

Examining Inequitable Talk and Perceived Status During Group-Worthy Tasks in an Undergraduate Topology Course

Brittney M. Ellis
Texas State University

Christine K. Austin
Texas State University

Candice Hamilton
Texas State University

Blake Dvarishkis
Texas State University

Kathleen Melhuish
Texas State University

We report on the initial findings from a project aimed at enhancing equitable group work in undergraduate proof courses through the design and implementation of group-worthy tasks. This mixed methods study reports on three group-worthy tasks implemented in a topology course. We investigated the extent to which student interactions in these small groups reflect participatory and relational equity in relation to perceived academic status. Quantitative results indicate patterns in which the group-worthy tasks may disrupt students with high status from dominating talk turns taken, the amount of talk turns taken, and number of words spoken by the students, or neither depending on the tasks. Qualitative results explored relational inequity via silencing, ignoring, or no talk during these three tasks. Our initial findings suggest that group-worthy tasks alone may not guarantee equitable participation or relations in group interactions.

Keywords: Status, Equity, Group-Worthy Tasks, Proof

There is a growing consensus that active learning approaches can benefit students taking undergraduate mathematics courses (Freeman et al., 2014). While there is some evidence that active approaches may lead to more equitable outcomes (e.g., Laursen et al., 2014; Theobald et al., 2020), it is important to recognize that such strategies do not automatically guarantee equitable participation among students. Research in inquiry-based settings suggests that inequities can persist, including in proof-based courses where group work is a central component (Brown, 2018; Ernest et al., 2019; Johnson et al., 2020). A possible cause for these inequities is how students perceive each other's status; that is, their assumptions about who is "better" or "smarter" at math influence their participation in group work.

One factor that can amplify or attenuate inequitable participation is the nature of group work tasks themselves (Shah & Lewis, 2019). Both content and structure can create opportunities for every student to engage and contribute mathematically. For example, well-designed group-worthy tasks can help address status-related dynamics that may otherwise hinder some students' participation (Cohen et al., 1999; Esmonde, 2009). Our understanding of the interplay between group-worthy tasks, status dynamics, and equity in undergraduate courses using active learning strategies is far from complete (Adiredja & Andrews-Larson, 2017), especially in advanced mathematics courses. This setting may exasperate status issues as the formal proof becomes the focal object of study (Weber & Melhuish, 2022). Here, we share analysis of a subset of data from the STructuring Equitable Participation in Undergraduate Proof (STEP UP) project aimed at supporting instructors in designing and implementing group-worthy tasks in proof-based courses. We investigated the following research question: To what extent do students' interactions in small groups reflect participatory and relational equity in relation to perceived status during three group-worthy tasks in a collaborative topology course?

Background and Framing

In our work, *equity* is when every student in a group has the necessary resources to support their learning (Shah & Lewis, 2019). *Inequity*, in contrast, occurs when power dynamics are structured in a way that prevents students from accessing these resources. Some resources might include physical access to the shared workspace (e.g., whiteboard, worksheet), opportunities to make written or verbal contributions to the group's work, and access to another group members' ideas. Two aspects of equity related to group dynamics are participatory and relational equity. *Relational equity* (Boaler, 2008) is characterized by interactions that allow each student in a group to have the opportunity to develop an identity as a capable learner. *Participatory equity* refers to the fair distribution of both opportunities to participate and the actual participation among all students involved in a learning interaction (Shah & Lewis). No situation is entirely equitable or inequitable; instead, relational and participatory equity fluctuate in subtle ways from moment to moment and can be influenced by social factors.

Inequities in group work can emerge due to imbalanced power dynamics rooted in differences in status. A status characteristic is defined as a "social ranking that everyone agrees is preferable to have a higher rank rather than a lower one" (Cohen et al., 1999, p. 84). In the classroom, academic status characteristics, such as perceived mathematical ability, are particularly influential (Cohen & Lotan, 2014) and can be affected by broader status characteristics like gender, race, and ethnicity (Cohen et al., 1999; Ridgeway, 2018). Students with higher status often have more chances to participate and contribute, which can lead to greater learning opportunities (Cohen & Lotan).

