

Inquiry into Suitability of a Qualitative Framework about Scaffolding Moves to Support Mathematical Modeling for the Purpose of Quantitative Analysis

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An important goal in research on mathematical modeling is to understand the influence of teacher scaffolding moves on students' model construction. This paper validates the use case of two common frameworks (one for scaffolding and one for student modeling processes) with STEM undergraduates. Study I investigated whether variability in scaffolding strategies could predict variability in participants' modeling processes. Findings suggested that the initial assumption—that scaffolding moves articulated by the framework are clearly aligned with specific modeling processes—was not supported. Study II further explored how scaffolding moves related to distinct stages of model construction and sought to improve cohesion of the categories of scaffolding moves. Our results challenge simplistic expectations about the relationship between scaffolding strategies and learner engagement, highlighting the need for more nuanced approaches to using these frameworks. The studies also demonstrate the importance of qualitative follow-up to refine our understanding of how scaffolding supports complex model construction processes.

Keywords: mathematical modeling, mixed methods, scaffolding

For STEM majors, mathematical modeling (hereafter: modeling) is a major instructional goal because it can authentically promote interaction between scientific and mathematical knowledge domains (Hallström & Schönborn, 2019). For research with a focus on blending knowledge (Hu & Rebello, 2013), a cognitive approach to studying students' engagement in mathematical modeling is often adopted (Kaiser, 2017). Over several decades, this has led to a fairly stable *descriptive* account of the processes attendant to modeling – called a mathematical modeling cycle (MMC) (Blum & Leiß, 2007) - which has, in turn, motivated research agendas like articulating features of learning environments that elicit those processes (Hankeln, 2020; Maaß, 2010) and improving modeling competence (Ärleback & Albarracín, 2019; Durandt et al., 2021). Naturally, that work has extended to understanding what teachers can do to support students' progress in constructing models. However, the descriptive accounts of student engagement are not predictive. On its own, an MMC does not indicate how an individual might engage in a specific task (Ärleback & Bergsten, 2010; Czoher, 2016) and it leaves out the role of the teacher. Separately, there are frameworks that describe teacher knowledge (e.g., Greefrath et al., 2021) and frameworks that hypothesize potential teacher moves that would be supportive of students' engagement in modeling (e.g., Stender & Kaiser, 2015).

Ideally, scaffolding moves would be linked to student engagement in modeling. To date, the existence and extent of these associations have not been systematically explored despite their heavy usage for describing student modeling engagement and teacher scaffolding moves separately. In this paper, we investigate whether a qualitative framework describing scaffolding moves aligns functionally with a framework describing engagement in modeling and justify modifications needed to use the frameworks to carry out studies capable of predicting which moves promote which processes. We report a pair of mixed methods studies that interrogate the utility and promise of two frameworks by addressing the following questions: (1) Are the descriptive categories of the frameworks naïvely associated? (2) Do theoretically similar codes

from the scaffolding moves framework function similarly? (3) What other considerations are there for using these frameworks to study the influence of scaffolding on modeling processes?

Theoretical and Analytic Frameworks

We conceive of a laboratory-based learning environment as a task-modeler-interviewer triad (Koichu & Harel, 2007), and in this study we focus on the modeler-interviewer interactions. We are interested in particular interactions that serve as *scaffolding* for the modeler's *engagement* in modeling. This means we required a framework for engagement, a framework for scaffolding, and a means for relating the two. To describe participant engagement in mathematical modeling, the field uses a mathematical modeling cycle (MMC) (Blum & Leiß, 2007). The cycle decomposes the mathematical modeling process into six sub-processes which connect stages of model construction: a modeler conceives a real-world problem, *understanding* it as a (mental) situation model, *simplifies* it to an idealized real model, *mathematizes* that to produce an externalized mathematical model, *works mathematically* to obtain a mathematical result, which is then *interpreted* in terms of the real-world context to yield real results, and then *validated* against the real-world situation.

