Assessing Mathematical Knowledge for Teaching College Algebra: Differences Between Community College Instructors and Math Majors Without Teaching Experience

Inah Ko
University of Michigan

Vilma Mesa
University of Michigan

Irene Duranczyk
University of Minnesota

The VMQI Research Team[1]
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This study compares mathematical knowledge for teaching college algebra at community colleges between instructors and math majors without teaching experience, aiming to show that the instrument assesses more than mathematical knowledge. We used an instrument designed to assess knowledge used in doing two tasks of teaching across three function types taught in college algebra, linear, rational, and exponential functions. The Multiple Indicators and Multiple Causes (MIMIC) model was applied to 416 community college mathematics instructors and 85 college students to estimate mean difference in the knowledge between the groups. The higher scores in the MKT-CCA suggests that instructors possess higher MKT than the students. As such, this instrument can be used to identify areas for focused faculty development.

Keywords: mathematical knowledge for teaching, college algebra, teaching experience, MIMIC

Our research group has been working on the development of an instrument to assess mathematical knowledge for teaching college algebra at community colleges. To measure this knowledge, we use an instrument organized by two tasks of teaching (understanding student work and choosing problems) with three function types (linear, exponential, and rational). An earlier study (Mesa et al., 2024) provided evidence supporting the validity of the inferences that can be made about instructors' knowledge, particularly, regarding its teaching-related features. The instrument provides a measure that differentiates the knowledge among community college instructors and the measure is correlated with both experience in teaching advanced college algebra and the frequency of doing a specific task of teaching-- choosing problems, a unique task teachers engage in on a routine basis.

To further support that the instrument assesses knowledge of teaching and not just mathematics, we compared the performance of two groups: math major students, who may have content knowledge but lack teaching experience, and community college instructors, who can reasonably be assumed to have both content knowledge and teaching experience. The analysis can further give evidence that the instrument is a viable tool for measuring this knowledge across the combined groups and to understand the characteristics of the knowledge it assesses. Specifically, we test whether the instrument can reliably differentiate knowledge levels between students and instructors, and whether instructors, those with teaching experience in the subject matter, perform better on the measured knowledge than students, those without such experience.

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[1] April Ström, Chandler-Gilbert Community College; Patrick Kimani and Laura Watkins, Glendale Community College; Mary Beisiegel and Camden Collins, Oregon State University; Claire Boeck, Rabia Kirikçilar, Inah Ko, Vilma Mesa and Melissa Valerie, University of Michigan; Bismark Akoto, Irene Duranczyk, and Dexter Lim, University of Minnesota.

After confirming the instrument's ability to distinguish knowledge levels among both groups, we compare their knowledge on a common scale. The significant difference in knowledge between instructors and students who lack teaching or tutoring experience provides additional evidence that our instrument measures an aspect of knowledge related to teaching. This led us to pursue the following question: How does the measured mathematical knowledge for teaching college algebra differ between community college instructors and college students who are majoring in mathematics?

Theoretical background and literature review

Central to the conceptualization of the instrument used in our work is that mathematical knowledge for teaching can best be elicited through the mathematical work embedded in activities that teachers do when they teach, and that are directly related to how they organize instruction (the interactions between content, students, and teachers) (Cohen et al, 2003). Herbst and colleagues developed an instrument measuring teachers' mathematical knowledge, structured by the types of work teachers do across different instructional situations (Herbst & Kosko, 2014; Ko et al., 2016). The data collected from their instrument have provided evidence that, at least in the context of teaching of high school geometry, it is possible to differentiate two distinct dimensions of the knowledge: one related to knowledge needed to choose givens for a geometry problem and another related to understanding students' work (Ko & Herbst, 2020).

The distinction of knowledge by different tasks of teaching is grounded on the concept of instructional exchange, which describes how knowledge is "transacted" in classrooms and the role of the teacher managing the exchange (Herbst & Chazan, 2011). Two key tasks – providing students with activities that would enable and support mathematical work and interpreting the work to assess student learning – represent a teacher's role in managing the exchanges. To manage this exchange effectively, teachers need to draw on their mathematical knowledge to make decisions guided by the instructional goals. By focusing on these instructional exchanges, Herbst and colleagues provide a unique lens for examining teachers' mathematical knowledge in key tasks of teaching that are applicable to any teacher, subject matter, and context.

