Derivative Process-Object Preferences of Post-Secondary Teachers

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Derivatives are often considered the central part of introductory calculus (Kidron, 2019). While there has been consideration of how non-mathematicians use the derivative differently than their mathematical colleagues (Dray et al., 2019), there has been no survey data to aggregate these differences over many respondents. This study begins to explore how such data can be gathered and analyzed using a survey generated by the researcher to determine the process-object conception of the derivative (Dray et al., 2019; Zandieh, 2000) favored by various post-secondary teachers. In its current form, the survey distributed showed few significant differences between favored process-object levels in the framework of Dray and colleagues (2019). Survey improvements are discussed.

Keywords: calculus, derivatives, STEM education

Calculus is often used as a "gatekeeper" or "filter" for STEM majors (Bressoud et al. 2015). In other words, students must first pass calculus before entering upper-level non-mathematics classes. Ostensibly, this is to ensure that students are prepared to use calculus concepts in non-mathematics classes. However, researchers have found that calculus may not be fulfilling this preparatory purpose, with many students seeing their mathematics classes as disconnected from their future careers. (Black & Hernandez-Martinez, 2016; Brown et al., 2008; Di Martino, 2019; Hernandez-Martinez & Vos, 2018; Rellensmann & Schukajlow, 2017; van der Wal et al., 2017; Vos, 2018). Graduates of various engineering programs have "perceived their mathematics courses as islands, with no relation to the rest of their education" (van der Wal et al., 2017, p. S98). Similarly, others have noted that "For too many students, [calculus] is either an insurmountable obstacle or—more subtly—a great discourager from the pursuit of fields that build upon the insights of mathematics" (Bressoud et al., 2015, p. v).

Many studies have explored how to improve calculus instruction, most notably the 2015 Mathematical Association of America (MAA) national study of college calculus (Bressoud et al., 2015). These studies have often focused on instructional methods, with less emphasis on aligning content with the needs of non-mathematics classes. Some researchers have sought to understand how different professionals understand and use derivatives (e.g. Dray et al., 2019), but there have been few large-scale examinations of the needs of students entering non-mathematics classes. A notable exception to this was a MAA workshop in 1986 to discuss how calculus could better serve students pursuing non-mathematics majors (Douglas, 1986). However, with more recent research showing students struggling with mathematics concepts in their major coursework, more work is needed to refine calculus curriculum to better meet students' mathematical needs.

The goal of this study was to pilot a survey to determine the aspects of the derivative most valued by teachers of non-mathematics courses and to compare those data with those of the mathematicians teaching calculus courses. The survey focused on the derivative since the derivative is the central concept covered in introductory calculus classes (Kidron, 2019). If this survey can be refined, validated, and deployed to a national or international audience, the results could be used to offer recommendations for how calculus curriculum could be improved to better serve all students.

Theoretical Framework

This survey was designed to align with the framework for representations of the derivative developed by Dray et al. (2019). These researchers sought to design a framework to describe the differences that had been observed in how mathematicians and non-mathematicians use the notion of a derivative outside of an introductory classroom context. This framework was itself an expansion of a framework developed by Zandieh (2000) to organize the facets of student understanding of the derivative.

The redevelopment of Zandieh's (2000) framework by Dray and colleagues (2019) stemmed from observation made by the researchers while mathematicians and non-mathematicians were asked to find a derivative on a "partial derivative machine." Using this machine, participants could adjust strings on one part of the machine to change the position of weights on another part of the machine. Participants were asked to find a derivative on the machine. The non-mathematicians suffered no difficulties in doing so and were able to quite easily fulfill the prompt. Mathematicians, however, struggled to develop a limiting process that could be used to replicate the limit definition of the derivative.

This difference led Dray and colleagues (2019) to define the notion of a "thick derivative," where non-mathematicians use "good enough" approximations as stand-ins for the theoretical limit process. This notion of a thick derivative emphasized how scientists use an average rate of change over a small change in an input value to approximate an instantaneous rate of change, something antithetical to the mathematicians who worked on the partial derivative machine in Dray and colleagues' study.

