

The Landscape of Research on the Meta-mathematical Aspects of Definitions

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Meta-mathematical aspects of definitions (MMAD) are the general notions held about the nature, roles, functions, and characteristics of mathematical definitions (Gilboa et al., 2023). Mathematics education research has produced several rich branches of research exploring the meta-mathematical conceptions that proof learners, educators, mathematicians, and researchers hold about mathematical definitions (Edwards & Ward, 2008; Gilboa et al., 2023; Torkildsen et al., 2023; Zaslavsky & Shir, 2005). Researchers have found the MMAD play a role in students' success in producing mathematical proofs and developing their conceptions of mathematical definitions (Gilboa et al., 2023; Torkildsen et al., 2023). While the MMAD have been discussed both implicitly and explicitly in mathematics education literature, we have anecdotally noticed a trend of conflation in these aspects that may lead to confusion for novice researchers. In this project, we seek to review MMAD literature produced by the top ten mathematics education research journals to characterize the conceptions held about the MMAD by the mathematics education community. We seek to answer the questions: What MMAD have been most and least explored by mathematics education researchers? How has extant literature characterized these MMAD? What group has been most represented in research exploring conceptions of MMAD?

We conducted a systematic review of the top ten mathematics education journals, as identified by Williams and Leatham (2017). These include *Educational Studies in Mathematics*, *Journal for Research in Mathematics Education*, *Journal of Mathematical Behavior*, *For the Learning of Mathematics*, *Mathematical Thinking and Learning*, *Journal of Mathematics Teacher Education*, *ZDM: The International Journal on Mathematics Education*, *Mathematics Education Research Journal*, *International Journal of Mathematics Education in Science and Technology*, and *School Science and Mathematics*. The search was performed by identifying articles with the keywords “function of definition,” “definition proof,” “mathematical definitions,” and “definition usage” in the body. A total of 108 articles were identified. The articles were coded separately by two researchers, searching for mentions of the meaning of mathematical definitions and concepts, with special focus on the meta-mathematical aspects (i.e. roles, functions, criteria, properties, and usage) of definitions. After disagreements were resolved, articles not implicitly or explicitly discussing the meta-mathematical aspects of definitions were excluded, resulting in 66 articles for further analysis in nine of the ten journals.

In this poster, we plan to share preliminary findings related to the implicit and explicit descriptions of MMAD as well as the meta-mathematical focus of the articles. These articles will be categorized according to the specific categories of the meta-mathematical aspects of definitions discussed. We offer a framework to clarify distinctions in the categories of MMAD, and thus aid in the streamlining of future MMAD research. Finally, we will describe the demographic (i.e. undergraduate students, pre-service teachers, mathematicians) that has been most-often featured in meta-mathematical definition research. During the poster session, we intend to gather feedback from the RUME community on future directions of the study and questions about mathematical definition research the community would like answered.

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