

“Why Would a Woman Put Themselves in This Situation:” A Discourse Analysis of Gender and Pronouns in Solving a Hat Problem

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The language we use to talk about, do, and learn mathematics informs how we socially construct our understandings thereof. This preliminary report discusses an instance from a different study where the use of feminine pronouns to describe smart logicians in a particular mathematical problem resulted in various reactions from participants. I analyzed participants’ interactions with the given task using an approach to discourse analysis that examines how the grammatical structures of language are used to construct particular meanings. Preliminary findings showcase how participants’ responses to the task reflect broader cultural understandings of who can be a smart logician. These findings are linked to the need for representation in curricular materials and the use of hat problems in the learning of logic and problem solving.

Keywords: word problems, logic, pronouns, gender, discourse analysis

Language is our primary medium for talking about, doing, and learning mathematics; as such, the mathematical language we use informs how we socially construct our understandings of mathematics (Planas et al., 2021). Knowing this, we may strive to use language to forward particular goals in mathematics. When teaching, we may use established formal mathematical language to enculturate students into a community of professional mathematical practice, or we may create new mathematical language to be culturally responsive to the backgrounds of our current students. When constructing mathematical word problems, we may choose to vary the demographic characteristics of characters to subvert the white cisheteropatriarchal and masculinized nature of mathematics (Leyva, 2017, 2021). One such variation through the subtle deployment of the feminine pronoun *her* is the catalytic element for this manuscript.

While conducting another study (see Bernier, 2024), I asked participating undergraduate students to engage with mathematical tasks from a collection by Roeder (2018). The focal task discussed in this manuscript is excerpted here (emphasis added):

Three *smart logicians* are standing in a line, so that they can only see the *logicians* in front of them. A hat salesman comes along and shows the three *logicians* that he has three white hats and two black hats. He places one hat on each *logician*’s head and hides the remaining hats.

He then says to the logicians, “Can anyone tell me what color hat is on *her* own head?” No one responds.

He repeats, “Can anyone tell me what color hat is on *her* own head?” Still no answer.

A third time: “Can anyone tell me what color hat is on *her* own head?” One of the logicians speaks up and gives the correct answer.

Who spoke, and what color hat is on *her* head? (Roeder, 2018, p. 21)

When I designed the study, the fact that the problem used feminine pronouns to refer to the smart logicians was not noted as significant or unusual. Yet, as students arrived at this task in their

respective interviews, they reacted to it in ways that seemed notable and reflective of broader mathematical and societal discourses.

After completing the earlier study, I decided to examine this phenomenon further. In this manuscript, I report preliminary findings from a discourse analysis examining how participants used gendered language in discussing and solving this problem. Before I discuss my analysis procedures and preliminary findings, I briefly discuss hat problems and the matter of representation in curricular materials.

What Color Hat is on Her Head? – Hat Problems

Hat problems, like the one included in Roeder (2018), are a common type of logic puzzle used across various works and contexts (Brown & Tanton, 2009; Howse et al., 2018). These problems involve the use of logical deduction and mathematical induction to determine the color of a hat on a particular character's head through the progressive dissemination of information and elimination of impossible scenarios. They can be made increasingly complex by altering the quantities of participants, colors, and hats, and the means of dissemination of information (Brown & Tanton, 2009). The iteration of the problem used in Roeder (2018) is isomorphic to one that has existed at least since 1995, with a previous iteration using male prisoners as the characters in place of smart logicians (Gerald & Mario, 2000).

Representation in Curricular Materials: It Matters

All learners deserve to see themselves and others reflected in the materials they use to learn (Gutiérrez, 2017). Indeed, many scholars have discussed at length the need for curricular materials to carefully and appropriately reflect a diversity of identities in order to promote equitable learning (Hickman & Porfilio, 2012; Macintyre & Hamilton, 2010; Mills & Mustapha, 2015). Given that cultural images of mathematical professionals are overwhelmingly masculine (Mendick et al., 2008), mathematical tasks which depict women as mathematically capable could be useful in encouraging women to participate in mathematics. The task as published in Roeder (2018) may (or may not) have been written with that intent, given its logical equivalence to but narrative distinction from earlier-stated hat problems. The problem was originally a reader submission (Roeder, 2017), but the provenance of the reader's version is unknown.

