Go-to Strategies Used by Undergraduate Developmental Mathematics Students to Support Their Fraction Understandings

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Difficulties with fractions are well-documented for undergraduate developmental mathematics students. As such, fractions are a known gatekeeping topic for this demographic. However, research on fraction understandings for this population is scarce. In this paper, I synthesize relevant literature regarding undergraduate developmental fraction understandings and related K–12 fraction literature. I then report findings from task-based clinical interviews to share examples of various go-to strategies these participants utilized as they worked through fraction tasks. I close with a discussion relating these findings to extant literature and propose future research directions.

Keywords: Fraction Understandings, Student Thinking, Undergraduate Developmental Mathematics

Many students experience difficulties when working with fractions, a gatekeeping topic for college-level mathematics courses (Mesa et al, 2014; Ngo, 2019). In this paper, I share examples of go-to fraction strategies that Nine, Vivian, and Sofia demonstrated as they worked with fractions in task-based clinical interviews (CI) (Ginsberg, 1997). These participants were undergraduate developmental mathematics students at a large research institution in the southern United States when they participated in their respective interviews during a spring semester between 2022 and 2024. Nine viewed fractions as division problems. Vivian considered fractions to see if they were in "the majority." Sofia simplified fractions and used known percentages as benchmarks when simplification did not help.

Literature Review and Theoretical Background

Research on the fraction understandings of undergraduate developmental mathematics students is scarce (Alexander, 2013; Mesa et al., 2014). Studies involving the mathematical thinking of undergraduate developmental mathematics students build on K–12 research since undergraduate developmental mathematics students study K–12 topics while being college students (Alexander, 2013). Fraction knowledge has been shown to function as a gatekeeper for algebra readiness, though further need for research has been suggested (Booth & Newton, 2012; Ngo, 2019; Siegler et al., 2012).

In this report, I draw on the fraction literature using three of Lamon's (2020) five fraction understandings as I examine what is productive in my participant's fraction thinking. Lamon's (2020) framework includes five fraction understandings: part-whole, operator, quotient, measure, and ratio. The part-whole understanding of fractions relates the number of equal parts of a whole to the whole (Lamon, 2020). The operator understanding views fractions as a set of instructions to be carried out (Lamon, 2020). The measure understanding is a geometric meaning for fractions and involves measures, or lengths, of intervals (Lamon, 2020). For the purposes of this study, I define fractions as numbers written in the form of *a/b,* where a and b are not necessarily integers, such that b is not equal to zero (Lamon, 2020; Empson & Levi, 2011). By strategy, I mean the approach the participant approached the problem when working on a task. The research question for this study is: What go-to fraction strategies do undergraduate developmental mathematics students tend to rely on as they encounter fraction tasks?

Methods

In this paper, I present data from a larger dissertation study. The overarching goal of the dissertation study is to get a picture of the way the participant is currently understanding fractions using frameworks from Lamon (2020) and Steffe & Olive (2010) and then to examine ways to create an environment to foster the strengthening of the measure understanding of fractions (Lamon, 2020). The data for this paper comes from initial task-based clinical interviews (CI) (Ginsberg, 1997). A CI is a semi-structured interview that focuses on describing the participant's current thinking and reasoning (Ginsberg, 1997; Steffe & Thompson, 2000). The CIs were conducted as the first phase in the dissertation study, in anticipation of the participant potentially continuing into a teaching experiment (Steffe & Thompson, 2000). The clinical interview data comes from video recordings, the participant's written work, and any field notes taken (Cobb & Steffe, 1983; Steffe & Thompson, 2000; Steffe & Ulrich, 2020).

Participants, Data Collection, and Analysis

Nine, Vivian, and Sofia (all pseudonyms), were recruited from randomly selected undergraduate developmental mathematics classes at a large research university in the southern United States. Each of them participated in a CI that included 12 fraction tasks, divided among two 45-minute sessions. Data was collected during the spring semesters of 2022, 2023, and 2024. I served as the interviewer, and my witnesses came from a pool of graduate students and faculty who had experience and knowledge about the CI methodology. Witnesses served based on availability. A primary camera captured the participant's written work, while a secondary camera recorded the interaction between the participant and the interviewer. Both ongoing analysis and retrospective analysis were utilized (Steffe & Thompson, 2000). For ongoing analysis, I wrote notes following each CI session to review what had taken place. During retrospective analysis, I watched the CI, wrote a thick description of each session, then analyzed the data using grounded theory techniques in the form of open and axial coding (Corbin & Strauss, 2015) to identify the strategies, or approaches, that the participants demonstrated in their sessions.