This notion of a thick derivative (Dray et al., 2019) influenced their modification of Zandieh's (2000) framework for understanding of the derivative. In both frameworks, the concept image (Vinner, 1983) of the derivative held by a person is divided into multiple process-object layers as well as different representations. The process-object layers represent how, as initially noted by Sfard (1991), a mathematical process is reified into an object that is later used by a person in other mathematical processes. In the case of the derivative, in the various representations of the derivative, a rate of change is first found using a ratio of changes in an output variable to those in an input variable. This process is then reified into an object of which a limit is taken to find an instantaneous rate of change. The result of this limit process is what mathematicians often term a derivative (Dray et al., 2019). These limits are reified into an object that is then used to generate derivative functions. Thus, this derivative understanding framework breaks the derivative down into ratio, limit, and function layers. These layers and then used within different representations of the derivative to understand and solve various problems. Dray and colleagues included five representations of the derivative in their framework, namely graphical, verbal, symbolic, and numerical, and experimental representations. Zandieh (2000) had only four representations, namely the graphical, verbal, symbolic, and physical representations.

Dray and colleagues argue that scientists use an approximation of the limiting process to generate a thick derivative that can be used as part of their work (2019). Thus, instead of a derivative only being defined at the "limit" layer of their framework, they argued that the *ratio* layer, over a small enough change in the input variable, is sufficient to produce a thick derivative that can be used in applications. As such, they argued for an increased focus on reasoning with differentials in calculus classes. This survey aimed to validate this call as well as use their framework to better understand what aspects of the derivative are most valuable for non-mathematics majors.

Research Question

This study sought to begin answering the question of whether there is a difference between how mathematicians and non-mathematicians value different process-object layers (Dray et al., 2019; Zandieh, 2000) of the derivative.

Significance of this Study

As noted by Dray and colleagues (2019), scientists may place their concept image of a derivative within the ratio layer of their framework, rather than in the limit layer as is done more formally within mathematics textbooks. If respondents in this survey produced placed more emphasis on the ratio and function layers, this would lend credence to the arguments of Dray and colleagues that a greater emphasis should be placed reasoning with differentials rather than with limits when discussing derivatives. Additionally, resulting differences between subject areas could inform institutional decisions on developing siloed or major-specific calculus classes, an initiative that has seen some success in increasing engagement and persistence in calculus classes (Ellis et al., 2021; Klingbeil & Bourne, 2013; Luque et al., 2022; Voigt et al., 2020).

Methods

This study was carried out using a survey designed by the researcher. On the survey, respondents were asked to name their primary subject area (i.e. mathematics, physics, chemistry, etc.) and then to respond to several Likert-scale items rating the importance of various items an instructor may find important for a student to understand at the end of an introductory calculus course. On each item, respondents could rank the item as, "Not at all important," "Slightly important," "Moderately important," "Very important," or "Extremely important." The items were broken up into sections according to the representations of the derivative discussed in the framework of Dray et al. (2019). Within each of these sections, there was one item corresponding to each process-object layer of the derivative. This resulted in five questions corresponding to each of the process-object layers within the framework. For each respondent, the five questions for each subject layer were then averaged to create a score on their value for each of the ratio, limit, and function process-object layers. These scores were considered scale variables, as they came from combined Likert-style questions (Field, 2018).

Data Analysis

I examined the relationship between subject area and the value respondents ascribed to the different process-object layers. I treated subject area as the independent variable, with the three different process-object layer scores as dependent scale variables. These variable types led to data being analyzed using multivariate analysis of variance (MANOVA) (Field, 2018; Huck, 2012). To find significant differences between responses of individual subject areas, Tukey's post-hoc test was used. Additionally, a one-way analysis of variance (ANOVA) was conducted to determine if there were significant differences between the values respondents in general ascribed to the three different process-object layers, as well as within specific subject area groups.

Participants

Wyoming post-secondary faculty were surveyed as part of this study. Specifically, faculty in majors requiring calculus were invited to participate. Although this does not represent a national random sample, Wyoming colleges all conduct nationwide hiring searches that require faculty to have similar educational credentials as those for other colleges throughout the United States

(Higher Learning Commission, 2019). Additionally, Wyoming state law requires that colleges throughout the state have identical course names and course outcomes (Common College Transcripts, 2018), ensuring that instructors from similar disciplines but at different institutions would have similar prerequisite calculus knowledge required for their courses.