Discourse Analysis

For this manuscript, discourse is taken to be the use of language, spoken or written, to communicate ideas and actions (Johnstone, 2018). Analyzing discourse is a worthy endeavor because discourse is both shaped by and shapes people and social structures (Johnstone, 2018; Planas et al., 2021). Learning in undergraduate mathematics classrooms involves discourse (Hetrick et al., 2023; Rios, 2023) – both the discourse occurring in the room and the *Discourses* (Gee, 2014a) which comprise what it means to be a student, a learner, or a mathematician.

Of course, given the broad definition of discourse above, most modes of qualitative analysis constitute an “analysis of discourse.” Indeed, many different forms and approaches to discourse analysis have been developed and pursued across the social sciences and within educational research (Johnstone, 2018; Warriner & Anderson, 2017). I largely adopt Gee's (2014a, 2014b) approach, where the structure of the discourse is critical to its analysis and where “meaning [is] an integration of ways of saying (informing), doing (action), and being (identity), and grammar [is] a set of tools to bring about this integration” (2014a, p. 8). In particular, I draw on the “tools” which Gee (2014b) characterizes as “specific questions to ask of data” (p. 2).

Methods

This manuscript examines a phenomenon that occurred while I was researching another phenomenon (see Bernier, 2024). As such, the data for this manuscript was not collected with the analysis presented here in mind. The relevant aspects of that study are described here, as well as the analytic approach I used for this manuscript.

Study Context and Data Extraction

The larger study from which the relevant data is extracted sought to deepen our understanding of how undergraduate students can engage in play when solving puzzles and mathematical problems (Bernier, 2024). To do so, I conducted a series of four clinical interviews (Hunting, 1997) one-on-one with six undergraduate students (Table 1). In these interviews, participants either played digital puzzle games or solved mathematical tasks taken from collections of “mathematical puzzles.” The second interview included three problems from Roeder (2018), including the task discussed in this manuscript.

Table 1. Summary of participants

Pseudonym	Gender Identity	Pronouns
Batman	Woman	she
Daniela*	Woman	she
David Wallace	Man	he
Pattakoby	Non-binary	he/they**
Shane	Man	he
Timothy Burns	Woman	she

*All participants chose their own pseudonym, except Daniela who declined to do so; the name Daniela was chosen arbitrarily.

**For readability, I use only masculine pronouns for Pattakoby.

For this manuscript, I pulled data from the transcripts already used for prior analyses. I located within each participant’s second interview transcript any discussion of the relevant task and extracted all such lines in chronological order. I then placed the extracted segments into a single Microsoft Word document for analysis.

Analysis Procedures, or, What’s in my Toolkit?

My approach to data analysis here was primarily guided by four tools from Gee’s (2014b) toolkit: the Deixis Tool (p. 16), the Fill In Tool (p. 18), the Why This Way and Not That Way Tool (p. 53), and the Figured Worlds Tool (p. 177). The Deixis Tool involves identifying and examining the shifting references (deictics) such as pronouns used by speakers. Since the gender of the logicians in the task was communicated via deictic, these seemed especially salient. I identified each use of a gendered pronoun and changed the font to an assigned color – orange for he/him/his, turquoise for she/her/hers, and purple for they/them/theirs. I also underlined other gendered words which took on “deictic like properties” (Gee, 2014b, p. 16), like guy and person. This process identified many (though not all) utterances worthy of further analysis.

While the Deixis Tool allowed me to identify critical moments in the data, the Fill In Tool and Why This Way and Not That Way Tool allowed me to get at the deeper meaning. According to Gee (2014b), these two tools examine the unstated implications of speakers' grammatical choices. With the Fill In Tool, I examined different segments for "[w]hat is not being said overtly, but is still assumed to be known or inferable" (p. 18). The Why This Way and Not That Way Tool led me to consider how else a participant might have said or phrased the same sentiment, and what the implications are of their chosen phrasing. To apply these tools, I read through the extracted segments, highlighting and annotating moments where the implications of their speech and choices were apparent. For example, this included highlighting each instance of an incongruous pronoun being used (e.g., **he** or **they** being used to describe a singular logicians). Comments then discussed the apparent implications of these grammatical choices.