Sub-Processes	Definition	Connects Stages
Understanding	Forming an initial idea about what the problem is asking for	real world → situation model
Simplifying & Structuring	Identify (un)important real-world entities and relationships	situation model → real model
Mathematizing	Represent idealized version of the real-world problem using mathematical conventions	real model → mathematical model
Working Mathematically	Analyze or solve mathematical problem	mathematical model → mathematical results
Interpreting	Re-contextualize mathematical results	mathematical results → real results
Validating	Verify results against constraints	real results → real situation

Table 1 Stages and processes of model construction from Blum and Leiß (2007)

Scaffolding is proffered support that aids a student “when performing a task that the student might otherwise not be able to accomplish” (van de Pol et al., 2010, p. 274); we focus on *contingent* scaffolding in-the-moment rather than longer-term scaffolding processes like fading and transfer of responsibility. In the context of modeling, an interviewer needs to provide that support contingent to the modeler's current mathematical reasoning and progress in model construction/evaluation. Stender (2016) developed a framework for scaffolding during modeling that aims to describe moves an educator can make to respond to students' reasoning and further their progress through the modeling cycle that retain student autonomy. Among the categories of moves in the framework are those labeled “B3” – these classify contingent moves targeting the cognitive dimensions of modeling (as opposed to moves that target affective factors). For example, the code B3.6 Prompt to Include Aspects labels moves in which the modeler is asked or encouraged to include something from the real-world she has not yet included in her model, for example posing the question “Do you think maybe there should be another [separate] population, like people who are recovered?” during a disease transmission task when the modeler was struggling with two meanings for the idea of “healthy people.” Due to space constraints, it is not possible to include in this manuscript the 26 B3 codes and 6 MMC codes we used, along with rules and examples. The codebooks are available in the following publications: the MMC

codebook was originally published in Czoher (2016) and the B3 codebook was originally published in German in Stender (2016).

Methods

Our interrogation of the analytic frameworks spans multiple mixed methods studies (Creswell, 2014). We used cognitive task-based interviews to generate data, cluster analyses and logistic regression to develop variables and explore associations among them, and then followed with interpretive methods to validate, refute, and explain patterns in the results.

Data were generated through a set of individual, task-based clinical interviews (Clement, 2000; Maher & Sigley, 2014) with 24 undergraduate STEM majors at a large southern university in the US who were studying differential equations. Each participant worked on between four and eleven modeling tasks across 10 hour-long sessions. The modeling tasks were based on canonical problems from differential equations and dynamical systems that call on feedback loops (e.g., disease transmission; predator-prey). Of the 11 tasks, the final six were open problem statements and contained only contingent scaffolding provided by the interviewer. The interviewer accepted all student productions as provisionally correct (Goldin, 2000) and attempted to make sense of why the participant reasoned as they did. The interviewer intervened in the participants' modeling process if the participant requested help, if it seemed that the participant got stuck, or to generate and test conjectures about the participants' ways of reasoning about the mathematical or real-world aspects of the task. Interviewer turns were formulated so they could map to the scaffolding moves framework.

In this report, we consider only the tasks without embedded scaffolding to isolate the influence of contingent scaffolding on participants' modeling processes. The dataset for this study comprised 52 events completed by 22 (distinct) participants working on six modeling tasks. Qualitative analysis proceeded with deductive coding procedures (Saldana, 2016), using the pre-defined, published codes for engaging in mathematical modeling processes (MMC codes), model construction stages (Stage codes), and contingent scaffolding moves for modeling (B3 codes). Due to the complexity of the codebooks, total duration of the 52 events, and planned quantitative models, our primary concerns about reliability were the chance of missing codable segments and consistent application of the codebooks across participants and tasks. Thus, two analysts independently coded each event. Pairs met regularly to reconcile codebook application to their assigned events and to resolve disagreements based solely on code definitions. In total there were 3,478 instances across 26 B3 codes and 5,596 instances across 6 MMC codes. Due to the interview protocol, *understanding* (233) and *working mathematically* (261) processes were rarely elicited compared to the others, and so were excluded from further analysis.