As our instrument is grounded on specific activities that the instructors are expected to do as they teach, we ascertain this knowledge is unique to those who teach the specific content. Consequently, people who presumably know the content (because they took the courses and have been using that content for other more advanced math courses) but who do not have teaching experience with the content would not perform well on an instrument assessing such mathematical knowledge for teaching, particularly in the context of community college algebra instruction.

Research on mathematical knowledge for teaching has consistently documented that as teachers teach more advanced mathematical courses (e.g., middle teachers with experience with high school mathematics; community college teachers teaching courses beyond calculus), they perform better on instruments measuring mathematical knowledge for teaching than those teachers with experience only on the mathematical content assessed by the instruments (Ko et al., 2021; Hill, 2007). Thus, we hypothesized that people who knew the content, but had not taken too many advanced mathematical courses, would not perform well on an instrument assessing such mathematical knowledge for teaching community college algebra. Consequently, we anticipated that a sample of people who could be seen as having the knowledge of the mathematical content that the instrument assesses but without teaching experience on that content and without knowledge of more advanced mathematics, would score lower on our instrument compared to instructors teaching the content in community colleges.

Methods

For the comparison, we chose undergraduate mathematics majors and a sample of community college instructors. The student sample was recruited from 47 large, public, and four-year colleges across all states that offered a mathematics degree and that had a high percentage of admitted students. Students were eligible for participating in the project if they had no tutoring or teaching experience beyond middle school. Of the 191 eligible students, 149 started the questionnaire and 110 completed the instrument. Out of 110 participants, 85 students' responses were used in the analysis after confirming their eligibility for participation. The majority (57%) were between 18 and 22 years old, 51% identified as male and 42% as female, and a minority (13%) identified as Hispanic or Latino, while the majority (76%) identified as White. All the participating students indicated that they took a calculus course, and 80% of the students indicated that they took a college algebra course. The instructor sample was recruited from two-year degree granting institutions in the United States by inspecting community college websites and then sending direct invitations to instructors for participation in this project. Among the respondents, 556 instructors from 314 institutions participated in the project. Among the 556, 416 instructors completed the instrument, and their responses were used in the analysis. Of the 416 instructors, 48% identified as male and 46% as female; in terms of race, 76% identified as White, 11% as Asian, 4% as Black, 2% as mixed, and 4% chose Other. Seventy-eight percent of the participants said they held full-time positions; 9% were on tenure track. The average number of years of teaching experience was close to 17 years (mean = 16.76, SD = 9 years; range: 1.5 to 47 years). The majority (68%) held a master's degree and 20% held PhDs in mathematics, mathematics education or another mathematics-related field (about 5% were PhD in Mathematics Education).

For this study, we used items of the MKT-CCA instrument designed to measure mathematical knowledge for teaching used in two tasks of teaching (understanding student work, choosing problems) college algebra in three function types (linear, exponential, rational) (Mesa, Ko, Boeck, et al., 2023). The instructors and students completed the exact same items. Following the prior studies that validated the instrument regarding the dimensionality of the measured MKT (Mesa, Duranczyk, et al., 2024; Mesa, Ko, et al., 2024), we used a unidimensional model estimating the amount of MKT-CCA as a single score. However, given the error correlations among the testlet sub-items that have the same stem and the possibility of the effect of different item formats (multiple choice or testlet), we used only multiple-choice items (19 items) in this study. A sample item is presented in Figure 1.

Instructor Jenkins wants to use the following set of rational functions to show that attending to the structure of the symbolic representation of a rational function can reveal properties of the function.

$$f(x) = \frac{x^2+2x-3}{x-1} \qquad g(x) = \frac{x^2+2x-3}{x^2-1} \qquad h(x) = \frac{x^2}{x^2-1}$$

Of the options below, select the option that is an **INACCURATE** description of what this set of functions affords.

- The set illustrates the difference between functions that have vertical asymptotes and functions that have holes.
- The set illustrates rational functions with different ranges.
- The set illustrates that when the degree of the denominator is the same as the degree of the numerator then the function will have a hole.
- The set illustrates that some rational functions can be simplified to a function that looks like a linear function.