Finally, I linked the local discourses in my data to larger discourses by applying the Figured Worlds Tool. To use the Figured Worlds Tool, one must "ask what typical stories or figured worlds the words and phrases of the communication are assuming and inviting listeners to assume" (Gee, 2014b, p. 177) In this case, the relevant Figured World seemed to be that of doing logic/mathematics and/or being a logician/mathematician

Preliminary Findings

Apparent throughout all interviews was a tendency of all participants – and myself as an interviewer – to at least sometimes refer to the logicians using incongruous pronouns. In some cases, these are fairly well explained by considering the aesthetics of the utterance. For example, Shane made the utterance: "there's three people, five different color hats, and one of **them** knows what color hat **they** have. In this utterance, Shane's use of **them** is congruous with the logicians being women, because it is being used as the third-person plural. The use of **they**, on the other hand, is incongruous, because its referent is "one of **them**," a singular smart logician. But, saying "one of **them** knows what color hat **she** has" could feel awkward or unintuitive.

On the other hand, some of the instances where participants used incongruous pronouns seem less innocuous. Pattakoby, for example, read the task aloud – but when he did, he verbally replaced "**her**" with "**their**" initially, and later used masculine deictics:

He places one hat on each logician's head and hides the remaining hats. **He** then says to the logicians, "Can anyone tell me what color hat is on **their** own head?" No one responds...

...and because the one in the middle knows that the **guy** standing in front of **him** has a white, possibly the **guy** in the middle can guess...

Pattakoby and Shane did not ever verbally acknowledge the use of feminine pronouns in the task description. Applying the Why This Way and Not That Way Tool and the Figured Worlds Tool leads me to conjecture that they did so because in their figured worlds a "smart logician" would be a man – perhaps to the point of not even accurately parsing the words in front of them, or implicitly assuming that the use of feminine pronouns was a mistake.

Several of the other participants did make note of the use of feminine pronouns, with many of them questioning the meaning of said use. Batman and David Wallace both phrased their questions similarly, with Batman asking, "Who is **her**?" and David Wallace asking "Why does **he** say **her** own head?" My responses to Batman and David Wallace did not acknowledge the gender of the characters and were intended to clarify the problem:

Yes, **they** were supposed to see if **they** can figure out what color hat is on **their** own head. So each person in that line is trying to, from whatever **they** can see, figure out if **they** know what color hat is on **their** head, which **they** can't see what color hat is on **their** head.

Using the Fill In Tool and Why This Way and Not That Way tool, it seems I misinterpreted these questions during the interview as being requests for clarification of the task goals. But, Batman and David Wallace both made the use of the word *her* central to their question, which implies that they were particularly confused about the use of a feminine pronoun to refer to the smart logicians. My own use of gender-neutral pronouns in my response is likely reflective of not having carefully considered the task's use of pronouns prior to the interviews and internalized subconscious biases I hold within my own Figured Worlds.

Finally, Timothy Burns and Daniela both noted the gender of the smart logicians explicitly. Timothy Burns asked the question in the title of this manuscript: "Which type of scenario is this and why would a woman put themselves in this situation?", while Daniela initially thought the use of *her* was meant to supply some sort of clue regarding the solution to the problem:

It does say that the *person* who spoke up and gave the correct answer was a girl because it says, who spoke and what color is on *her* head? I don't know if that's important though because it doesn't mention anything about gender besides that. Or, wait, actually *they* might all be girls because *he* says, the salesman asks all of the logicians. Can you tell me what color is on *her* own head?

Applying the Fill In Tool, both Timothy Burns and Daniela observe that the logicians are meant to be women. However, neither seems to be evaluating this problem as a plausible scenario in the first place, with Daniela seeming to interpret the task like a language riddle while Timothy Burns seemed to reject the whole problem as absurd. In other words, their reaction to the task did not relate as much to their understanding of *who* can do mathematics, but instead, of *what* features of a mathematics task are and are not important.

Limitations and Implications

Given preliminary findings based on a secondary and unplanned-for analysis to observe a phenomenon, I must be extremely cautious in suggesting implications of this work. Especially important is my lack of use of the Frame Tool (Gee, 2014b, p. 44), which notes the importance of considering ever-increasing layers of context when conducting discourse analysis. I have not fully considered, for example, how what I know about participants' backgrounds and prior experiences may have informed their reactions to the task, nor how the interview context might be different than the classroom context in informing these reactions.

Still, I believe this analysis illustrates a broader notion that work to subvert inequities in mathematics is never easy or free. I cannot speak to the intent of whoever decided that the logicians in the problem should be women, but I know that I myself have made similar changes in producing curricular materials. The intent of this, on my part, has been that small changes to make materials more representative might help to undermine established cultural images bit-by-bit. What we see in this analysis, though, is that changes which are too small may be overlooked or ignored. While it is compelling to not want to make a "big deal" about representation – and a desire to avoid tokenism or otherwise "singling out" representation of women and other marginalized groups – it seems there is a danger too in being too subtle. At a minimum, instructors and researchers should be careful – more careful than I was – in examining the potential implications and anticipating the reactions of learners when interacting with tasks that include characters from marginalized groups and subvert cultural expectations or norms.