Study I: Are the Two Frameworks Naïvely Associated?

By meaningful association between frameworks, we mean that variability across the set of MMC codes should be accounted for by variability across B3 codes, when aggregated across the events. At each instance of a B3 move and for each MMC process, we determined if the target process was observed within a two-minute window subsequent to the B3 move. If the process was observed at least once in the window, we said the B3 move was taken up by the participant. Combining results across events, we estimated the probability of uptake for each process and set of B3 codes. To account for variation in base rate of occurrence of MMC codes, we estimated a base probability under the null or no effect model, where the null assumption is that MMC codes are uniformly distributed across the interview sessions. Under the null model, we let X_k be the number of instances of MMC code k in a given two-minute window. Then X_k follows a binomial

distribution with size equal to the total number of instances of code k and probability equal to 2 divided by the total combined time of the sessions. The base probability is $p_{b_k} = P(X_k > 0)$. We then normalized the probability of uptake, by computing an odds ratio:

$$OR = \frac{(p/(1-p))}{(p_{b_k}/(1-p_{b_k}))}$$

If the uptake probability equals the expected value under the null model, then $OR = 1$, and indicates the contingent scaffolding move is not associated with an increased uptake of process k . On the other hand, $OR > 1$ indicates that the contingent scaffolding move promotes process k . The results table was a daunting $18 \times 4 = 72$ rows long without clear patterns emerging. While the reader might be muttering “no duh,” we note that this initial analysis was necessary for exploring whether each B3 category had a cohesive nature with respect to the MMC’s. There were some pairs of B3 and MMC codes with a strong association, for example, B3.11u Attention to Units was often followed by *validating* ($OR = 1.463$), but in general, the B3 codes were not clearly tied to specific phases of modeling, which washed out any aggregate impact of scaffolding moves on participant modeling processes. We took this result as evidence against the naïve hypothesis that the scaffolding moves framework was aligned with the modeling processes framework. We determined that the hypothesis rested on an implicit assumption that, in the aggregate, each B3 code would contain scaffolding moves that were largely homogenous with respect to the modeling process induced in the participants. This called for qualitative follow-up.

Upon inspecting the instances of B3 codes, some of the codes seemed to operate in a general descriptive way, supporting a wide range of participant modeling activities. Such B3 codes had very high usage and because the data corpus was quite large, we had opportunities to see more specific usage patterns arise. For example, the code B3.12 Interviewer Explaining or Instructing had 254 instances, but the nature of what was being explained or instructed was not uniform across all of the instances. An inductive coding process within B3.12 yielded 6 subcodes (instructing real world, instructing mathematics, instructing modeling, considering alignment between model and reality, explaining distinctions and similarities between scenarios, explaining meaning of representations). We carried out a similar process for 3 other B3 codes. This yielded a new total of 47 B3 codes. Again, inspecting instances of B3 codes, most of the B3 categories did not correspond to a single aspect of model construction. This explained our results from Study I. If the categories of scaffolding moves are not homogenous with respect to what aspects of model construction they are targeting, then we ought not expect a consistent response from the modeler in terms of what process is elicited. Thus, we further hypothesized that the proportion of uptake of scaffolding moves depended on which stage of model construction the B3 codes referred to. Our next step was to examine the extent to which each B3 code could be said to refer to a single stage of model construction or a Profile of stages.

Study II: How Does the Scaffolding Framework Connect to Model Construction?

Our next step was to explore the distribution of B3 codes across model construction stages. We conducted a deductive qualitative coding procedure inspired by Çakmak Gürel (2023) and Tropper et al. (2015) that mapped each instance of a scaffolding move to (1) the stages of model construction presented in the modeling cycle it referred to (see Table 1, third column) or (2) to no stage. For example, the move “Let’s work on just the susceptible and infectious. And we’ll pick back up the removed later” used with a participant who was working on modeling disease transmission was coded as B3.23 Narrowing Scope because the interviewer suggested the participant to ignore the removed population. Because the move referred to the distinct

populations identified by the participant (susceptible, infectious, recovered), we inferred it to refer to the Real Model stage of the MMC. For example, the code B3.24 Documenting Writing was an interviewer move that merely sought to clarify what the participant wrote down (e.g., Does that say “ x ” or “ α ”?) and was not coded as referring to a stage of model construction.

