Developing CALC: Developing Conceptual Assessments of Learning in Calculus

Beth Cory
Sam Houston State University

Taylor Martin
Sam Houston State University

Michael Oehrtman
Oklahoma State University

William Robertson
West Virginia University

Vicki Sealey
West Virginia University

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Introductory calculus is a gateway to study in STEM disciplines. It introduces concepts about rate of change, accumulation, and co-variation, which are foundational concepts for modeling across STEM fields. Thus, assessing student understanding of calculus concepts is important to researchers and practitioners. Currently, no curriculum-independent concept inventory for introductory calculus has been validated and found to conform to accepted educational testing standards (American Educational Research Association, 2014; Gleason, White, Thomas, Bagley, & Rice, 2015). In 2007, Epstein and colleagues presented the first Calculus Concept Inventory (CCI) to evaluate the effectiveness of teaching methods on conceptual learning in calculus. However, the inventory was not research-based, was narrowly designed as a single multiple-choice summative exam (Epstein, 2007, 2013), and Gleason et al. (2015) found that the CCI failed a thorough psychometric analysis. Our purpose is to develop a reliable, well-validated curriculum-independent concept assessment to test productive understandings of calculus.

Instead of developing one assessment for all of calculus, we decided smaller, modular assessments for the various calculus topics would be more helpful. Initially, we focused on the topic of definite integrals and accumulation. We created a taxonomy of concepts related to definite integrals and further narrowed our initial focus to definite integrals involving area or other applications. Literature on student understanding of definite integrals shows two main themes. First, students who view integrals as adding up pieces and understand the pieces quantitatively are more successful in problem solving (e.g. Sealey, 2014; Simmons & Oehrtman, 2023). Second, many students over-rely on integrals as area without being able to unpack the meaning of the quantities that form the area (e.g. Sealey, 2006; Jones, 2015).

Productive understandings involve procedural and conceptual understandings. We utilize Hiebert and Carpenters' (1992) distinction between *procedural understanding*, where students solve problems using a predetermined sequence of actions, and *conceptual understanding*, where students solve problems by relying on their networks and relationships of ideas within the problem. Since many seemingly conceptual calculus problems are familiar to students and can be solved with only a procedural understanding, we aimed to create conceptual questions which, in particular, test students' ability to reverse-engineer why their problem-solving methods work.

We developed and refined seven initial assessment items which contained both standard tasks and slight variations, which were intended to disrupt application of memorized procedures and required application of conceptual knowledge. Based on clinical interviews with 15 students at 3 institutions, we observe that students' responses to these variations reveal insight that may further distinguish productive conceptual understanding from memorized procedures. Future research will involve designing additional small suites of conceptual questions on derivatives and limits. We will eventually invite other researchers to contribute to the development of these assessments in other areas of calculus.

References

American Educational Research Association. (2014). *Standards for educational and psychological testing.* Washington, DC: American Educational Research Association.

Epstein, J. (2007, September). Development and validation of the Calculus Concept Inventory. In *Proceedings of the ninth international conference on mathematics education in a global community* (Vol. 9, pp. 165-170). Charlotte, NC.

Epstein, J. (2013). The calculus concept inventory – Measurement of the effect of teaching methodology in mathematics. *Notices of the American Mathematical Society, 60*(8), 2-10.

Gleason, J., White, D., Thomas, M., Bagley, S., and Rice, L. (2015) The calculus concept inventory: A psychometric analysis and framework for a new instrument, *Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education,* 135-149.

Hiebert, J. & Carpenter, T. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 65Y97). New York: Macmillan.

Jones, S. (2015): The prevalence of area-under-a-curve and anti-derivative conceptions over Riemann sum-based conceptions in students' explanations of definite integrals, International Journal of Mathematical Education in Science and Technology, DOI: 10.1080/0020739X.2014.1001454

Oehrtman, M., & Simmons, C. (20 23). Emergent Quantitative Models for Definite Integrals. International Journal of Research in Undergraduate Mathematics Education, 9(1).

Sealey, V. (2006). Definite Integrals, Riemann Sums, and Area under a Curve: What is necessary and sufficient? In Alvatorre, S., Luis Cortina, J., Saiz, M. and Mendez, A. (Eds.). *Proceedings of the Twenty-Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* [CD-ROM].

Sealey, V. (2014). A Framework for Characterizing Student Understanding of Riemann Sums and Definite Integrals. Journal of Mathematical Behavior, 33, 230–245.