"Trust me, I am an Expert!": Combinatorial Problem Solving and Thinking Processes in Researchers With High Mathematical Preparation

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Combinatorics plays a key role in mathematics and its teaching: on the one hand, it represents a core in discrete mathematics while, on the other hand, in the school curriculum, it plays a role in the development of topics both in mathematics and in other disciplines (Batanero et al., 1994), offering students the opportunity to develop and train problem solving skills in an environment that does not rely on specific mathematical prerequisites (Kapur, 1970). In this sense, the research literature presents several studies that focus on the problem solving processes developed by students from elementary school (e.g., English, 1991) to university (e.g., Lockwood & Gibson, 2016). We note, however, that most of the research has focused on solvers without high combinatorial expertise. With our work, we propose a further step in the development of this line of research by focusing on combinatorial problem solving processes as developed by what we call experts, who in this work are to be considered as academics whose field of research and/or expertise is or includes combinatorics (e.g., statistics, probability, etc.).

Framing our research within Schoenfeld's model of problem solving (1985), the focus of our work is on analyzing the different phases that occur during the resolution of some combinatorial tasks, paying particular attention to the heuristics involved (based on the comprehensive description presented in Lamanna et al., 2022) and the dynamics that develop across the dimensions of combinatorial reasoning (formulas/expressions, set of outcomes, counting processes - Lockwood, 2013). We proposed an elementary task (i.e., solvable with a single application of a combinatorial formula) and a compound non-standard task (i.e., a task for which a closed direct solution formula does not exist) to observe how experts approach the two tasks and if/how they change and adapt their approaches between the two tasks. In summary, our research question is: Which heuristics and dimensions of combinatorial reasoning are involved by a group of experts when solving an elementary and a non-standard combinatorial task?

To answer our research question, we involved five experts in individual interviews in which they were proposed two combinatorial tasks to solve in a think-aloud process explaining each step of their reasoning. The results showed that in the case of the elementary task, experts tend to provide a solution that includes heuristics for direct counting of all configurations, thus involving only two of the three dimensions of combinatorial thinking and often skipping the problem solving phases of exploration of validation. On the other hand, when dealing with a non-standard task, experts rely on a broader set of heuristics (in particular involving enumerative processes), thus shifting the dynamics of their thinking towards the inclusion of all dimensions of Lockwood's model – which are often involved in the problem solving phases that were skipped in the solution of the elementary task. Comparing the two solving processes, we observe that while there are cases in which the experts seem to use efficient counting techniques, there are also situations in which they demonstrate an expertise that is built on a base that includes a variety of different heuristics – including enumerative processes that are often discouraged, especially in the case of high school combinatorics (Lamanna & Branchetti, 2024). We observe that the results of this work provide some key insights on the learning objectives that one should pursue to enhance a structured development of students' combinatorial thinking.

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