This qualitative analysis along with descriptive statistics enabled us to examine the characteristics of the B3 scaffolding moves in two ways. First, for each scaffolding move, independently from the B3 code it is assigned to, reflects a combination of model construction stages it refers to. These combinations can be aggregated across B3 codes to give a sense of the relative frequency each stage is referred to within a given B3 code. For example, the proportion of B3 instances referring to both the Real Model and the Math Model was 39.31%. Second, each B3 code can be said to have a *profile* for the stages of model construction its instances refer to. For example, the code B3.7 Encourage to Make Assumptions or Simplify contained instances that typically referred to Real stages of model construction. We noticed that some profiles referred to stages of model construction associated with the Real World (i.e., situation model (SM), real model (RM), and real results (RR)) others referred only to stages of model construction associated with the Math World (i.e., mathematical model (MM) and mathematical results (MR)), and some profiles referred to both. Hence, we used the analysis of B3 code instances in terms of model construction stage to collapse to four broad categories: Real (scaffolding move refers to situation and/or real model), Math (scaffolding move refers to mathematical model), Both (scaffolding move refers to both Real and Math), and Neither (scaffolding move refers to neither Math nor Real). To fully characterize the differences in uptake of MMC codes according to whether each B3 category contained instances that primarily referred to Real, Math, Both, or Neither stages of model construction, we developed a set of regression models. For each process we fit a mixed logistic regression model with a participant random effect, u_i , to account for dependence (Agresti, 2012):

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 \text{Real} + \beta_2 \text{Math} + \beta_3 \text{Both} + u_i$$

To test our hypothesis that the induced MMC fingerprint would depend only on the stage profile of the B3 codes (i.e., that in the aggregate, the B3 codes have clear associations with sets of elicited modeling processes) we conducted a two-step cluster analysis. We first clustered the B3 code instances based on their Stage profiles. Then, within each Stage Profile cluster, we clustered the B3 codes according to the MMC processes elicited by the codes in that sub-cluster.

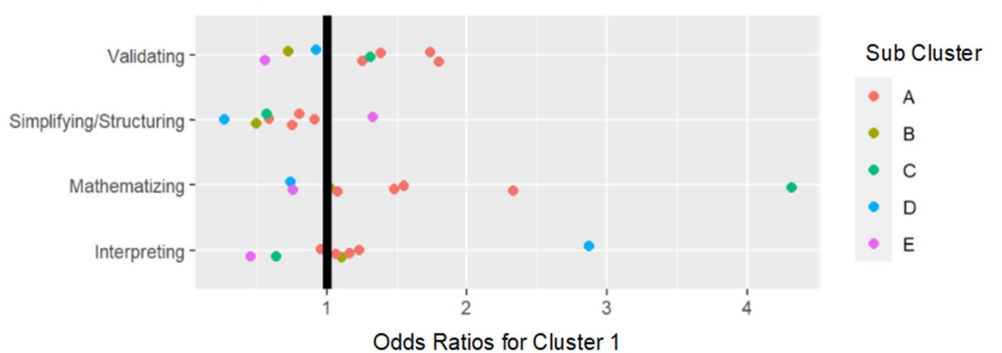


Figure 1 Sample of odds ratios for Cluster 1, from the two-step cluster analysis. Each dot represents a B3 code category. Cluster 1 contains B3 codes whose instances tend to target Both mathematical and real-world stages of model construction. Subcluster 1-A tended to promote validating, mathematizing, and interpreting while suppressing simplifying/structuring.