Small group interactions impact opportunities to learn during group work because students who have more opportunities to participate often participate more, and as a result, learn more (Esmonde, 2009). Research has shown that the nature of the task influences the extent to which participation is inequitable between group members (Chizhik, 1999, 2001). One strategy to balance participation and status is to utilize group-worthy tasks, a term borrowed from Complex Instruction (Cohen et al., 1999). A task is "group-worthy" if it affords various ways to approach the task, includes mechanisms to ensure that every participant is engaged and learning, and promotes a positive sense of interdependence among group members (Cohen & Lotan, 2014). Such tasks can elicit more opportunities for each member to participate in mathematically meaningful ways, potentially disrupting imbalanced participation due to status differences.

Methods

Participants and Procedures

The data comes from an NSF-funded collaboration between mathematics educators and mathematicians to design and implement group-worthy tasks in proof contexts. The research team video and audio recorded student interactions during three group-worthy tasks in a Topology course at a Hispanic-serving institution with a high first-generation population in the southern United States. The class was taught by a topologist during the Fall 2023 semester and consisted of 17 undergraduates and 3 graduate students (see Table 1). Daily classroom activity included randomized group work, usually with students working in groups of 3 or 4 at whiteboards. The three group-worthy tasks had different content and structure from the usual group activities. The three tasks were implemented on non-consecutive class days in weeks 8, 9, and 12 of the 16-week semester. All audio was transcribed and cleaned alongside the video data.

Task 1 focused on proof comprehension of the Universal Property of Quotient Spaces Theorem, with four groups of four students studying two examples to understand the quotient

space and its induced map. Each individual student worked on one part (a-d), then collaborated on part (e) to create a composite function. Task 2 focused on constructing a proof of the proposition stating, “The diagonal is closed if and only if X is Hausdorff”, with groups of four investigating two examples, answering questions for each, and recording their findings on the front whiteboard to identify patterns. Each group member was assigned a unique role¹ with specific information needed to complete the task. Task 3 focused on proof analysis of the proposition stating, “Let $f: X \rightarrow Y$ be homeomorphisms. If X is connected, then Y is connected.” In this task, pairs of students compared two proofs (A and B). Each student studied one proof, then explained it to their partner to compare and identify similarities and differences.

Table 1. Participating Student Demographics, Perceived Status, and Task/Group Information.

<u>Task #.Group Name</u>	<u>Name (Perceived Status)</u>	<u>Gender</u>	<u>Race/Ethnicity</u>
Group 1.A, Group 2.C, Pair 3.B	Lily (low)	Woman	Hispanic
Group 1.A, Group 2.B, Pair 3.C	Zahir (low)	Man*	Hispanic*
Group 1.A, Group 2.A, Pair 3.A	Pink (mid)	Woman	White, Hispanic
Group 1.A, Group 2.B, Pair 3.D	Kieran (mid)	N/A	South Asian
Group 1.B, Group 2.C, Pair 3.A	Kendra (high)	Woman*	Black*
Group 1.B	Brandon (mid)	Man	White
Group 1.B, Group 2.A, Pair 3.D	Joe (mid)	Man	Spanish
Group 1.B, Group 2.C, Pair 3.E	N (mid)	Woman	Mexican
Group 2.A	Alexis (high)	Woman	White-Caucasian
Group 1.C, Group 2.A, Pair 3.B	Luke (mid)	Man	White
Group 2.B	Ariella (low)	Woman*	Hispanic*
Group 1.C, Group 2.B, Pair 3.C	Xander (high)	Man*	White*
Group 1.C, Pair 3.E	Julie (high)	Woman	White
Group 1.C	Samantha (low)	Woman*	White*

*Indicates researcher assumption; all other data is student self-identified. Note: Group 2.C was not analyzed qualitatively due to a non-consenting fourth member.

