Mathematical Modeling and Quantitative Reasoning: The Case of Task Design

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Mathematical modeling, the process of representing a real-world situation through mathematics, has gained increasing interest across K-16 education. Engaging in modeling requires certain competencies that are unique for solving modeling tasks. Despite the prevalence of modeling across curricula, numerous studies have documented the challenges students face as they engage in modeling (e.g., Klock & Siller, 2020). One promising approach for supporting learners' modeling competencies is to have students engage in quantitative reasoning as they construct models for real-world situations (Czocher et al., 2022). Quantitative reasoning entails conceiving a situation consisting of quantities and quantitative relationships (Thompson, 2011). Scholars have argued that quantitative reasoning not only supports students' understanding of *mathematical content* (e.g., Carlson et al., 2015) but also *mathematical processes* such as proofs and modeling (e.g., Belin & Akar, 2020). However, the first step in systematically supporting students' modeling competencies via their quantitative reasoning involves developing modeling tasks that are the ideal learning ecologies for students' quantitative reasoning to emerge. The central question that guides this study is: *In what ways can quantitative reasoning be integrated into the design of modeling tasks?* We address this question by designing a model development sequence (Lesh & Doerr, 2003) that integrates modeling principles with quantitative reasoning.

The model development sequence has three stages *Model Eliciting* (phase 1), *Model Exploration* (phase 2), and *Model Adaptation* (phase 3). In phase 1, students will develop strategies for building models and constructing quantitative relationships. In phase 2, students will analyze the models they constructed through examining the relationship between quantities and their variations. In phase 3, students will apply and refine the techniques developed in phase 1 and phase 2 to model a more complex, open-ended situation with limited information. We developed four sequences each consisting of three tasks. The sequences are *optimization*, *proportionality*, *flow*, and *growth*. In this poster we highlight the optimization sequence, where students use their algebraic knowledge to construct, analyze, and interpret functions that model real-world situations. Phase one (*food truck task*) involves modeling the relationship between demands and profit of a food truck business. Phase two (*catering*) involves modeling the profit for a catering company based on seating capacity. The instructional goal for the students is to construct a piecewise profit function for varying seating capacities. Phase 3 (*honey business*) involves modeling local business' profit changes due to COVID. The instructional goal for the students is to model the relationship between price, demand, and profit through building on and expanding on the quantitative reasoning strategies developed in earlier tasks.

To test the feasibility of our tasks, we intend to implement our tasks with undergraduate students in a clinical setting. Following a design-based experiment (Brown, 1992) methodology, results from the clinical interviews will be used to modify the existing tasks or design new ones if needed. This study ultimately will provide insights into the characteristics modeling tasks that have the capacity to elicit students' quantitative reasoning.

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