

Understanding-Oriented Lectures in Calculus:
Practical Strategies for Enhancing Students' Mathematical Learning

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Many instructors observe that students often rely on memorization in introductory college mathematics. However, existing literature on effective teaching strategies does not fully address the need to cultivate both conceptual understanding and deeper orientations towards understanding in students' learning. This study presents a case analysis of calculus lectures delivered by an experienced instructor at an R1 institution, revealing significant opportunities for enhancing conceptual learning and promoting stronger orientations towards understanding among students. The analysis identified four key strategies employed by the instructor: 1) framing instructional activities as a pursuit of understanding; 2) examining underlying meanings while connecting students' intuition; 3) providing motivations, justifications, and multiple perspectives; and 4) creating cognitively challenging tasks and encouraging students to embrace discomfort in learning. These simple, practical strategies do not impose significant demands on instructors yet hold great potential for helping more students approach mathematics through deep understanding.

Keywords: Conceptual Understanding, Orientations, Calculus, Lectures

Effective teaching in undergraduate mathematics classrooms is often associated with clear explanations, approachable instructors, prompt feedback, high expectations, fair assessments, interactive teaching methods, well-organized course structures, and the use of relevant examples and applications (Chickering & Gamson, 1991; Harper et al., 2019; Mesa et al., 2015). While these strategies are crucial, they may not fully address the need to foster both conceptual learning and orientations towards understanding (Huang, 2022, 2023) in students' mathematical learning. Despite recent advances in undergraduate mathematics education research (Artigue, 2016), many instructors continue to observe that students tend to rely heavily on memorizing formulas and procedures in introductory college mathematics (Burn & Mesa, 2015). This reliance on rote learning undermines students' ability to engage meaningfully with mathematics and develop mathematical proficiency (Schoenfeld, 2022).

Such a persistent issue raises a critical question: How can we better support students in developing both conceptual understanding and understanding-oriented approaches to learning mathematics? Given that most undergraduate mathematics courses continue to be delivered in lecture formats (Artigue, 2016; Johnson et al., 2018), it is unlikely that this mode of instruction will disappear anytime soon. Rather than dismissing lectures entirely, this study seeks to identify simple, practical techniques that instructors can integrate into their daily teaching practices without imposing significant additional demands. While transformative teaching methods, such as inquiry-based instruction (Laursen & Rasmussen, 2019), have made significant contributions to the field, they often require substantial resources and institutional support for large-scale implementation (Weaver, 2016). This paper specifically explores opportunities within calculus lectures, delivered by an experienced and well-recognized instructor at an R1 institution, that promote students' conceptual understanding and understanding-oriented approaches to learning mathematics. These opportunities offer great potential for low-cost, large-scale implementation

that could effectively support more students in approaching mathematics through deep understanding and enhancing their learning outcomes.

Literature Review

Mathematics education researchers have conceptualized mathematical understanding in various ways (Hiebert & Carpenter, 1992; Perkins & Blythe, 1994; Piere & Kieren, 1994; Schoenfeld, 1985; Shapiro & Stolz, 2019; Sierpinska, 1994). In this paper, I integrate Hiebert and Carpenter's (1992) view of understanding as a well-connected web of knowledge with Schoenfeld's (1985) operational definition, which emphasizes the ability to solve challenging problems. This paper focuses on the first two strands of mathematical proficiency identified by the National Research Council (2001)—conceptual understanding and productive disposition—while acknowledging the importance of the other three strands: strategic competence, adaptive reasoning, and procedural fluency.