We refer to the elicited MMC processes as the induced MMC Fingerprint associated with a given B3 code's Stage Profile.

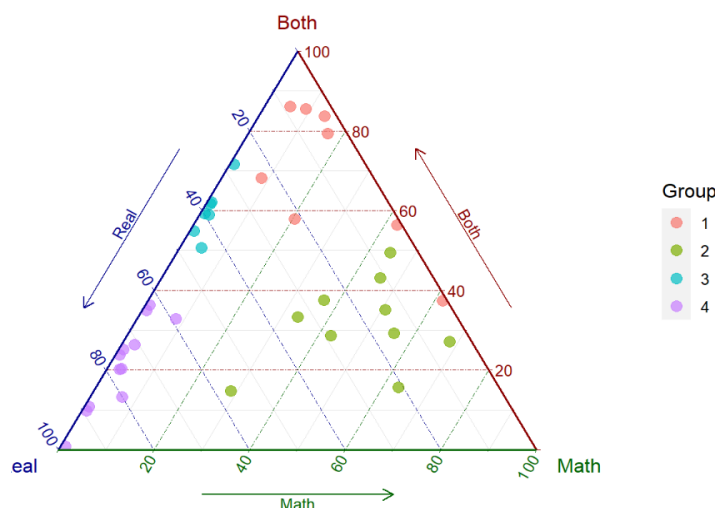


Figure 2 Ternary graph depicting clusters of B3 codes according to Stage Profile. Colors are re-used and do not indicate the same cluster membership as the colors in Figure 2. Figure 1 contains only codes from Group 1 in Figure 2.

The result was four clusters each containing between three and five sub-clusters. Odds greater than 1 indicated that, in the aggregate, moves in the given B3 code elicit the given modeling process at a higher rate than the null model would predict (see Figure 2 for Cluster 1 results). For example, the code B3.14 Suggestion for action, related to content (next mathematical step or analysis) strongly promotes validating (odds = 2.21) while the code B3.11p suppresses simplifying/structuring (odds = 0.14). We observed good separation between the clusters based on Stage profile, as illustrated in the ternary graph in Figure 1. Each point within the ternary graph can be thought of as a triple corresponding to a Stage profile (% Real, % Math, % Both). For example, the apex which is labeled Both corresponds to a (fictional) B3 code whose moves were always classified as Both and never classified as either Real (only) or Math (only) and the Cluster 3 codes tend to contain moves that do not refer to Math (only).

This seemed a good sign and so we qualitatively characterized all clusters and all 16 subclusters (impossible to report here and it would bore the reader if we did). We found that while quantitative analysis grouped B3 codes together based on the similarity of their profiles of model construction stage, moves within each cluster induced dissimilar MMC fingerprints and moves within the subclusters were often difficult to characterize. This suggested we should further analyze the similarities and differences of B3 codes within each Stage Profile cluster as well as examine the expression of B3 moves within each B3 code to explain each cluster's capacity to elicit (or suppress) multiple modeling processes. For example, both B3.22 Modify Scenario and B3.23 Narrowing Scope are B3 moves which were grouped into the same Stage Profile, and both moves elicit the *simplifying/structuring* and *mathematizing* modeling processes. Thus, while Study II did indicate that the modeling processes elicited by the various scaffolding moves depended, at least in part, on the Stage the moves in the category tended to target, the grouping criteria Real/Math/Both/Neither was too coarse to facilitate theory-driven interpretations for many of the sub-clusters. We discuss the next steps in the discussion, next.

Discussion

Our investigations confirmed that the MMC Fingerprint induced by categories of scaffolding moves does depend on model construction Stage. This finding partially supports previous assertions (see, for example Schukajlow et al., 2015; Stender & Kaiser, 2015) that successful scaffolding depends on which stage or phase of the modeling cycle a move is relevant to. However, Stender and Kaiser (2015)'s assertion implies that the key to understanding successful scaffolding lies in diagnosing which phase of the modeling cycle the modeler is *in* and conditioning the contingent facilitator response on that information. In contrast, we observed that many of the interviewer's scaffolding moves -- when contingent to the modeler's mathematical reasoning and accounted for what had been produced so far by the modeler -- may or may not be related to the stage or phase of modeling cycle the modeler finds themselves in by the time the interviewer intervenes.