Results

After the survey responses were cleaned to remove incomplete responses, there were 75 complete responses that were analyzed as part of this study. Since there were only one or two respondents from some of the original subject areas included on the survey, respondents were grouped as is seen in table 1.

Table 1. Respondents Grouped by Subject Area

Subject Area	Number of Respondents
Mathematics	23
Physics & Chemistry	8
Biology	6
Engineering & Computer Science	12
Other	26
Total	75

Reliability Results

Table 2 shows the reliability scores on each of the sets of items used to assess the ratio, limit, and function layers. As is seen in the table, the values for Cronbach's alpha were relatively high for each layer, suggesting high internal reliability in the results.

Table 2. Reliability Results

Layer	Cronbach's Alpha
Ratio	0.769
Limit	0.848
Function	0.876

Test Results

Based on Roy's Largest Root, the results of the MANOVA were statistically significant, showing that there is a difference in the value of the three different process-object layers by teachers of various subject areas, $V = 0.294, F(4, 70) = 5.146, p = 0.001$. However, when results were broken down using Tukey's post-hoc test, the only significant difference found between subject are groups were between the "other" group and the mathematics group on each layer. When one-way ANOVA tests were run on the overall ratings for each process-object layer, the results were not statistically significant. This lack of significance suggests that overall, teachers value each of these three layers equally.

Discussion

Although the MANOVA yielded a statistically significant result, the small number of significant differences between individual subject groups is disheartening. However, as this was a pilot study for the survey used, the results suggest several ways the survey can be improved. Additionally, the results that were significant offer clues to how this research could be applied.

Survey Improvements

As noted previously, this study piloted the survey used. Since the intergroup significant values were found only between the mathematics group and the “other” group, the options for subject area on the survey need to be expanded. Some respondents to the survey noted in a comment section that subjects such as economics, geology, and chemical engineering were not included in the suggested subject areas. The intent of the survey was to have respondents who marked “other” on the survey state their subject area in a text box, but as no respondents filled in that text box, this feature was not employed properly.

There is also no validity data for this survey at this point. Interviews should be conducted with respondents to the survey to make sure respondents understand the intent of the questions. These cognitive interviews would help improve questions to better assess which of the layers the respondents value more. Additionally, in discussions with some other researchers, there are concerns that the survey questions may not have aligned as closely with the framework of Dray et al. (2019) as was originally intended. Refinements to questions will help better delineate differences in values in these layers.

Most importantly, a larger sample size needs to be collected to better validate these results. While the current sample size of 75 is enough to run some analyses, a larger sample size would allow the use of tools such as factor analysis to determine validity (Field, 2018). With a larger sample size, significant differences between the values ascribed to the three process-object layers may emerge, allowing for more specific feedback on the suggestion of Dray and colleagues (2019) to focus more on differentials in calculus classes.

Possible Applications of Current Results

With the understanding that these results come from a pilot study, some suggestions can be made from these data. Firstly, since significant differences were observed between the mathematics and “other” group, the nature of the known members of the “other” group can inform some instructional choices. Some members of the “other” group, as seen in some comments in the survey, came from subject areas such as accounting and business majors. These students often take some form of “business calculus” where certain aspects of calculus, such as trigonometric derivatives, are removed from the mainstream calculus curriculum. These results, where the mathematicians valued all the process-object layers more than those in the “other” group, seem to confirm that such a reduced calculus curriculum may be desired by teachers in accounting and business majors.

However, as the sample size for this survey was so low, and the known number of accounting and business majors in the survey was so low, such claims are preliminary at best.

Future Applications of this Survey

Once this survey is validated and refined, this survey could be deployed to a larger audience of post-secondary teachers to better make recommendations for how calculus curriculum can be modified to better suit the needs of the non-mathematics students in calculus classes. This larger audience could include college faculty, research scientists, and professional engineers just to name a few. Such results could help calculus better serve students entering STEM fields, rather than serving as a barrier to success.

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