Schoenfeld (2010) introduces the notion of orientation, which includes beliefs, values, dispositions, and biases that fundamentally shape people's behaviors in well-practiced domains such as teaching and learning. Building on this work, Huang (2023) elaborates on the construct of *orientations with respect to understanding in mathematics*, defining it as “the degree to which students exhibit an inclination towards and demonstrate an earnest concern for understanding in mathematics learning” (p. 49). Huang (2023) develops a methodological tool for measuring this construct with established validity and reliability. According to Huang, understanding-oriented students “care deeply about how things work, prioritize goals and actions for developing understanding, and are willing to put in much effort to understand underlying principles” (2023, p. 49). It is important to note that an orientation towards understanding is closely associated with a growth mindset (Dweck, 2006, 2013; Dweck & Yeager, 2019).

Previous work on orientations with respect to understanding has focused primarily on students. However, to design interventions that support the development of students' orientations towards understanding, we must first describe the types of opportunities available in various learning structures (e.g., lectures, curricula, small-group interactions) and analyze the nature of these opportunities. As a first step, this paper provides a case study of a semester of promising calculus lectures that appear to offer substantial opportunities for both enhancing students' conceptual understanding of core calculus concepts and fostering strong orientations towards understanding in mathematics learning.

Methods

This study is part of a larger investigation into students' calculus learning in a large-enrollment, first-semester calculus course at a public R1 institution. The course consists of three hours of lecture and three hours of discussion per week over a 16-week semester. Lectures are typically delivered by an instructor in a large auditorium twice a week, while graduate teaching assistants lead discussion sections of approximately 30 students on other weekdays. Each semester, over 1,400 students enroll in the course. The larger study involves observing all lectured-based instruction and three randomly selected discussion sections weekly, conducting surveys on students' learning experiences, interviewing the course instructor and graduate teaching assistants, and carrying out three rounds of task-based interviews with 57 students to gain an in-depth understanding of their calculus learning in Fall 2023. The primary data sources for this paper are 48 hours of video recordings of lecture-based instruction, supplemented by my field notes. The course instructor is a tenured mathematician who has been teaching mathematics at the university since earning his Ph.D. 12 years ago.

I used grounded methods (Charmaz, 2006; Vollstedt & Rezat, 2019) to explore the opportunities in calculus lectures that promote students' conceptual understanding and foster their orientations towards understanding in mathematics learning. Given that the connection between mathematics lectures and students' orientations towards understanding is largely unexplored in existing literature, grounded methods allowed for an iterative and flexible process where empirical data significantly informed the development of new concepts and categories (Vollstedt & Rezat, 2019). To distill video recordings into analyzable segments, I created activity logs at 20-minute intervals, partitioned the recordings into episodes, and developed verbatim transcripts with notes on the mathematical ideas discussed. In the next phase, I wrote analytic memos for each transcript, documenting emerging themes that could support students' conceptual understanding or orientations towards understanding, along with their related mathematical discourses in the classroom. I also connected these themes to relevant literature in this phase. Based on an initial list of themes developed from the previous phase, I coded all the transcripts and trained an undergraduate research assistant to independently code them as well. Disagreements in coding led to discussions on how to distinguish different codes and cluster similar ones, prompting us to revisit all transcripts to refine our code list and definitions. The coding process was iterative, requiring us to re-analyze all transcripts as new codes or revised definitions emerged. We resolved all disagreements in the process. In the final phase, we conducted a comprehensive review of all video recordings and field notes to ensure that the emerging categories aligned with our initial impression of the data and captured the whole story. This review guided our revisions to the categories to ensure accuracy and thoroughness.

Results & Discussion

The analysis revealed four key strategies that the calculus instructor employed to foster students' conceptual learning and nurture their orientations towards understanding in mathematics. These strategies are: 1) explicitly framing instructional activities as a pursuit of understanding; 2) examining underlying meanings while actively connecting students' intuition; 3) providing motivations, justifications, and multiple perspectives; and 4) creating cognitively challenging tasks while encouraging students to embrace discomfort in learning.

Explicitly Framing Instructional Activities as a Pursuit of Understanding

A central strategy employed by the instructor was the consistent and explicit framing of instructional activities as a pursuit of understanding. By using phrases such as "let's try to understand" and "I really want to understand what's going on," the instructor signaled that the goal extended beyond procedural mastery to cultivating both deep conceptual understanding and orientations towards understanding in students' mathematical learning.

