

Undergraduate Students' Conceptions of the Function of Mathematical Definitions in Proving

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In mathematics education, researchers have attempted to elaborate on many meta-mathematical aspects of mathematical definitions by exploring novice and expert conceptions of the function of definitions in proving (Edwards & Ward, 2004, 2008; Gilboa et al., 2023; Parameswaran, 2010; Torkildsen et al., 2023; Zaslavsky & Shir, 2005). We define *students' conceptions of the function of mathematical definitions* in proving as the work a mathematical definition does for a student during proving (i.e. what a student uses or believes they can use a definition for during the proving process). While studies have suggested that novice provers (undergraduates) may have differing conceptions from expert provers (graduates or mathematicians) about the function of mathematical definitions in proving (Edwards & Ward, 2004; Zaslavsky & Shir, 2005), these studies are almost entirely observational, interpreted through the lens of the proof educator/researcher (Gilboa et al., 2023; Haj-Yahya et al., 2023; Tabach & Nachlieli, 2015). To date, there exists a paucity of research characterizing students' conceptions of the function of mathematical definitions in their own words: the language of the novice prover. In this project, we sought to answer the question: What conceptions do novice provers have about the function of definitions in mathematical proving?

Methods

This study was conducted in two introduction-to-proof (ITP) courses at two primarily undergraduate universities in the Eastern United States. Written reflection data were collected from a total of 26 undergraduate students enrolled in the ITP courses at two different points in the semester. Following in-class collaborative proving activities relating to logical equivalence and rational numbers, participants were prompted to write a response to, "What function did the mathematical definition for [topic] serve in your proof construction attempts?" Responses to this question were first open coded (Saldaña, 2016) and then open codes were sorted into thematic categories. Finally we deductively coded the data with these categories. All data were coded by both authors, and we discussed any disagreements until 100% consensus was met.

Findings

Through our deductive analysis we identified ten different conceptions that students held regarding the function of mathematical definitions in proving. The conception most frequently cited by participants (n=19, 73.1% of reflections) involved how definitions provided something operable within their proving. For example, Shawn wrote, "the definition was our cornerstone[...] it was about manipulating and applying them to form a coherent and sound proof." We will include a complete summary of the findings in our poster presentation, including examples of students' conceptions from each theme. Through improving the RUME community's knowledge of students' conceptions about the function of definitions in their proofs, this project can help proof instructors and researchers leverage students' conceptions into their development as authentic provers. During the poster session, we will solicit feedback for how to continue this investigation and questions about students' conceptions of the function of mathematical definitions that the RUME community would like to see answered in the future.

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