Fraction Tasks Overview

The following list contains a subset of the tasks that were used in the CI. The interviews began with two open-ended questions that asked what $\frac{2}{3}$ and $\frac{7}{3}$, respectively, meant to the participant. These questions allowed the participants to share their fraction thinking before encountering tasks that targeted specific fraction understandings. The participants were also asked to read these questions out loud, as they may be read in multiple ways.

1. *What does 2/3 mean to you?*
2. *What does 7/3 mean to you?*
3. *This figure* (a picture of a hexagon) *represents a whole. Show me* (blank). This blank can be filled in with $\frac{15}{6}$ and/or $\frac{4}{6}$, depending on the difficulty participants experienced with the previous tasks.
4. Number line task
    a. Give the participant four index cards marked with $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, and $\frac{7}{6}$ respectively. *Without using decimals, put these fractions in order from least to greatest.*

b. Give the participant a piece of paper with a line across it lengthwise. *If this is a number line, where would you place the fractions as accurately as you can?*

5. *Jade shared 5 candies among 3 friends. What amount of candy did each friend get?* Note: Alternative questions included sharing 12 candies among 5 friends and 2 candies among 3 friends.

Results

Nine, Vivian, and Sofia demonstrated varied go-to strategies as they worked on the fraction tasks in the CI. Nine viewed fractions as division problems. Vivian considered fractions to see if they were in "the majority." Sofia simplified fractions and used known percentages as benchmarks when simplification did not help. In the following I will explain each go-to strategy with a few illustrative examples.

Nine

Nine's go-to strategy was to primarily think of these tasks as division problems. Below, I will give examples from three tasks that illustrate this line of fraction thinking. When asked to explain what two thirds means to her in Task 1, Nine shared, "When I think two thirds, I just think two over three or two divided by three." Nine gave two examples for scenarios a teacher might give for two thirds. The teacher might give information about having pizza or brownies and the need to divide the food so they can be shared fairly. This lent to Nine viewing fractions as instructions for dividing the numerator by the denominator. She gave a similar response for seven thirds in Task 2, saying that seven thirds was "seven divided by three." This time she added that her "first instinct was generally to turn [seven thirds] into a mixed fraction." Then Nine talked through her process of using division to turn seven thirds into two and one third. Mixed numbers were not the only results of Nine's viewing fractions as division problems. When Nine was figuring out how she would share twelve candy bars among five friends in Task 5, Nine's immediate reaction was to divide 12 by five, this time using long division to find the decimal equivalent for 12 divided by five, shown in Figure 1. While we know that this problem refers to the fraction $\frac{12}{5}$, there is no evidence that Nine viewed this particular task as a fraction.

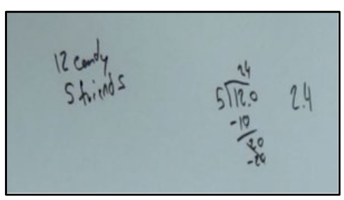

Figure 1. Nine finds how much candy each person gets when sharing 12 candies among five friends.

Vivian

Vivian's go-to strategy was to check to see how a fraction compared to "the majority," which is greater than one half. Below, I will give examples of two tasks that illustrate this comparison. In response to Task 1, two thirds meant to Vivian that "the majority of something is taken up or something has happened to it." She gave an example of three buildings in a location where two of the three buildings have people in them, shown in Figure 2. When interpreting her drawing, Vivian stated, "So, the majority of the three buildings, two thirds of the three buildings, are occupied." A second way that Vivian could describe what two thirds meant to her was to measure two thirds of a cup. As with the buildings, Vivian referred to the two thirds of a cup as "the majority of the cup."

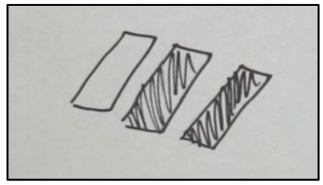

Figure 2. Vivian shared an example of what $\frac{2}{3}$ means to her.