Our analysis challenges two other naïve expectations about associating interviewer moves and modeler engagement through the modeling cycle. First, one might expect that if the interviewer provides scaffolding for phase N of the modeling cycle that the modeler would respond by progressing to phase $N + 1$. That does not seem to be the case. For example, there were sub-clusters whose MMC Fingerprints were characterized by *interpreting* that followed from interviewer moves that focused on the situation model. Underlying the first expectation is a deeper assumption that the appropriate thing to do if a modeler needs scaffolding for stage M of model construction is that the proffered support should target stage M . This also seems not to be the case. Our analysis of uptake of scaffolding moves was based on the antecedents to participants' elicited modeling processes. The moves were contingent to the participants' in-the-moment needs and often targeted Stages of model construction that did not correspond to the contemporaneous stage that the participant was in. Thus, paralleling well-known results that students' routes through modeling cycles are nonlinear and idiosyncratic (Ärleback & Bergsten, 2010; Borromeo Ferri, 2006; Czoher, 2016), it seems that one should also not expect that promoting student progress in their model construction would be as simple as targeting specific stages or phases of the cycle.

The quantitative analysis established associations between scaffolding moves and the modeling processes they elicited. Some interviewer moves that mapped to the same sub-cluster elicited very similar MMC fingerprints suggesting that the moves either functioned similarly or co-occurred frequently enough that their MMC uptake was similar. We also found that interpreting these associations was complicated by a lack of theoretical coherence within some clusters. In particular, moves that seemed similar based on the code descriptions or move expression may not have similar Stage profiles and therefore did not cluster together. This observation knocked down a third naïve hypothesis that moves that are behaviorally described similarly would have a similar impact on modeling processes. Additionally, our results suggest that the number of B3 codes could be streamlined by examining co-occurrences of similarly functioning code categories. As our investigation has laid the groundwork for the two-step clustering approach, future work can examine the uptake resulting from a streamlined B3 codebook. This next step would bring the goal of estimating the likelihood that a given scaffolding move would induce a particular modeling process within reach.

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References

- Agresti, A. (2012). *Categorical data analysis* (Vol. 792). John Wiley & Sons.
- Ärleback, J., & Albarracín, L. (2019). The use and potential of Fermi problems in the STEM disciplines to support the development of twenty-first century competencies. *ZDM: the international journal on mathematics education*, 51, 979-990.
<https://doi.org/10.1007/s11858-019-01075-3>
- Ärleback, J., & Bergsten, C. (2010). On the Use of Realistic Fermi Problems in Introducing Mathematical Modelling in Upper Secondary Mathematics. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies* (pp. 597-609). Springer. https://doi.org/10.1007/978-1-4419-0561-1_52
- Blum, W., & Leiß, D. (2007). How do Students and Teachers Deal with Modelling Problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling: Education, Engineering and Economics* (pp. 222-231). Woodhead Publishing.
<https://doi.org/https://doi.org/10.1533/9780857099419.5.221>
- Borromeo Ferri, R. (2006). Theoretical and Empirical Differentiations of Phases in the Modelling Process. *ZDM*, 38(2), 86-95. <https://doi.org/10.1007/BF02655883>
- Çakmak Gürel, Z. (2023). Teaching mathematical modeling in the classroom: Analyzing the scaffolding methods of teachers. *Teaching and Teacher Education*, 132.
<https://doi.org/10.1016/j.tate.2023.104253>
- Clement, J. (2000). Analysis of Clinical Interviews: Foundations and Model Viability. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research methodologies for science and mathematics education* (pp. 341-385). Lawrence Erlbaum.
- Creswell, J. W. (2014). *Research Design: Qualitative, Quantitative and Mixed Methods Approaches* (4th ed.). SAGE Publications, Inc.
- Czocher, J. A. (2016). Introducing Modeling Transition Diagrams as a Tool to Connect Mathematical Modeling to Mathematical Thinking. *Mathematical Thinking and Learning*, 18(2), 77-106. <https://doi.org/10.1080/10986065.2016.1148530>
- Durandt, R., Blum, W., & Lindl, A. (2021). Fostering mathematical modelling competency of South African engineering students: which influence does the teaching design have? *Educational Studies in Mathematics*, 109(2), 361-381. <https://doi.org/10.1007/s10649-021-10068-7>
- Goldin, G. A. (2000). A Scientific Perspective on Structured, Task-Based Interviews in Mathematics Education Research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517-545).
<https://doi.org/10.4324/9781410602725.ch19>
- Greefrath, G., Siller, H.-S., Klock, H., & Wess, R. (2021). Pre-service secondary teachers' pedagogical content knowledge for the teaching of mathematical modelling. *Educational Studies in Mathematics*, 109(2), 383-407. <https://doi.org/10.1007/s10649-021-10038-z>
- Hallström, J., & Schönbörn, K. J. (2019). Models and modelling for authentic STEM education: reinforcing the argument. *International Journal of STEM Education*, 6(1).
<https://doi.org/10.1186/s40594-019-0178-z>
- Hankeln, C. (2020). Mathematical modeling in Germany and France: a comparison of students' modeling processes. *Educational Studies in Mathematics*, 103(2), 209-229.
<https://doi.org/10.1007/s10649-019-09931-5>

