Students' Limit Conceptions and Representational Approaches

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Investigating students' thinking about the limit concept in calculus through representational approaches supports the need for students to learn mathematical concepts with deep understanding and actively build new knowledge (NCTM, 2000/2014). This study takes an asset-based perspective and focuses on students' reasoning, views, and understandings of the limit concept rather than focusing on students' misconceptions and lack of understanding, as some past research has done (Bezuidenhout, 2001; Denbel, 2014; Fernández, 2004; Liang, 2016; Nagle et al., 2017). Additionally, some studies have investigated students' abilities to reinvent the conventional $\varepsilon - \delta$ limit definition (Swinyard, 2011; Swinyard & Larsen, 2012) but have not explicitly addressed how students' reasoning with representational approaches plays a role in advancing to the more formal limit definition. This poster presentation shares a portion of the results from a larger study on students' limit conceptions (Intuitive, Informal, and Formal) and their representational approaches (Graphical, Numerical, and Algebraic) to understanding the limit concept. The two research questions are:

1. *What limit conceptions do students possess?*
2. *What connections exist between students' limit conceptions and representational approaches?*

To address these research questions, pre-assessment surveys were collected from 63 students enrolled in Calculus II from a public university in the southeastern United States. The surveys have six limit-view statements (adapted from Williams, 1991/2001) that constitute the three main classifications of limit conceptions (Intuitive, Informal, Formal) and five multiple/short answer limit problems that constitute the three main representational approaches (Graphical, Numerical, Algebraic). To address research question 1, we analyzed the frequency counts for students' preferred choice of the limit-view statements that best expressed their understanding. To answer research question 2, we compared students' responses to the six limit-view statements to their choice of representational approaches to the five multiple/short answer problems.

We found that the intuitive limit-view had the most significant frequency, which could suggest that students begin to conceptualize limits from an intuitive perspective. Additionally, we found that students' *intuitive limit conceptions* are tied to graphical reasoning, and students' *informal limit conceptions* are tied to numerical reasoning. The results of this study are consistent with Adiredja et al.'s (2020) suggestion that the teaching and learning of abstract concepts in more tangible ways should be done based on students' intuitive reasoning.

Implications of this study offer both calculus instructors and curriculum reformers ways to integrate students' ways of reasoning into calculus teaching and learning. Specifically, results suggest that calculus instructors may leverage different representational approaches to provide opportunities for students to deepen their conceptual understanding of limits.

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