Figure 1. A sample MKT-CCA item involving Choosing problems in Rational function

To compare MKT-CCA scores between instructor and student groups, we applied the Multiple Indicator Multiple Cause (MIMIC) model that compares latent mean between the groups while detecting item bias. Specifically, in this model, we estimated the effect of the group variable on MKT-CCA while estimating the unidimensional MKT-CCA score for both instructors and students. The group variable indicated whether a sample is from a group of instructor (0) or student (1).

Results

Establishing unidimensional construct MKT-CCA

A one-parameter (1PL) and two-parameter (2PL) item factor analysis was conducted on the 19 items using the weighted least squares with mean and variance adjusted (WLSMV) estimator for the whole sample, including instructors and students. Both models fit the data well. In the 1PL model, factor loadings of all 19 items were set to be the same, while in the 2PL, they were freely estimated. The fit indices for 1PL and 2PL were as follows: 1PL (RMSEA=0.038, CFI=0.941, TLI=0.941l) and 2PL (RMSEA=0.011, CFI=0.996, TLI= 0.995). In all models, factor mean and variance was set to 0 and 1, respectively. All the analyses were performed using Mplus v.7.4 (Muthén & Muthén, 1998–2015). All item loadings were significant and greater than 0.3. A comparison of the 1PL and 2PL models using DIFFTEST suggested that 2PL is significantly better than 1PL (DIFFTEST(18) = 71.016, $p<0.001$), indicating that item discriminations are significantly different across different items. Consequently, in the subsequent analyses, we used a 2PL approach.

Comparing MKT-CCA scores between instructor and student groups

The MIMIC model fits the data well (RMSEA=0.021, CFI=0.979, TLI=0.977) and the result suggests that being in the student group has a significantly negative effect on MKT-CCA. Specifically, on average, the student group scored 0.40 standardized deviations lower on the

MKT-CCA than the instructor group. This effect size can be considered as medium according to Cohen's criterion (small, medium, and large for values of 0.20, 0.50, 0.80) (Cohen, 1988). The estimated model diagram is presented with standardized estimates in Figure 2.