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References

- Bernier, J. (2024). *Exploring characteristics of play in the puzzle and mathematical problem solving of undergraduates* [Doctoral dissertation, Arizona State University].
- Brown, E., & Tanton, J. (2009). A dozen hat problems. *Math Horizons*, 16(4), 22–25. <https://doi.org/10.4169/194762109x468517>
- Gee, J. P. (2014a). *An introduction to discourse analysis: Theory and method* (4th ed.). <https://doi.org/10.1353/jsh.2010.0055>
- Gee, J. P. (2014b). *How to do discourse analysis: A toolkit*. Routledge. <https://doi.org/10.4324/9781315819662>
- Gerald, E., & Mario, U. (2000). The prisoner problem--a generalization. *The Mathematics Teacher*, 93(3), 192–193.
- Gutiérrez, R. (2017). Living Mathematx: Towards a vision for the future. *Proceedings of the 39th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Indianapolis, Indiana
- Hetrick, C., Herbst, P., Ion, M., & Milewski, A. (2023). Building instructional capacity across difference: Analyzing transdisciplinary discourse in a faculty learning community focused on geometry for teachers courses. In S. Cook, B. Katz, & D. Moore-Russo (Eds.), *Proceedings of the 25th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 915–924).
- Hickman, H., & Porfilio, B. J. (Eds.). (2012). *The new politics of the textbook: Problematizing the portrayal of marginalized groups in textbooks*. Sense Publishers.
- Howse, J., Stapleton, G., Burton, J., & Blake, A. (2018). Picturing problems: Solving logic puzzles diagrammatically. *CEUR Workshop Proceedings*, 2116, 12–27.
- Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *Journal of Mathematical Behavior*, 16(2), 145–165. [https://doi.org/10.1016/s0732-3123\(97\)90023-7](https://doi.org/10.1016/s0732-3123(97)90023-7)
- Johnstone, B. (2018). *Discourse Analysis* (3rd ed.). John Wiley and Sons, Inc.
- Leyva, L. A. (2017). Unpacking the male superiority myth and masculinization of mathematics at the intersections: A review of research on gender in mathematics education. *Journal for Research in Mathematics Education*, 48(4), 397. <https://doi.org/10.5951/jresmetheduc.48.4.0397>
- Leyva, L. A. (2021). Black women's counter-stories of resilience and within-group tensions in the white, patriarchal space of mathematics education. *Journal for Research in Mathematics Education*, 52(2), 117–151. <https://doi.org/10.5951/jresmetheduc-2020-0027>
- Macintyre, T., & Hamilton, S. (2010). Mathematics learners and mathematics textbooks: a question of identity ? Whose curriculum ? Whose mathematics ? *The Curriculum Journal*, 21(1), 3–23. <https://doi.org/10.1080/09585170903558224>
- Mendick, H., Moreau, M.-P., & Hollingworth, S. (2008). Mathematical images and gender identities. In *Research Report Series for UKRC* (Issue 6).
- Mills, S., & Mustapha, A. S. (Eds.). (2015). *Gender representation in learning materials: International perspectives*. Routledge.
- Planas, N., Morgan, C., & Schütte, M. (Eds.). (2021). *Classroom research on mathematics and language: Seeing learners and teachers differently*. Routledge.

- Rios, J. (2023). Resources for participation: Examining multilingual students' perspectives on what it means to participate in active learning. In S. Cook, B. Katz, & D. Moore-Russo (Eds.), *Proceedings of the 25th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 502–510).
- Roeder, O. (2017, April 28). Can you solve these colorful puzzles? *FiveThirtyEight*. <https://fivethirtyeight.com/features/can-you-solve-these-colorful-puzzles/>
- Roeder, O. (2018). *The Riddle: Fantastic puzzles from FiveThirtyEight*. W.W. Norton & Company.
- Shea, S. (2012). Teaching the blue-eyed islanders puzzle in a liberal arts mathematics course. *Primus*, 22(7), 538–544. <https://doi.org/10.1080/10511970.2011.647257>
- Warriner, D., & Anderson, K. T. (2017). Discourse analysis in educational research. In K. A. King, Y.-J. Lai, & S. May (Eds.), *Research methods in language and education* (pp. 297–309). Springer International Publishing.