conceptual question results paying greater attention to the explanation part of reading material, and whether answering a procedural question results in greater attention to the example part. Seeking differences in how conceptual or procedural questions drive attention can shed light on how question types might affect the depth of their comprehension of mathematical ideas.

Constructing Even and Odd Functions

A given function $f(t)$, defined in the set of real numbers $\mathbb{R}$, can always be split into two parts, one of which is even and one of which is odd. To do this, write $f(t)$ as:

$$f(x) = \frac{1}{2}[f(t) + f(-t)] + \frac{1}{2}[f(t) - f(-t)]$$

The term $\frac{1}{2}[f(t) + f(-t)]$ is even because when t is replaced by $-t$, we have:

$$\frac{1}{2}[f(-t) + f(t)] = \frac{1}{2}[f(t) + f(-t)]$$

which is the same as the original.

However, the term $\frac{1}{2}[f(t) - f(-t)]$ is odd since, on replacing t by $-t$, we have:

$$\frac{1}{2}[f(-t) - f(t)] = -\frac{1}{2}[f(t) - f(-t)]$$

which is the negative of the original.

Example Suppose we need to split the function $f(t) = e^{-t} + \pi$ into even and odd parts.

The even part is:

$$f_{\text{even}}(t) = \frac{e^{-t} + \pi + e^{t} + \pi}{2} = \frac{e^{-t} + e^{t}}{2} + \pi$$

The odd part is:

$$f_{\text{odd}}(t) = \frac{e^{-t} + \pi - (e^{t} + \pi)}{2} = \frac{e^{-t} - e^{t}}{2}$$

Figure 1. The reading text: explanation on the top and example on the bottom.

Methods

Participants. We recruited 45 engineering and science students from various year levels through flyers, announcements, and class presentations. Participants received a £10 voucher as a gesture of gratitude for their contribution.

Materials. For the reading material, we adapted page 12 from *Helping Engineers Learn Mathematics* (HELM) Workbook 6, as shown in Figure 1. The HELM series, available online, was developed from 2002 to 2005 by a consortium of five UK universities led by Loughborough University and have been widely adopted. The page we used comprises two parts. The first introduces and explains the concept that every function can be written as the sum of two components: one symmetrical to the *y*-axis (the even part) and one symmetrical to the origin (the odd part). The second part illustrates how we can find the odd and the even part of a function. We call these the *explanation text* and *example text* respectively.

The first author developed two questions as shown in Figure 2. The conceptual question was intended to relate to the ideas presented in the explanation text, and the procedural question was intended to relate to the example text. We then developed two visual stimuli for eye-tracking. Both contained explanations and example texts. On the right of this text, one of the mathematical questions was presented.

Conceptual Question

Suppose $f_{even}(t)$, and $f_{odd}(t)$ are respectively the even and the odd part of a function $f(t)$ defined in the set of real numbers.

If $f_{even}(\pi) = f_{odd}(\pi) = 2023$, then $f(-\pi) = 0$.

a) Correct

b) Wrong

Procedural Question

To find the odd part of the function $f(t) = t^3 + 3^t$, we write:

$$f_{\text{odd}}(t) = \frac{1}{2}\left(t^3 + 3^t - \left((-t)^3 + 3^{-t}\right)\right) = \frac{1}{2}\left(t^3 + t^3 + 3^t - 3^{-t}\right)$$

Therefore:

$$f_{\text{odd}}(t) = \frac{1}{2}\left(3^t - 3^{-t}\right) + t^3$$

a) Correct

b) Wrong

Figure 2. The conceptual and procedural questions

Eye-tracking measures. Eye-tracking determines gaze point coordinates and eye movement velocity when a person views a visual stimulus (Bojko, 2013), which are then classified into fixations and saccades. Fixations are continuous low-velocity data points, while saccades are high-velocity sequences of eye movements between fixations. Researchers define regions of a visual stimulus to obtain eye-tracking metrics that address their research questions. These regions are called Areas of Interest (AOIs). The interpretation of these metrics is based on the Eye-Mind Assumption (EMA), which posits that eye movements and visual focus are aligned with attention and cognitive focus (Just & Carpenter, 1980).

Here, we used as our metric fixation counts, where a high fixation count serves as a proxy for increased attention directed towards an AOI (Skaramagkas et al., 2023; Strohmaier et al., 2020). We defined two AOIs: one for the explanation text and one for the example text (Figure 1).

Open-text questions. The post-eye-tracking questions included “Did you find the questions you had to answer complex or difficult?” and “Were the questions anyhow unexpected to you?”

Procedure. The study followed a within-subjects experimental design. Participants were randomly allocated into two groups. The participants in one group encountered first the visual stimulus with the procedural question, and then the visual stimulus containing the conceptual question. For the other group the sequence was reversed, i.e. conceptual then procedural question. Participants completed the eye-tracking and then the open-text questions one by one in an eye-tracking lab where the researcher unobtrusively observed through a second screen behind the participant.

Results

The distributions of fixation counts across question type (conceptual, procedural) and text type (example, explanation) are shown in Figure 3. As can be seen the data are skewed, with some participants not looking at some text types at all for a given question type (zero fixation counts), and a few participants with much higher fixation counts than most.

Overall, the conceptual question resulted in higher fixation counts than the procedural question, and this was the case for both the explanation and the example text. For the explanation text the median fixation count when participants were shown the conceptual question was 57, and when shown the procedural question the median was 24. For the example text these medians were 16.5 for the conceptual question and just 3 for the procedural question. Therefore, the conceptual question resulted in more attention in general than the procedural question.

