Exploring Student Strategies for Matrix-Vector Multiplication in a Linear Algebra Video Game

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In this work, we investigate the second level of the video game *Vector Unknown: Echelon Seas* (*VUES*, Zandieh, et al., 2018; Mauntel et al., 2019; Mauntel et al., 2020; Mauntel et al., 2021), which was designed to integrate elements of Game-Based Learning (GBL; Gee, 2003; Williams-Pierce & Thevenow-Harrison, 2021), Inquiry-Oriented Learning (IOI; Rasmussen & Kwon, 2007; Zandieh, Wawro, & Rasmussen, 2017), and Realistic Mathematics Education (RME; Gravemeijer, 1994; Freudenthal, 1991). The game, *VUES*, is set within a 3-dimensional environment, providing a unique platform for players to engage with concepts of vectors in a dynamic way. In this level, players use matrix-vector multiplication to aim and fire cannonballs at targeted ships.

Players are given a target vector that is represented by a ship. Players pick one of four matrices, represented by different cannonballs, and one of four cannon vectors, represented by different cannons. After choosing a cannonball, there is a geometric support that shows the player visualize a lattice generated by the column vectors of the matrix. The goal of the game is to choose the matrix and cannon vector that multiply together to form the ship vector. Through this process, players can explore the calculation of different matrix-vector multiplications from a transformation point of view. The game also provides a geometric visualization related to the column vectors of the matrix. By exploring these different interpretations of matrix multiplication, students can deepen their understanding of how matrix transformations influence vector directions in a practical and engaging context. Our research question is: What strategies do students use to complete the second level of *VUES*?

To investigate students' strategies in playing this level, we conducted three paired-teaching experiments (Steffe & Thompson, 2000), each lasting approximately one hour and 15 minutes. These students are currently enrolled in a linear algebra course and have prior experience with 3D games. During the interviews, students were asked to explain their strategies for choosing matrices and vectors and describe how they anticipated the cannonball's trajectory. This approach allowed us to explore their strategies for solving and completing the video game related to matrix-vector multiplication. The interviewers scaffolded information about how to use the grid lines at various levels of gameplay.

In the poster, we will present our preliminary analysis of student strategies. In each interview, the student fired at least 10 cannonballs. When firing the first few cannonballs, most students used guess and check. Some students used a numeric form of guessing and checking by choosing one cannonball (matrix) and multiplying the first row of the matrix by a vector to check whether the result matched the first element of the target vector or not. Others used a geometric form of guessing and checking that involved the geometric support from the game, which included a provided trajectory line based on their choice of cannonball and cannon, to check the multiplication results.

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