Data Analysis

This study used a convergent mixed methods design, combining quantitative and qualitative results (Creswell & Plano Clark, 2018). The goal was to gather complementary data on students’ interactions in a topology course to assess participatory and relational equity in group tasks (Morse, 1991). Phase one involved coding video transcripts of group work for three group-worthy tasks, calculating turn and talk point ratios, and conducting a Spearman Rho analysis. Phase two merged these results to explain group interactions and interpret the findings.

Quantitative Analysis. The constructs captured in the quantitative analysis are: Perceived Status, Turn Ratio, and Talk Ratio. Perceived Status was determined through the results of a status survey that each participating (and present) student completed the class period following the third group-worthy task implementation. For the status survey, students ($n = 10$) circled peers they thought were “best at math” which captured perceived academic status via mathematical

¹ (1) Linguist assisted group with provided key definitions of Hausdorff, diagonal, and closed. (2) Zoologist assisted group’s understanding of provided “strange and exotic” topologies: trivial, discrete, finite-complement, and poset topologies. (3) Product Manager assisted group with provided definition of the product topology. (4) Summoner assisted group by “enlisting additional resources” for the group.

ability. Those circled four or more times were “high” status (coded as 2), two to three times were “mid” status (coded as 1), and less than two were “low” status (coded as 0).

We adapted Reinholz and Shah’s (2018) ratio approach to quantifying participatory equity to the individual level. We calculated Turn Ratios and Talk Ratios. Turn Ratios were calculated based on each student’s talk turns divided by the total talk turns and then by the ideal portion (e.g., 0.25 for a group of four). A turn is defined as a continuous segment of uninterrupted speech by a student. A new turn starts when a different student speaks, or is addressed by the instructor. Talk Ratios were calculated using a word count total by summing words per talk turn, coding as 1 for < 5 words, 5 for ≤ 20 words, and 20 for > 20 words (Reinholz & Shah, 2018). The overall Talk Ratio for everyone was then calculated in a similar manner to the Turn Ratio.

Turn Ratios and Talk Ratios both provide information about participatory equity. Spearman Rho was calculated for Task 1 ($n = 12$), Task 2 ($n = 11$), and Task 3 ($n = 10$) to evaluate the correlation between Turn and Talk Ratios. Given that Turn Ratios are based on talk turns and Talk Ratios on word count, they are expected to reflect each other. A Spearman Rho was chosen over Pearson correlation due to the analysis being more appropriate for a small sample size and ordinal data (Siegel & Castellan, 1988). Additionally, Spearman Rho requires, and considers, the variables to be ordinal for the objects to be ranked (Siegel & Castellan, 1988).

Qualitative Analysis. We used Stovall et al.’s (2023) concepts of Silencing, Ignoring, and No Talk to analyze relational *inequity* in group interactions. “Silencing” occurred when one or more students interrupted or talked over another so that the former student did not finish their idea. “Ignoring” occurred when one or more students did not verbally acknowledge another student’s verbal contribution. “No talk” occurred when a student did not verbally contribute during the entire activity and the other students and/or instructor did not address this. We reviewed videos together to resolve disagreements and only finalized codes after reaching a consensus. After coding, we wrote memos summarizing group dynamics (including physical space and student actions), inequitable talk instances, and initial inferences. The memos also included each group member’s perceived status to further link the qualitative summaries with the quantitative results. A research team member, also a student in the class, provided insider knowledge on classroom dynamics, which helped resolve coding conflicts and clarify norms.