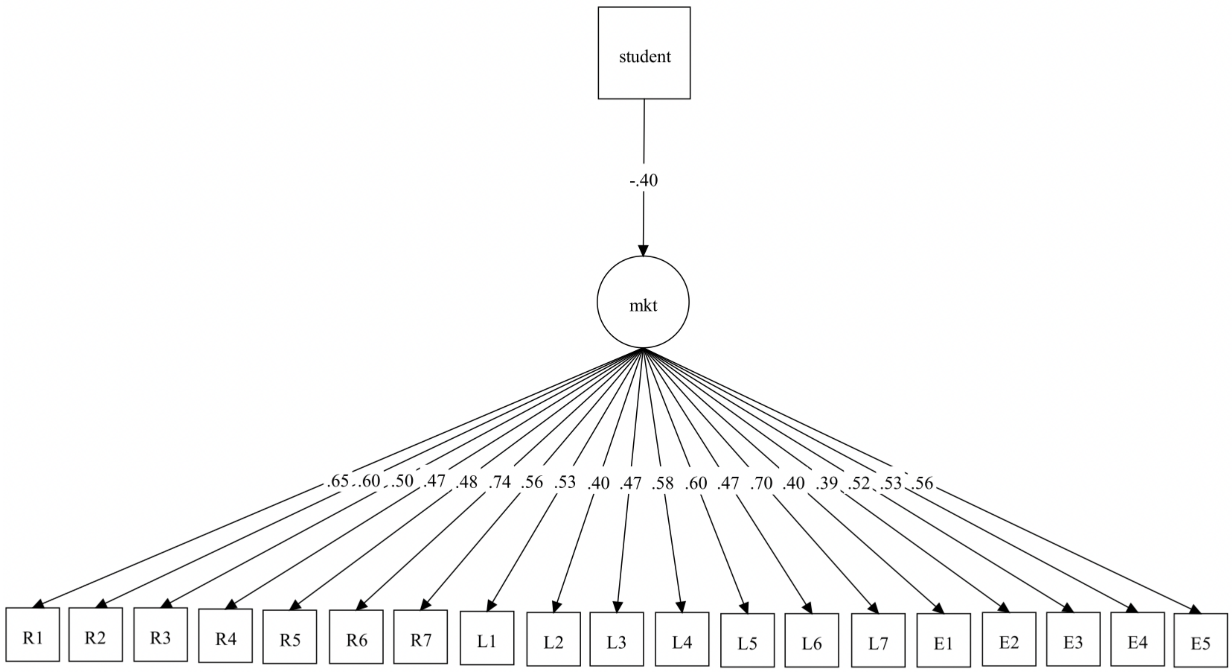

Figure 2. MIMIC model with student group indicator (standardized & significant estimates)

Identifying differential item functioning (DIF) items

Next, we used the MIMIC model to identify potentially biased items across the two groups in how each item measures the MKT-CCA. The difference in MKT-CCA score should stem from the differences in their MKT-CCA, not from items favoring one group over the other. In other words, items with significantly different item intercepts, even after adjusting for latent mean differences, indicate bias that may not relate to the knowledge commonly measured by the 19 items. We tested whether the two groups responded similarly to each item measuring MKT-CCA while controlling for their MKT-CCA. Those items showing differences after adjusting for latent mean differences are considered differential item functioning (DIF) items (Kaplan, 2008). This analysis showed that instructors are more likely to answer correctly than students for two items Item L7 and Item R1 after controlling for their MKT-CCA. Item L7 is an item about choosing problems in linear functions. The stem of the item stated a professor's intended goal for the students and asked participants to choose the option that presents a form of linear equation that will enable his or her students to achieve the learning goal. The correct answer is one form of a linear equation that is not as commonly introduced in textbooks as other forms like slope-intercept or point-slope form. The form is more likely to be found in advanced mathematics courses or specialized texts dealing with analytic geometry. Thus, students might be less familiar with the form than instructors, who have more experience teaching various courses related to algebra, and it might have caused students to score lower than instructors at the same level of MKT-CCA. Item R1 provides an example of incorrect student work and asks the respondent to select the option that is likely to be produced by the student who solved the problem. To choose the correct answer, the participant must be able to identify the error the student made in the example and choose the option that shows the same error. Thus, familiarity with similar kinds of errors or experience in seeing the error would help a participant in choosing the correct answer. As instructors have more experience in analyzing a variety of student errors, the experience may benefit them in getting a correct answer, even if their mathematical knowledge level is the same as a student's.

We conducted an adjusted MIMIC model that allows estimating the effects of the two items, which are suggested to have DIF, on MKT to account for group differences in the measurement of biased items. Specifically, we regressed MKT-CCA on the group indicator while also adding suggested regression paths from the group indicator to each of the two items. This approach allowed us to estimate the mean difference while also determining the remaining item variance explained by the group indicator. The estimated diagram with significant effects is presented in Figure 3.