During Task 4, Vivian decided to place the $\frac{2}{3}$ card to the right of the $\frac{1}{2}$ card, stating "Two thirds is the majority. So, that $\left(\frac{2}{3}\right)$ would probably be bigger." The $\frac{7}{6}$ card posed an issue for Vivian. To Vivian, fractions exceeding 1 did not exist. Specifically, with $\frac{7}{3}$ in Task 2, and again with $\frac{7}{6}$ in this task, she stated "seven sixths doesn't exist." Despite this issue, Vivian continued to try to analyze $\frac{7}{6}$ in this task, stating "that's (pointing to the $\frac{7}{6}$ card) more than the majority, that's more than the whole." This helped her decide to place the $\frac{7}{6}$ card to the right of the $\frac{2}{3}$ card, as two thirds is less than a whole. Vivian later described seven sixths as overflowing when she placed the fraction to the right of one on a number line. The $\frac{3}{8}$ card was placed to the left of the $\frac{1}{2}$ card because "three eighths is less than the majority." When asked about $\frac{3}{8}$ being less than the majority, Vivian elaborated that "it's less than half, so it's the smallest." This statement provides evidence that Vivian views one half as the dividing line for what is in the majority. The only time when Vivian used a benchmark other than one half was when she used one whole with seven sixths.

Sofia

Sofia's go-to strategy was to simplify a fraction and then to compare the fraction against a known percentage. Below, I will present three examples of Sofia simplifying fractions and one example of using known percentages.

Simplifying fractions. When asked what $\frac{7}{3}$ meant to her in Task 2, Sofia said that her "first thing [reaction] was to simplify" the fraction, which in this case involved working to turn $\frac{7}{3}$ into a mixed number. After determining that there were two wholes, with three in each whole, and one left over, shown in Figure 3, Sofia took out a hexagon and seven triangle pattern blocks. She arranged six triangles on the hexagon, remarking that she had one more left because she had used six of the seven triangles. She returned to her original work and interpreted her "two whole and one left" to mean that seven thirds is two and one third.

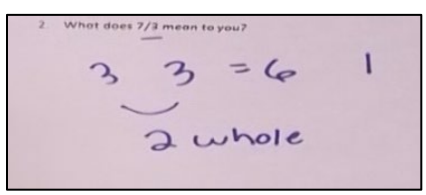

Figure 3. Sofia interprets 7/3 as two whole and one left.

Next, Sofia was presented with Task 3, where she read $\frac{15}{6}$ as "fifteen over six." Sofia's first action with this task was to simplify $\frac{15}{6}$ to $\frac{5}{2}$. Given her answer of $\frac{5}{2}$ in Figure 4a, I interpreted

Sofia's written justification to mean that she was dividing both the numerator and denominator by three in her simplification process. She reinterpreted five halves as "two wholes and one left over." Sofia initially interpreted this simplification as $2\frac{1}{6}$, but quickly changed her answer to $2\frac{1}{2}$. When she expressed that she was unsure how to continue, I asked Sofia to explain more about the "one left over." Sofia explained that since there is a two in the denominator of $\frac{5}{2}$, the one left over would indicate one half, since that is the simplified part of $\frac{15}{6}$. She still seemed to be unsure of her simplification. So, I asked Sofia to draw or represent what her answer might look like. Sofia worked to find two pattern blocks that work together to add up to the whole given in the prompt, which was the hexagon. After a bit of experimentation, Sofia chose to represent her answer with two trapezoid blocks, which she arranged on top of a hexagon block. Sofia clarified that the two trapezoids represented the two wholes as she pointed to the simplified fraction. Then she added a triangle block beside the stack, shown in Figure 4b, to represent the remainder of one sixth from the original task prompt. I interpreted Sofia's actions to mean that she was using the pattern blocks to check her fraction simplification process.

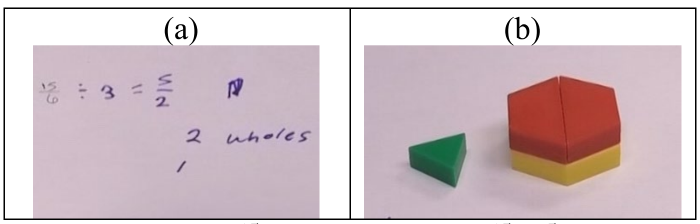

Figure 4. (a) Sofia simplifies $\frac{15}{6}$. (b) Sofia represents $\frac{15}{6} = \frac{5}{2}$ with pattern blocks.