- Hu, D., & Rebello, N. S. (2013). Using conceptual blending to describe how students use mathematical integrals in physics. *Physical Review Special Topics - Physics Education Research*, 9(2). <https://doi.org/10.1103/PhysRevSTPER.9.020118>
- Kaiser, G. (2017). The Teaching and Learning of Mathematical Modeling. In J. Cai (Ed.), *Compendium for Research in Mathematics Education* (pp. 267-291). National Council of Teachers of Mathematics.
- Koichu, B., & Harel, G. (2007). Triadic Interaction in Clinical Task-Based Interviews with Mathematics Teachers. *Educational Studies in Mathematics*, 65, 349-365. <https://doi.org/10.1007/s10649-006-9054-0>
- Maaß, K. (2010). Classification Scheme for Modelling Tasks. *ZDM - Mathematics Education*, 31, 285-311. <https://doi.org/10.1007/s13138-010-0010-2>
- Maher, C. A., & Sigley, R. (2014). Task-Based Interviews in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer Science+Business Media Dordrecht. <https://doi.org/10.1007/978-94-007-4978-8>
- Saldana, J. (2016). *The coding manual for qualitative researchers*. SAGE Publications Ltd.
- Schukajlow, S., Kolter, J., & Blum, W. (2015). Scaffolding Mathematical Modelling with a Solution Plan. *ZDM Mathematics Education*, 47, 1241-1254. <https://doi.org/10.1007/s11858-015-0707-2>
- Stender, P. (2016). *Wirkungsvolle Lehrerinterventionsformen bei komplexen Modellierungsaufgaben*. <https://doi.org/10.1007/978-3-658-14297-1>
- Stender, P., & Kaiser, G. (2015). Scaffolding in Complex Modelling Situations. *ZDM Mathematics Education*, 47, 1255-1267. <https://doi.org/10.1007/s11858-015-0741-0>
- Tropper, N., Leiss, D., & Hänze, M. (2015). Teachers' Temporary Support and Worked-Out Examples as Elements of Scaffolding in Mathematical Modeling. *ZDM Mathematics Education*, 47, 1225-1240. <https://doi.org/10.1007/s11858-015-0718-z>
- van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in Teacher–Student Interaction: A Decade of Research. *Educational Psychology Review*, 22, 271-296. <https://doi.org/10.1007/s10648-010-9127-6>