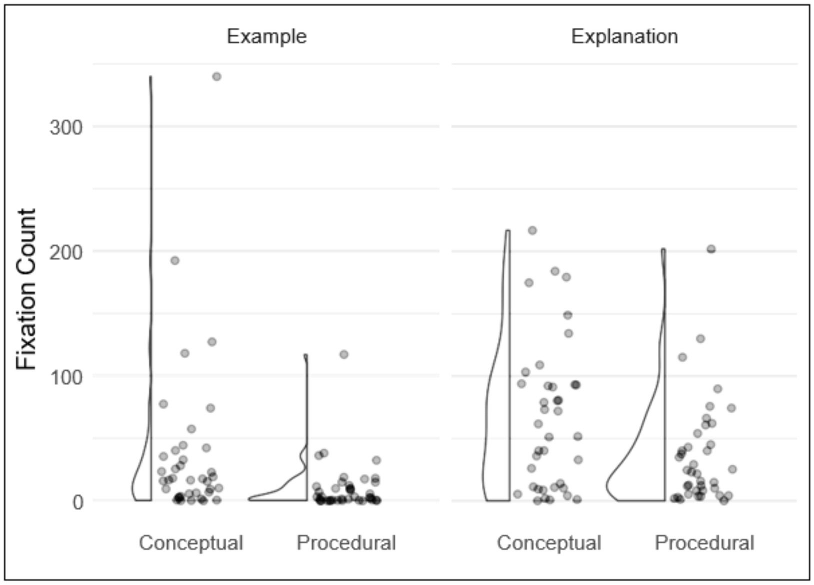

Figure 3. Jitter and raincloud plots of fixation counts. Each point shows the mean fixation count for each participant within each question type (conceptual, procedural) for each text type (example, explanation). The rainclouds represent the density of points along the fixation count axis.

These medians also show that the conceptual question followed the pattern we expected: higher fixation counts for the explanation text than the example text, and a paired Wilcoxon test revealed that this difference was significant, $W = 558.5$, $p = .002$. In other words, when presented with the conceptual question the participants paid more attention to the explanation text than the example text. However, the medians also contradicted our expectations for the procedural question that participants would pay more attention to the example than the explanation text. In fact, they paid more attention to the explanation text, and the difference was again significant, $W = 649.5$, $p < .001$. Therefore, for both question types, fixation counts were higher for the explanation text than the example text.

This greater attention on the explanation text might reflect that it is larger than the example text, as shown in Figure 1. We therefore considered the ratio of the median fixation counts. For the conceptual question the ratio of median fixation counts for the explanation to example texts was 3.5, and for the procedural question it was 8.0. Therefore, the procedural question resulted in attention being paid to the explanation rather than the example text relatively more than the conceptual question, again contradicting our expectations.

Discussion

We conducted an eye-tracker experimental study in order to understand how relevance manipulation might be productively applied to independent study materials. Specifically, we investigated how what we called conceptual and procedural questions might direct attention to the explanation and example parts of a mathematical text.

The conceptual question was associated with higher fixation counts on the explanation text than on the example text as we expected. However, many participants still paid reasonable attention to the example text when presented with the conceptual question. Part of the reason for this may be that participants were looking for a comparison example to help them answer the question. Some of the responses to the open-text questions supported this, with one participant stating, “There was not an example to compare with” and another stating “No example given for a case like that”.

We were surprised that the procedural question was also associated with higher fixation counts on the explanation text than the example text. We wonder if part of the reason is that the true-false format may have resulted in participants attempting to validate the question with reference to the explanation text. In other words, what we intended as a question requiring calculation may not have been perceived as such by the students. Some participants' open-text comments supported this, for example "I was expecting something to work out, not just mental thinking" and "There was no calculation".

Another consideration is the relative difficulty of the two questions. The participants in our study were more successful with the procedural question (51% answered correctly) than the conceptual question (21% answered correctly). There may be something more inherently difficult about conceptual questions and, as discussed earlier, this may arise due to procedural questions being more common in tests and exams. Consistent with this one participant commented "I have not encountered a question like this".

It is possible that the unfamiliarity some participants stated, which, as aforementioned, may have contributed to the question's perceived difficulty, also reduced their appreciation of the relevance of the explanation text. The conceptual question required participants to combine pieces of information from the explanation text and make inferences based on these, rather than directly referring to any single text segment that is immediately recognizable as corresponding. Some participants did not recognize any relevance in the specific parts of the explanation text that we intended as pertinent. Evidence of this appears in some participants' open-text comments, such as "The text did not mention such a scenario", and "Nothing similar in the text". Therefore, the ability to appreciate relevance when reading, particularly for answering conceptual questions, may need to be cultivated in undergraduate students.

Our findings also bring forward the complexity of designing questions to function as conceptual or procedural and the confounding role of difficulty. In a follow up study, we are using the method of comparative judgement (Jones & Davies, 2023) to investigate whether the conceptualness or proceduralness of a question are independent or belong to the same spectrum. We are also investigating the extent to which the relative difficulty of a question relates to conceptualness and proceduralness. This will enable us to better interpret the results reported here, and also to conduct future studies to investigate further the role of relevance manipulation in the design of mathematical texts and questions for students.

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