Graphing as a Tool to Support Covariational Reasoning

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There are limitations to using graphing tasks to assess students' covariational reasoning since students' nonnormative graphing schemes can impede their ability to graphically re-present their images of covarying quantities (Frank, 2018; Moore et al., 2024). Thus, Drimalla et al. (2020) encouraged researchers to attend to students covariational reasoning in non-graphing contexts. In this poster we document the challenges in demonstrating advanced levels of covariational reasoning without normative mathematical representations, such as graphs.

Covariational reasoning entails an *anticipation* to track how two quantities magnitudes' change together. This involves three essential constructions: (1) constructing two quantities - measurable attributes of a situation, (2) imagining each quantity to vary (variational reasoning) and (3) coupling these images of variation through a multiplicative object (Thompson and Carlson, 2017). The multiplicative object is an essential construction in **co**variational reasoning. As Saldanha and Thompson (1998) explain, "As a multiplicative object, one tracks either quantity's value with the immediate, explicit, and persistent realization that, at every moment, the other quantity also has a value" (pp. 1-2). That is, as the student imagines variation in one quantity they necessarily imagine variation in the other.

One can conceptualize a point in the Cartesian coordinate system as a multiplicative object that simultaneously entails the magnitudes of two quantities. One can then construct a graph by tracking this point in the plane as they imagine the quantities to co-vary. Through this construction the graph is a record of the multiplicative object – the point – under variation. The physical construction of tracking point in the plane allows one to present and coordinate images of simultaneous variation. In contrast, there are linguistic difficulties in expressing one's anticipation to simultaneously attend to two quantities' magnitudes as they vary together: at any moment in time the language one uses will be focused on one quantity or another. One cannot simultaneously communicate variation of the constructed multiplicative object. This is a methodological limitation to studying students' covariational reasoning outside conventional representations, such as graphs.

In this poster we use the work of Sam, an undergraduate first semester calculus student, to document how his graphing activity supported and inhibited his construction of quantities, images of variation, and a multiplicative object. We report that while Sam's nonnormative graphing schemes inhibited him in graphing the gross covariation of two quantities, the activity of try to construct a graph supported him in constructing quantities from a dynamic situation and coordinating gross images of variation. We have no evidence that Sam constructed a multiplicative object to unite these images of variation. We caution this might be evidence of a linguistic challenge, and not a limitation to Sam's covariational reasoning. Sam's work illustrates some challenges of assessing students' covariational reasoning outside the context of graphing and suggests that the action of graphing might be necessary to enact advanced levels of covariational reasoning.

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