This emphasis was evident in a lecture on September 12, 2023, when the instructor encouraged students to think beyond simply obtaining the correct answer. He remarked, "Many of you might know the answer, okay? But that's not the point here. *I want to really understand* how the answer relates to the definition and how we could maybe solve other problems based on this example" (emphasis added). This framing aligns with Huang's empirically based characteristics of students strongly oriented towards understanding, such as "spending time understanding confusing ideas even after getting an answer" (2023, p. 51).

In another lecture on September 21, 2023, the instructor challenged students to understand the limit of $\arctan(x)$ as x approaches infinity without relying on the graph of $y = \arctan(x)$. He said, "*I really want to understand* what's going on geometrically. I would like to get an explanation of what this limit is, without having to know the graph of the function. Based on

what you've learned about how we typically deal with trig expressions, what is something we can draw *to understand this?*" (emphasis added). When a student suggested drawing a right triangle, the instructor responded, "Very good idea, *let's try to understand* what $\arctan(x)$ is in terms of a right triangle. How can we construct a right triangle using the given information?" (emphasis added). The conversation led to the creation of a visual representation, shown in Figure 1, which the class then used to explain what happens when x approaches infinity.

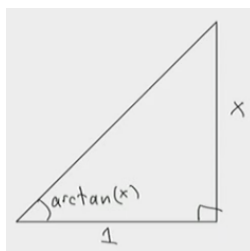


Figure 1. Understanding $\arctan x$ with a triangle

This deliberate focus on understanding was also evident in a lecture on October 10, 2023, before introducing logarithmic differentiation. The instructor invited students to explore how logarithms might help finding the derivative of $f(x) = (x + 1)(x + 2)^5(x + 3)^7(x + 4)^9$ and to understand why the strategy worked. He emphasized,

I want to understand why this is working. This looks like magic. It's so much easier than the product rule—but if something in math seems like magic, that just means we don't understand it well enough. So let's really try to understand what's going on here and why this method is working. To make sure we understand this method, let's go one step further and really see what's happening. (emphasis added)

The consistent and explicit framing of instructional activities as a pursuit of understanding fostered a learning environment where students were encouraged to engage deeply with the material, thinking critically, and build a strong conceptual foundation in mathematics. This approach aligns with what Huang (2023) observes to be key indicators of strong orientations towards understanding, supporting students in viewing mathematics as a sense-making activity and approaching mathematical learning through more understanding-oriented methods.

Examining Underlying Meanings While Actively Connecting Students' Intuition

The second strategy employed by the instructor involved examining underlying meanings of mathematical concepts while actively connecting with students' intuition. For instance, in a lecture on October 17, 2023, the instructor modeled digging deep into the underlying meanings of complex formulas and searching for intuitive explanations. He remarked:

I want to think about what this means and how we could explain this formula in a more intuitive manner. Yes, you could use the methods from the previous class to derive this formula, but it's complicated and maybe not very intuitive. So I want to show you a way to think about it in terms of differential equations—you'll see that *this actually makes sense* and that it has to be the result. (emphasis added)

This approach aligns with Bruner's (1960) notion of uncovering "fundamental structures" in

learning. By focusing on the underlying meanings of mathematical concepts, the instructor fostered meaningful connections and supported students in seeing ideas through core disciplinary principles, both of which are crucial for developing mathematical expertise (Bransford et al., 2000; Hiebert & Carpenter, 1992; Schoenfeld, 1985). The practice of searching for intuitive explanations made abstract concepts accessible to students while nurturing their mathematical intuition, leading to more connected learning and the development of flexible mathematical thinkers (Agarwal & Sengupta-Irving, 2019; Fischbein, 2005; Schoenfeld, 2020). In the next example, the instructor clarified the meaning of derivatives using Leibniz’s notation and connected it to students’ visual and physical intuition in a lecture on October 5, 2023. He said:

Let me say one more interpretation of *what this actually means*. The idea here is that dy/dx is a quotient. We can think of dy/dx as very small quantities, like Δx or Δy . Imagine you’re standing on the point (x, y) ; then dy/dx tells you how much you need to change the y-coordinate slightly if you change the x-coordinate by one infinitesimal unit. (emphasis added)

This explanation exemplifies how the lectures supported both genuine sense-making and conceptual understanding of abstract concepts. Relating derivatives to students’ common experiences, such as standing on a point and looking towards a particular direction, aligns with Boaler et al.’s (2016) work that emphasizes making abstract mathematical ideas tangible and relatable. Additionally, the instructor’s use of phrases such as “I want to give you a bit more intuition of what this means,” “we should see what it means visually,” “I would like to find, in the real world, intuition for this theorem,” and “put all the theorems apart; just think about your intuition—what would it say?” underscores his commitment to connecting mathematics with students’ repertoires of knowledge (Bernstein, 1996) and fostering genuine sense-making of challenging concepts.

Providing Motivations, Justifications, & Multiple Perspectives

The third instructional strategy centered on offering motivations, justifications, and multiple perspectives to enrich students’ conceptual understanding and encourage deeper engagement with mathematical ideas. Research suggests that presenting the motivations behind definitions, theorems, and proofs enhances students’ ability to see how these ideas fit into the broader mathematical landscape, fostering more meaningful connections and deeper reasoning (Abramovich et al., 2019). In a lecture on September 14, 2023, the instructor exemplified this approach by motivating the formal definition of limits as well as its core components. He explained:

I’ve explained the limit by saying that $f(x)$ is approximately equal to L as long as x is sufficiently close to a . But this is not a mathematically precise definition. This is not how you can base your mathematical theory on. We want to have a definition that, whatever you prove later on based on this definition, is going to be 100% true, guaranteed. So we need to make this more precise. First of all, we should probably talk about what we mean by *approximate*. Well, we can think of an error in an approximation, which is basically the difference between $f(x)$ and L , which is just $|f(x) - L|$. The smaller the error, the better the approximation. Then we also need to precisely define what is *sufficiently close*. Well, closeness—we talked about this in relation to x being approximately a . We could

also call it approximation, but somehow it's often called closeness. It's the distance between x and a , which is $|x - a|$. Okay, these are intermediate steps. Now we are ready to make a mathematically precise definition of limits.

By breaking down the abstract concept into comprehensible parts and providing step-by-step rationales, the instructor transformed a potentially daunting definition into a logical sequence of ideas. This process of demystifying abstract concepts helped students appreciate the rigor and necessity of precise definitions, which are crucial for developing positive disciplinary identities and a sense of belonging to the broader mathematics community (Huang, 2024).

Moreover, the instructor consistently emphasized the importance of justifying ideas and understanding them from multiple perspectives. For instance, in a lecture on October 3, 2023, after algebraically proving the product rule and connecting it to more basic principles, the instructor proceeded to justify the rule geometrically. This approach aligns with research showing that connecting different mathematical representations—such as algebraic and geometric—reinforces students' conceptual understanding and encourages flexible thinking in mathematical problem-solving (Schoenfeld et al., 2023). The instructor reiterated the significance of multiple perspectives in a lecture on October 31, 2023, stating: “As I always say, it's always good to get several perspectives on the same problem because sometimes some perspectives are more useful than others. And that's the case here. So let me explain, one more time, why this limit is 2, using a different method.” This emphasis on multiple perspectives and justifications reflects evidence-based practices in mathematics education, where students benefit from seeing problems approached in multiple ways (Schoenfeld et al., 2023).

