Undergraduate Students' Nature of Mathematics in an Introduction to Proof Course

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What is mathematics? This question, and the study of the nature of mathematics more broadly, are of interest to many mathematicians. However, many universities do not include dedicated coursework to studying the nature of mathematics. Reflecting on the nature of mathematics is important for both personal development as a mathematician (i.e., Hersh, 1998; Lakatos, 1976), as well as for the development of mathematical concepts itself (i.e., Ernest, 1985; Lakoff & Nunez, 2000).

Transition to proof courses offer a time for students to reflect on the nature of mathematics due to the shift from computational calculus courses to more abstract proof courses. Boyle and colleagues (2015) argued a transition to prof course may be the first time students are asked to become creators of mathematics rather than reproducing what they have previously seen. This course presents an opportunity to study what views these students hold of the nature of mathematics. In this study, I asked students in an introduction to proof course to respond to the prompt, "In your own words, what is math?" This question was asked multiple times through the semester after several intentional activities to force reflection were completed. Ten students responded to three prompts each over the course of the semester. Each batch of 10 responses were open coded for common responses. Nine of the ten students were math majors in their third or fourth years of the major, all having completed the calculus sequence. The remaining student was a computer science major with a mathematics minor.

As of the writing of this poster, the final writing prompt is being given to student participants. Initial results indicate an overwhelming majority (8 of 10) of students initially viewed the nature of mathematics as simply a "toolbox" to be used for other disciplines, or for problem solving. Seven of the 10 students remarked they had never formally considered the question "What is mathematics" before answering this prompt. Halfway through the semester when the question was asked again, responses started shifting slightly away from a toolbox approach to include more reasoning for validation or to answer questions mathematicians themselves ask.

Students were asked to reflect on what contributed to their evolving understanding of the nature of mathematics. Two students mentioned watching YouTube series, while the other students mentioned working with proofs in a general sense to shift their nature of mathematics. By February, I will have analyzed the third group of responses to see how this shift across the semester continues.

In this poster session, I hope to gain insight from attendees into what aspects of this research are most convincing, and any future directions such as intentional readings to include or classroom activities to include to help foster growth in the nature of mathematics.

References

Boyle, J. D., Bleiler, S. K., Yee., S. P., & Ko, Y. –Y. (2015). Transforming perceptions of proof: A four-part instructional sequence. *Mathematics Teacher Educator*, *4*(1), 32-70. https://doi.org/10.5951/mathteaceduc.4.1.0032

Ernest, P. (1985). The philosophy of mathematics and mathematics education. *International Journal of Mathematical Education in Science and Technology*, *16*(5), 603-612. http://10.1080/0020739850160505

Hersh, R. (1997). What is mathematics, really? *Mitteilungen Der Deutschen Mathematiker-Vereinigung*, *6*(2). http://10.1515/dmvm-1998-0205

Lakatos, I. (1976). Proofs and refutations. *The British Journal for the Philosophy of Science*, *14*(53), 1-25.

Lakoff, G., & Nunez, R. (2000). Where mathematics comes from: How the embodied mind brings mathematics into being. Basic Books, New York.