

Introductory Students' Definition and Views Regarding Proof

Christina Koehne
SUNY New Paltz

Anthony Cooper
SUNY New Paltz

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In order to meet students where they are, we must first understand where they are at. As Rouhani and Zazkis (2023) noted in their RUME paper, “the existing research on conception of what a proof is and what counts as a proof is primarily focused on expert mathematicians (e.g., Inglis, & Aberdein, 2016, Weber & Czoher, 2019)”. Complementary to Rouhani and Zazkis’s work on students’ conceptions about *proofiness*, we looked at how Introduction to Proof students defined proof and why they thought proofs were important.

Methods

The participants were undergraduate students in a public university, who were enrolled in an introduction to proofs course. Twenty students voluntarily agreed to participate and completed at least the initial reflective thought prompts: 1) How would you define or describe the concept of proof? 2) Why do you think proofs are important in mathematics? We merged the students’ initial and final reflective thought prompt answers to form our unit of analysis, since several students did not answer both the initial and final prompts. The authors independently analyzed and open-coded each entry before reaching an agreed upon final set of codes and themes. Independent coding was then revised before having a discussion to resolve differences.

Findings

These preliminary findings provide evidence that Introduction to Proof students reflected on four major components when defining proof and its importance: *Coherence*, *Community*, *Building Concepts*, and *Mental Processes*. Each of these four components is comprised of subcategories which will be discussed below.

In terms of *Coherence*, students primarily reflected on the importance of illustrating the validity of statements, the sequencing or logical ordering of statements, and argumentation. Although generalization, claims, and abstractions were mentioned, they were done by only a handful of students. Moreover, proof *coherence* was present for all students who reflected on communication to a *community*. However, the participants were not always explicit in the intent for communication; rather, they were vague in the notion that “You are proving...” for some purpose or would speak in terms of the collective “We”, such as, “...concepts that we already know to be true”. Additionally, of the students who mentioned using previous theorems, axioms, or definitions to help establish ideas, three (out of eight) did so without notions of communications. Similar to coherence, students would vaguely speak about the ideas of *Building Concepts*, which we noted as the regard of proof as a foundational component that mathematics is built upon or using known concepts to advance mathematics. Lastly, *Mental Processes* was prevalently seen as students noting the usefulness of proofs. However, several students also noted on their emotional experiences regarding proofs.

In conclusion, there is evidence to suggest that students find proofs important and useful for the foundation and expansion of the mathematical community knowledge. However, when it comes to communication and ideas of what a proof is, the notions, although present, could be more nuanced. Special attention in proof courses could be given for the importance of proof.

References

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