Creating Cognitively Challenging Tasks & Encouraging Embrace of Discomfort

The fourth instructional strategy involved designing cognitively challenging tasks and encouraging students to embrace the discomfort inherent in deep learning. Research shows that engaging with tasks of high cognitive demand fosters deep learning and robust understanding (Schoenfeld, 2017). The instructor implemented this strategy by regularly presenting tasks that pushed students beyond their comfort zones, leading to some concerns about the difficulty of assignments. In a lecture on September 14, 2023, the instructor addressed these concerns:

There are problems on the homework where you cannot apply the solution strategy that I told you, where you have to think deeper and somehow come up with your own solution strategy based on the intuition that I try to convey in lectures. It's not that we're giving you homework problems and we kind of forgot to give you the solution strategy. We do it on purpose. These are problems where you will actually learn something because you really have to go back to the material and learn the concepts—even though, of course, they may take a much longer time. But the goal should always be to learn the concept, not just finish the exercise.

This response illustrates the instructor's deliberate intent to challenge students and promote deep engagement with mathematical concepts. By designing tasks that required students to think deeply and critically, the instructor supported students' robust conceptual understanding.

In addition to presenting challenging tasks, the instructor emphasized the importance of embracing discomfort as a natural part of the learning process. In a lecture on August 24, 2023, he remarked:

Mathematics can be tough, and it's normal to feel frustrated when you're trying to understand an abstract concept or working through complex problems. Don't shy away from that discomfort; instead, use it as a signal that you're pushing the boundaries of your understanding. Sometimes, the most significant learning happens when you persist.... Some students work on hundreds of problems but fall short of expected success because they don't try to really understand the main concepts. Doing more and more problems is sometimes easier than getting at the root of a misunderstanding. My recommendation to you is to embrace the feeling of unease—this is when you really learn.

This emphasis on discomfort as a catalyst for growth aligns with Dweck's (2006, 2013) work on growth mindset, which suggests that viewing challenges as opportunities for growth can significantly enhance students' learning outcomes. By framing discomfort as a natural and valuable part of the learning process, the instructor cultivated a classroom culture of resilience and growth, fostering stronger orientations towards deep understanding (Huang, 2023) among students.

Concluding Remarks

This paper has examined the opportunities within calculus lectures, delivered by an experienced and well-recognized instructor at an R1 institution over a semester, that promoted students' conceptual learning and orientations towards understanding. The findings highlight four key strategies that the instructor used to achieve these goals: 1) explicitly framing instructional activities as a pursuit of understanding; 2) examining underlying meanings while actively connecting students' intuition; 3) providing motivations, justifications, and multiple perspectives; and 4) creating cognitively challenging tasks while encouraging students to embrace discomfort in learning. Importantly, these strategies are not isolated but can be employed simultaneously within lectures and potentially other instructional structures.

This study contributes to the broader discourse on mathematics education by demonstrating that, even within traditional lecture formats, there are opportunities that instructors can leverage to support more meaningful and understanding-oriented mathematics experiences for students. Notably, the identified strategies do not require significant changes to existing instructional structures or significant time commitments from instructors. Instead, they are practical, low-cost techniques that instructors can easily learn and incorporate into their everyday teaching practices.

While this research focuses on calculus instruction in a large-enrollment setting, the strategies identified have broader implications for undergraduate mathematics education. Whether delivering lectures, following inquiry-based instruction, or employing a flipped classroom model, instructors can adapt these strategies to their contexts. These strategies offer a pathway towards transforming students' mathematical experiences by fostering both robust conceptual learning and stronger orientations towards understanding, which can directly address the concern raised by Burn and Mesa (2015) that students often rely heavily on memorization in introductory college mathematics. Future research could explore how these strategies can be scaled across different mathematical contexts and course structures, as well as how institutional support can be leveraged to sustain these instructional improvements. By implementing these strategies and cultivating a classroom culture that prioritizes understanding, we can help students develop deeper mathematical understanding and move away from unhealthy habits of memorization in favor of meaningful learning.

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