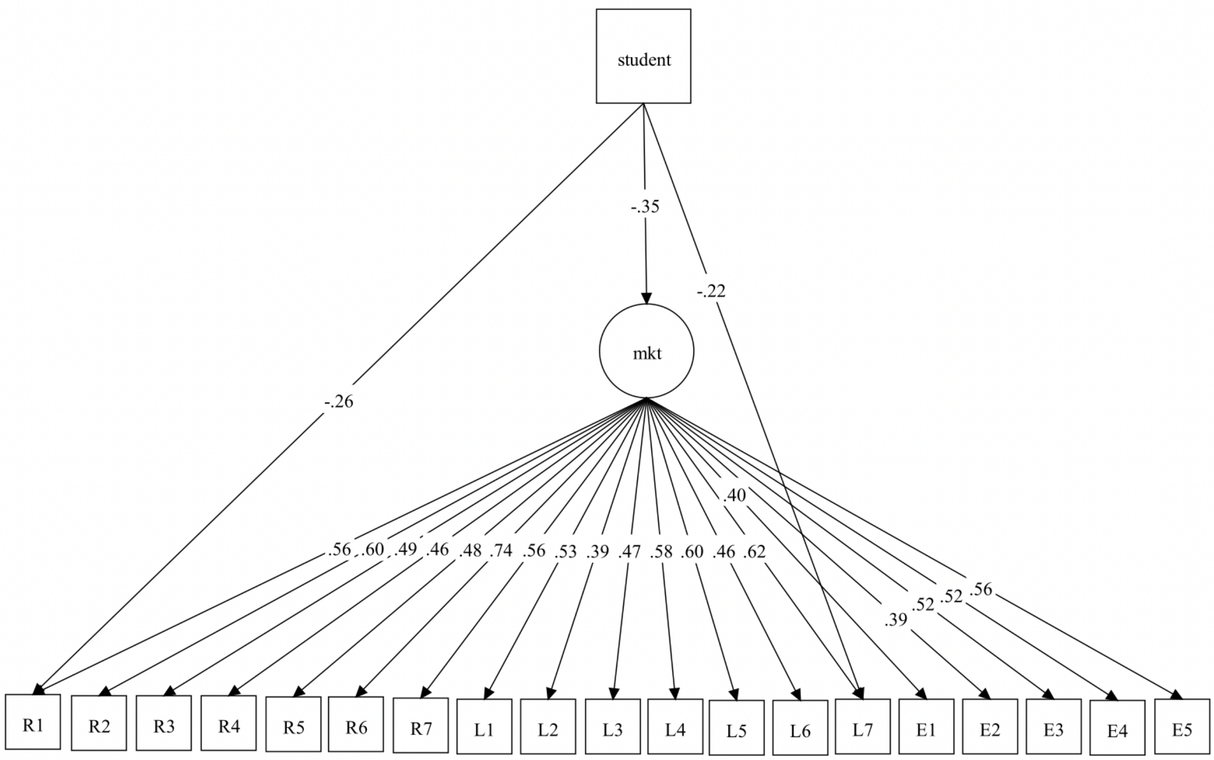

Figure 3. Adjusted MIMIC model with student group indicator and DIF effects (standardized & significant estimates)

The adjusted MIMIC model yielded improved fit statistics (RMSEA=0.012, CFI=0.993, TLI=0.993). As expected, considering the direction of DIF items, the estimated difference in MKT-CCA between instructors and students remained positive and significant (standardized beta=-0.35). This indicates that instructors tend to possess higher MKT-CCA than students, after controlling for DIF items.

Discussion

This study examined the differences in performance on developed instruments consisting of 19 items designed to measure mathematical knowledge for teaching college algebra in community colleges between community college mathematics instructors and college students.

We hypothesized that students would score lower because the participants in the student sample included in this study did not have teaching or tutoring experience. The results confirmed this hypothesis – estimated MKT-CCA was lower for students, and this pattern was consistent across all items. Our hypothesis was based on the characteristics of the knowledge construct we intended to measure through our instrument. As our instrument items were designed to capture the mathematical knowledge used in teaching mathematics, higher scores for instructors with teaching experience imply that the knowledge measured by our instrument is related to teaching. However, it remains unclear whether the instructors' higher scores are due to knowledge acquired from teaching experience or from taking more advanced mathematics courses. Future analysis could examine item-by-item response patterns among instructors and identify qualitative features of the items significantly affected by teaching experience versus those affected by advanced mathematics coursework.

In this study, we applied a MIMIC model rather than multiple group models given that it had a lower sample size requirement, and it is suggested to be an efficient way of comparing latent constructs with adjusting item intercept bias (Diemer, 2023). However, as we did not apply a measurement invariance model that tested the extent to which each of the different components of item estimates across groups, including invariance in factor structure and item discrimination, there is a possibility that we are missing some differences in how each item works for the groups regarding the strength in discriminating the knowledge. In a future study, we will apply a full measurement invariance model and test equal factor loadings for the groups and examine whether the model provides the same result.

By providing a tool that can be used for both instructor and student (including graduate students interested in teaching) populations, our study contributes to the field in identifying potential areas for interventions, training, and curriculum development to enhance not only current community college instructors' mathematical knowledge for teaching college algebra, but also that of students, who might eventually be interested in teaching mathematics at the community college or possibly at other post-secondary institutions.

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