However, Sofia seemed uneasy with her answer to Task 3, expressing that she "is so bad at fractions." To foster more confidence in Sofia, I gave her another copy of the Task 3 prompt and asked her to show me four sixths. Sofia took out a hexagon pattern block and five triangle blocks. She arranged four of the triangle pattern blocks on the hexagon block, shown in Figure 5a. Next, she took out two rhombus blocks and placed them on top of the triangle blocks, shown in Figure 5b. Sofia said that "if this (pointing to the triangles) represents four sixths, the two rhombi represent two thirds." Sofia could have left her answer in the form of the task prompt, as shown in Figure 5a. But to her, the answer was incomplete. This time, Sofia used the pattern blocks as a means of not only representing two thirds, but in finding the simplified fraction.

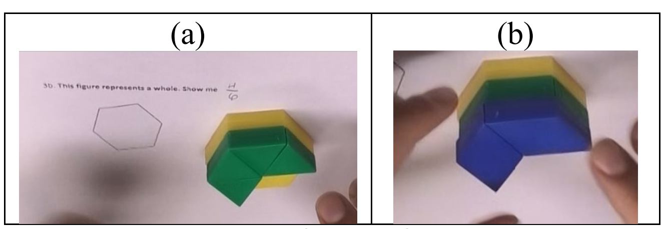

Figure 5. Sofia shows (a) $\frac{4}{6}$ and (b) $\frac{2}{3}$ using pattern blocks.

Comparing fractions to known percentages. During the number line task (Task 4), Sofia initially moved the $\frac{2}{3}$ card to the left of the $\frac{1}{2}$ card. Next, she stated that she needed to see what three and eight had in common to make sense of the fractions. I interpreted this to mean that Sofia was deciding where to place the three eighths card, and she was attempting to simplify the fraction. After being offered scratch paper, Sofia began comparing fractions to known percentages. She wrote the fraction $\frac{2}{3}$ and set it equivalent to 66%, shown in Figure 6. I took this to mean that in that moment, Sofia's known percentage for two thirds was 66%. Next, she set $\frac{1}{2}$ equal to 50%, shown in Figure 6. Sofia is unsure about her placement of the $\frac{7}{6}$ card, so she decided to work from a percentage that she knew. She began by saying that one fifth of 100 was 20%, writing $\frac{1}{5}$ 100 = 20%, as shown in Figure 6. From there, she reasoned that $\frac{1}{6}$ as a percent was less than $\frac{1}{5}$ as a percent, adding "I'm not sure by how much, but I'm sure that it's less than 20." At this point, Sofia moved the $\frac{7}{6}$ card to the right of the $\frac{2}{3}$ card but before the $\frac{3}{8}$ card, as shown in Figure 7. She said that she was unsure she placed the $\frac{7}{6}$ card far enough to the right, but she tabled her placement of the $\frac{7}{6}$ card and moved on to considering the $\frac{3}{8}$ card. Sofia went back to her work for $\frac{7}{6}$, and noted that if she went to one eighth (writing $\frac{1}{8}\ 100\ = \downarrow$, shown in Figure 6), "then that would be even lower than from where we started from at one fifth." Sofia also noted that one fifth was "her only reference point, since she [knew] that clearly." Using this comparison to her known percentage of one fifth being 20%, one sixth being less than 20%, and one eighth being less still, she moved the $\frac{3}{8}$ card to the left of the line of fractions, indicating that three eighths was less than one half. After further contemplation, Sofia decided to leave the $\frac{7}{6}$ card at the right end of the line of cards since it was the only number that is a whole with a remainder, as shown in Figure 6 with her $1\frac{1}{6} = \frac{7}{6}$ notation.

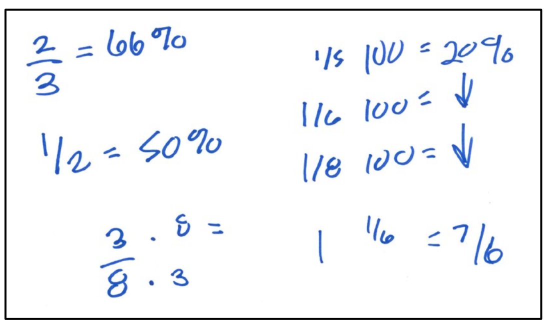

Figure 6. Sofia found known percentage equivalencies for $\frac{2}{3}$ and $\frac{1}{2}$, then reasoned to find how to order $\frac{3}{8}$.

