

Sudoku: A Bridge to Proof Validation

Brittney Fahnestock
Texas State University

Sarah Hanusch
SUNY Oswego

Keywords: proof writing, proof validation, curriculum

It is well established in the literature that undergraduate students struggle when first learning how to write proofs, writing, and validating the proofs of others (e.g. Alcock & Weber, 2005; Selden & Selden, 2003; Stylianides, et al., 2017). The literature base hypothesized that explicit instruction on proof validation would help students improve their proof validating skills, and this study attempted to measure whether that hypothesis is true.

Curriculum Components

The curriculum introduced proofs through Sudoku, inspired by Greenhill's (2011) activity. This low-barrier experience allowed all students to engage equitably in the lessons. The second lesson focused on counterexamples and encouraged collaboration. The third lesson centered on identifying assumptions and conclusions as used in proof frameworks (Selden, et al., 2017).

Lesson four focused on proof validation and was emphasized due to its application in upper-level proof courses and everyday logic. This segment of the curriculum used the language of the Toulman scheme (1958) and included a handout that walked students through two guided examples on validating proofs. The students then tried examples on their own or in small groups.

Methodology

A pre-post test model was implemented. Twenty-three participants completed the pre-test, however only five participants had paired pre- and post-tests. The pre-test and post-test were identical assessments and consisted of 10 questions. Five questions provided proofs with unknown validity and asked the participants to choose if the proof was valid or invalid and explain why. One question gave the participants both a proof and disproof of the same statement and they had to determine which one was valid. The students were also asked to write their own proof of a statement and to then discuss the validity of their proof.

Results

On the pre-test, the participants in this study performed in ways consistent with the literature (Alcock & Weber, 2005; Ko, 2010; Weber & Ramos, 2014). The one result that counters previous research findings is that these participants demonstrated a conceptual understanding of counterexample with each counterexample question validated correctly at least 60% of the time. One notable finding is that although a majority of the participants wrote a valid proof, more than 50% of the participants did not answer the question asking them to validate their own proof.

Although the sample size is insufficient to make a statistical conclusion, there is some evidence that the students effectively learned proof validation techniques from the curriculum. Across the 50 questions, the scores decreased in two instances, improved in 20 instances, and stayed the same the remaining 28 times. Even more compelling is the changes in vocabulary observed from the textual analysis. On the post-test the participants made several references to the “warrants” for claims and to the “proof framework.” This result is triangulated by the survey given at the end of the course where the majority of the respondents said they used the proof framework every time or almost every time when writing their proofs.

References

- Alcock, L., & Weber, K. (2005). Proof validation in real analysis: Inferring and checking warrants. *Journal of Mathematical Behavior*, 24, 125-134. doi: 10.1016/j.jmathb.2005.03.003
- Greenhill, C. (2011). Sudoku, logic and proof. *Parabola*, 47, 1–6.
- Ko, Y.-Y. (2010). *Proofs and counterexamples: Undergraduate students' strategies for validating argument, evaluating statements, and constructing productions* [Doctoral dissertation, University of Wisconsin-Madison].
- Selden, A. & Selden, J. (2003). Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem? *Journal for Research in Mathematics Education*, 34, 4-36.
- Selden, A., Selden, J., & Benkhalti, A. (2017). Proof frameworks-a way to get started. *PRIMUS*, 28. <https://doi.org/10.1080/10511970.2017.1355858>
- Stylianides, G. J., Stylianides, A. J., & Weber, K. (2017). Research on the teaching and learning of proof: Taking stock and moving forward. In Jinfa Cai (Ed.), *Compendium for research in mathematics education* (pp. 237-266). National Council of Teachers of Mathematics.
- Toulmin, S. (1958). *The uses of argument*. Cambridge University Press.
- Weber, K., & Mejia-Ramos, J. P. (2014). Mathematics majors' beliefs about proof reading. *International Journal of Mathematical Education in Science and Technology*, 45, 89-103. <http://dx.doi.org/10.1080/0020739X.2013.790514>