Results

Quantitative Results

Since both Turn Ratio and Talk Ratio reflect participatory equity to some degree, we used Spearman’s Rho to assess their correlation, expecting to find a relationship between the number of talk turns and the number of words spoken. As suspected, the relationship between a student’s Turn Ratio and Talk Ratio was found to have a strong positive correlation that was statistically significant for both Task 1 ($\rho = .874, p = < .001$) and Task 2 ($\rho = .897, p < .001$). This means that students with a high Turn Ratio are likely to have a high Talk Ratio, meaning the more talk turns a student has the more likely they will say more words. However, unexpectedly, the relation between the two ratios was not found to be statistically significant ($\rho = .530, p = .115$) for Task 3. To potentially explain the relationship between the ratios, we parsed the relation via perceived academic status (see Figure 1). In Figure 1, the red circles represent a student with high status, the blue diamonds represent a student with mid status, and the green squares represent a student with low status. We note three patterns from that data when accounting for status.

First, with Task 1, the Turn Ratio and Talk Ratios are highly associated, and perceived status is consistently spread throughout. This suggests that, when regarding Task 1, status did not

influence a student's Turn or Talk Ratios. Second, with Task 2, Turn Ratios and Talk Ratios are highly related, however, those with high status are clustered at the top right meaning that those with perceived status talked more in both the amount of talk turns taken and words spoken. This relationship would not be captured by Spearman Rho. Furthermore, other than one green square on the right (low status), those with low status are clustered at the bottom left. Lastly, with Task 3, we note that most of the students had a Turn Ratio between 0.90 and 1.05, however students with high status (red circle) have higher Talk Ratios than those with low status (green square). Our initial findings suggest that the group-worthy tasks may disrupt high status students from dominating talk turns taken (only), talk turns taken and amount of talk, or neither depending on the task. Thus, while the Talk Ratios and Turn Ratios may be related in some instances, our tasks may not be disrupting status as originally conjectured. These patterns may not be generalizable and could result from various factors such as task design (e.g., Task 3 was paired), course culture, timing, or student interactions.

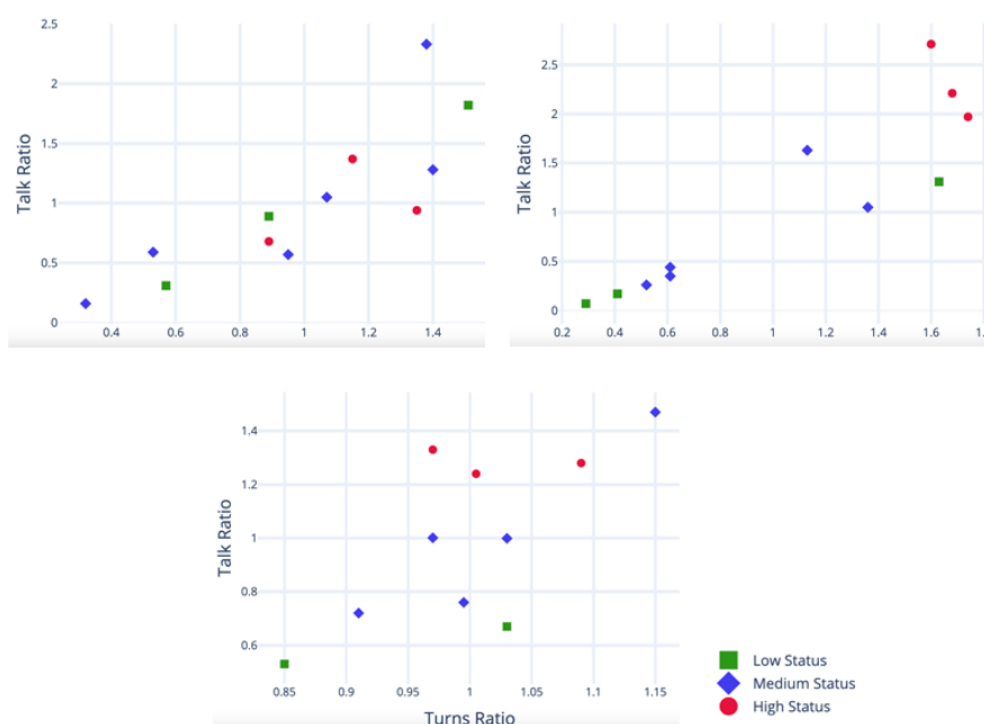


Figure 1. Correlations between Turn Ratios (horizontal axis) and Talk Ratios (vertical axis) Parsed by Perceived Status for Task 1 (Top Left), Task 2 (Top Right), and Task 3 (Bottom Middle).