From Figure 7, we see that the fractions have now been ordered as $\frac{3}{8}, \frac{1}{2}, \frac{2}{3}$, and $\frac{7}{6}$. For the second part of the task, Sofia created a number line that included the fractions she had ordered. First, she marked the line with zero on the left and 100 near the right end, before adding 50 approximately halfway between them. Then she further segmented this space in increments of 10 (i.e., 10, 20, 30, 40, 50, 60, 70, 80, 90). She wrote the fractions on sticky flags and used her calibration and previously worked percent relations to place the sticky flags where she thought they should go, as seen in Figure 8. An orange flag representing $\frac{3}{8}$ was placed near the 10. A pink flag representing $\frac{1}{2}$ was placed above the 50. A green flag was used to represent $\frac{2}{3}$ and placed just to the left of the 70. At this point, Sofia added 120 at the end of the printed line then 110 halfway between the 100 and 120. A blue flag went above the 110 to represent $\frac{7}{6}$.

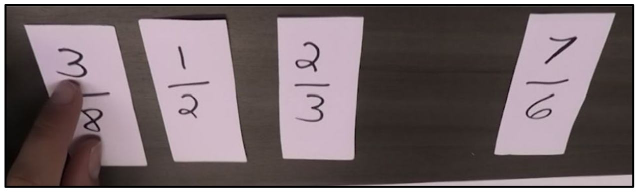

Figure 7. *Sofia's final order of the fraction cards for* $\frac{3}{8}$*,* $\frac{1}{2}$*,* $\frac{2}{3}$*, and* $\frac{7}{6}$*.*

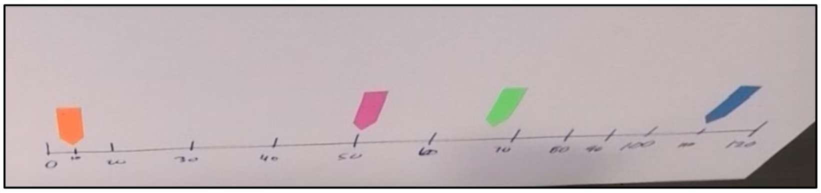

Figure 8. Sofia creates a number line with her fractions. The orange flag represents $\frac{3}{8}$*. The pink flag represents* $\frac{1}{2}$*. The green flag represents* $\frac{2}{3}$*. The blue flag represents* $\frac{7}{6}$*.*

Discussion

Nine, Vivian, and Sofia exhibited different go-to fraction strategies with both the open-ended task for what two thirds and seven thirds mean to them. These strategies also appeared when the solution path for fraction tasks were not straightforward for these participants. Their go-to strategies could be mapped onto fraction understandings and schemes from the literature. Nine tended to go to her operator understanding of fractions (Lamon, 2020), in which she viewed the fraction as a set of instructions for a division problem of numerator divided by denominator. This was carried out both to find mixed number and decimal solutions. Vivian used a benchmarking strategy. Most often, her benchmark was one half as she contemplated whether fractions were in the majority. She used a benchmark of one, or the whole, when she said that $\frac{7}{6}$ was overflowing. Sofia may have found mixed numbers, but she was doing so for a different reason than Nine did. Where Nine tended to go straight for division, Sofia wanted to find fractions she viewed as being easier to work with. Thus, Sofia's go-to strategy used commensurate fractions (Hackenberg et al., 2016; Steffe & Olive, 2010) as she sought to simplify the fractions presented to her, generally before working further with them. She also used a benchmarking strategy as she compared fractions with "known" percentages in the number line problem.

K–12 mathematics research has worked hard to provide an environment that is conducive to preparing students for college mathematics. And yet, there are students who are testing below college readiness for mathematics and being placed into developmental courses. While the K–12 curriculum often focuses on the part-whole understanding of fractions (Lamon, 2020), the go-to strategies in this paper are not within that silo. This paper provides a few examples of fraction thinking of undergraduate developmental mathematics students; it is not seeking to make a generalization of the go-to fraction understandings that are used by undergraduate developmental mathematics students. Rather, the purpose of this paper is to provide examples of what an instructor may witness as they work with students in this demographic. Sharing this information may serve to inform those working with developmental mathematics students by providing examples of the fraction understandings that developmental mathematics students hold. Future research may yield other go-to strategies used by undergraduate developmental mathematics students.

Acknowledgments

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