Qualitative Results

The qualitative analysis occurred concurrently and focused on the relational equity element. In this section, we report an overview of these results for each task and consider how relational and participatory equity may relate.

Task 1: Proof Comprehension. Task 1 showed a more equal distribution of talk and turns during the task. There were no instances of “no talk”, meaning everyone in each group verbally contributed to the group’s mathematical work. However, there were some instances of ignoring and silencing. In Group 1.B, there were two cases of ignoring directed toward N (mid status) and Joe (mid), three instances of Brandon (mid) silencing N, and two instances of Brandon silencing

Kendra (high). Although Kendra spoke often, she asked many questions and listened to her peers' responses without talking over them. Kendra was the only member of this group that attempted to engage collaboratively with all members. Comparatively, Group 1.A, Kieran (mid) was ignored by everyone else in the group many times, and silenced (both physically and verbally) several times by Lily (low). Interestingly, Lily, who had low perceived status, dominated this group interaction and engaged in inequitable talk the most compared to her group mates. At one point Kieran stated, "So... this is something—" then Lily interjected, "We know this because of this," which cut Kieran off and he did not get a chance to finish his idea. In Group 1.C, inequitable talk occurred as students of equal or lower status silenced or ignored others, while a high-status student encouraged collaborative engagement. This suggests that relational inequities may account for the more evenly distributed participation ratios.

Task 2: Proof Construction. In this task, there were students who dominated the talk time and talk turns according to the quantitative analysis. The two focal groups had different patterns of interaction relationally. Group 2.A's interactions were largely respectful and inclusive with no instances of ignoring. Alexis (high status) made sure everyone contributed to the work by passing the marker and explicitly saying things like, "Okay. Who wants the marker next?" There were three instances of silencing directed towards Joe (mid) by Alexis and Luke (mid). For example, after Alexis made an observation, Joe tried to make a statement ("Yeah. I think it-") but was interrupted by Luke, and Alexis continued talking. Alexis and Luke's verbal contributions took up a majority of the overall talk time.

In contrast, "no talk" was indicated for Group 2.B because Zahir (low) contributed only one-word responses (e.g., "yeah", "oh okay") and only spoke more than one word to the instructor when he joined the group to ask how things were going. There was one instance of Kieran (mid) and Xander (high) ignoring Ariella (low), and one instance of Xander (high) silencing Ariella. For example, Ariella started to say something (although it is unclear if she was reading to herself or talking to the group), and Xander interrupted her to ask for the definition of a product topology (she was the "Product Manager"). She stopped talking, gave him the paper with the definition on it, and said, "It's right over here. I guess it's the product topology." She did not continue speaking. Kieran and Xander contributed almost all of the mathematical work, with Xander writing on the whiteboard nearly the entire time. Together, Xander and Kieran's verbal contributions took up about 84% of the total talk time. This group exemplified relational inequities in which higher status students ignored and silenced lower status students. That is, the participatory inequities may be at least partially explained by the relational inequities.

Task 3: Proof Analysis. The quantitative results pointed to this task having balanced turns in talk, but an imbalance in how much talk occurred. There were no cases of ignoring or "no talk" across all five pairs. There were few instances of silencing across each pair (except Pair 3.C), Xander (high) and Zahir (low), in which Xander repeatedly silenced Zahir. For example, while Zahir was explaining his proof, he stated "And then—" but Xander interrupted him by saying, "We know F is a homeomorphism..." barely giving Zahir time to talk before he started explaining to Zahir how to do the proof. Xander's verbal contributions took up about 67% of the total talk time, while Zahir's took up only 33%. Comparatively, there were some instances of silencing between partners in Pair 3.D, Joe (mid) and Kieran (mid), however one person was not overly silencing the other throughout the interaction, unlike with the previous pair.

Overall, the interactions across the remaining pairs were respectful and collaborative. In Pair 3.A, Pink (mid) and Kendra (high), there were no instances of silencing. The two made sense of both proofs together throughout the task, with Kendra asking a lot of questions when she was

confused and Pink helping Kendra process through the reasoning behind both proof approaches. Kendra's verbal contributions made up 62% of the talk time and Pink's took up 38%. Then, in Pair 3.B, Luke (mid) and Lily (low), there was only one instance of Lily silencing Luke. They noticed the proofs were similar, and while Luke shared what he noticed, Lily interrupted him to share her own noticing. Luke's verbal contributions made up 73% of the pair's verbal contributions, meaning Lily's verbal contributions made up only 27%. For Pair 3.E, Julie (high) and N (mid), there were very few cases when Julie silenced N. For instance, at one point N stated, "So then because of the surjection or the bijection, we know it's—" then Julie interjected, "It's a bijection, just cover both bases." N did not have a chance to finish saying what she knew. Julie's verbal contribution took up 64% of the total talk time while N's took up 36%. In four of the pairs, the Turn Ratios were overall balanced, yet the partner with higher perceived status spoke more words than the other partner. This trend did not apply to the only status-balanced pair, Joe (mid) and Kieran (mid), in which both Turn and Talk Ratios were balanced. We found that in the low-high pairing, there were substantial relational inequities that may account for the imbalanced talk, but this did not account for the imbalance in the mid-low status pair.

Discussion

This study set out to examine the extent to which group interactions reflected participatory and relational equity during three group-worthy tasks in an inquiry topology course. Task 1 seemed to disrupt inequitable status and participation patterns, despite some instances of relational inequities. In fact, the inequities appeared to disfavor those with higher status. Task 2 seemed to reproduce inequitable status and participation patterns and was the most relationally inequitable of the three tasks, with the only instance of "no talk" occurring, and higher-status students ignoring and silencing lower-status students. Task 3 seemed to balance the number of talk turns, but high-status students still dominated the conversations in words. There were some relational inequities with silencing in a low-high pairing, but this was not sufficient to account for the overall imbalance in talk across the pairs.

Reflecting on the task design, we can form further conjectures about participation differences. Task 1, which had the most balanced participation, was structured for group members to become experts on specific parts and assist others, culminating in collaborative work for the final part. This design fostered individual accountability and interdependence, making each member's contribution essential for the group's success. In comparison, Task 2 used group roles and it was observed that students largely ignored these roles. Each role card had important information on it, but students did not have to become "experts" on their role. In Group 2.B, we saw that Ariella was assigned the "Product Manager" role and was meant to be the expert on the product topology definition. However, since the definition was written on a card, Ariella simply handed it to Xander, rendering her role obsolete. It may be that students perceived these types of group roles as unnecessary to complete the task. Finally, Task 3 was a paired task with distributed expertise of two different proofs, which necessitated student interaction and contributions, leading to more balanced Turn Ratios. However, Talk Ratios showed that higher-status students still dominated the conversation.

What is clear is that group-worthy tasks alone may not guarantee more equitable group interactions. In the qualitative analysis we noticed the impact the instructor had on disrupting inequities. Instructors can engage a quiet student or highlight a contribution that may otherwise be ignored. Future work will explore the relationship between instructor interventions during group work (e.g., assigning competence; Cohen & Lotan, 2014), as well as how the culture of the classroom and the interplay of race and gender may also impact group work dynamics.

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