Case Studies of Undergraduate Students' Agentive Participation in the Parallel Spaces of Calculus I Coursework and Peer-Led, Inquiry-Oriented, Complementary Instruction

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Calculus has long been known as a "gateway course" to STEM fields in postsecondary education. To address this issue, researchers in the Math Department at Montclair State University designed a model of complementary instruction that features peer-facilitated workshops where Calculus I students work in groups on inquiry-oriented, groupworthy tasks. The purpose of this multiple-case study is to seek answers to the question, "How do undergraduate Calculus I students experience and navigate their learning of calculus in the parallel spaces of coursework and inquiry-oriented complementary instruction?" The findings of one case study are presented here and include characterizations of the different forms of agentive participation afforded to students in each of the two spaces, as well as their complementary nature relative to learning calculus with understanding. Implications for dismantling the persistent barriers imposed by calculus on access to postsecondary STEM fields are also discussed.

Keywords: Calculus, Complementary Instruction, Agency, Participation

Calculus has historically operated as a "gateway course" to STEM fields in postsecondary education (Hagman et al., 2017). In the hopes of transforming calculus education to be "lean and lively," the calculus reform movement in the 1990s called for a change in calculus instruction to include fewer topics and utilize an active and engaging approaches to teaching and learning (Johnson et al., 2014). Two decades later, the President's Council of Advisors on Science and Technology (2012) proposed a similar suggestion in an effort to provide students with the time necessary to develop deep and conceptual understandings of calculus. Unfortunately, despite the ongoing efforts to reform calculus education, calculus maintains its gate-keeping role.

The Insights and Recommendations from the Mathematical Association of America (MAA) (Bressoud et al., 2015) suggested seven essential practices for establishing a successful calculus program: (1) attention to the effectiveness of course placement procedures; (2) proactive student support services, including the fostering of student academic and social integration; (3) construction of challenging and engaging courses; (4) the use of student-centered pedagogies and active-learning strategies; (5) coordination of instruction, including the formation of communities of practice for instructor learning; (6) effective training of graduate teaching assistants; and (7) regular use of local data to guide curricular and structural modifications. Informed by these recommendations, researchers at Montclair State University designed a peer-led (Roth et al., 2001), inquiry-oriented *complementary* workshop that runs parallel to students' learning in class in order to address this pressing issue. Calculus I students in these workshops (Yu & Seventko, 2015) work collaboratively on deliberately designed groupworthy tasks (Buell et al., 2016; Cohen & Lotan, 2014) that address calculus concepts.

A review of the literature on peer-led cooperative learning models in postsecondary education confirms their effectiveness in various undergraduate mathematics courses in relation to students' academic achievement and other outcomes (Altomare & Moreno-Gongora, 2018; Liou-Mark et al., 2015; Trenshaw et al., 2019). As this body of literature evaluates effectiveness using quantitative methods, little is known as to how and why peer-led cooperative learning models yield the outcomes found in these studies. The study reported here aims to fill this research gap as it seeks to answer to the question, *How do undergraduate students experience their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?*

Perspectives and Methods

This multiple-case study (Merriam, 1998) is framed from a situated perspective (Lave & Wenger, 1991), and uses Holland et al.'s (1998) concept of *figured worlds* to analyze changes in agentive participation and in relation to identity formation (Vågan, 2011). The unit of analysis is forms of agentive participation enacted by students in class and in workshop. A grounded theory analytical approach (Corbin & Strauss, 2014) is used to characterize students' participation in order to answer the research question. With the agentive participation codes as clusters, a word cloud for each class and workshop space was created for each case study participant to depict a summary overview of their enacted participation in each of the two spaces.

Two cohorts of Calculus I students who attended the workshops as part of their course requirements participated in the study. Each cohort consists of four participants from the same class taught by the same instructor. The data corpus consists of video recordings of 24 classes, six workshops, and three focus group interviews (Creswell, 2012), all of which were transcribed and subject to analysis.

Findings

The table in Figure 1 lists the various forms of participation that emerged from the grounded theory analyses of the two cohorts. These participation actions were further sorted into three categories of interactivity: *high*, *moderate*, and *nominal* activity. *Interactivity* describes students' interaction with their peers, tasks, or material resources. The *high interactivity* category describes agentive actions involving a high level of interaction among students, such as inquiring, sharing, and explaining. Agentive actions in this category involve the conceptual practices of making associations and connections among mathematical concepts (Pickering, 1995), which are the kinds of high cognitive demand (Stein et al., 2000) practices that support learning with understanding (Hiebert & Grouws, 2007). The *moderate interactivity* category describes agentive actions that entail independent work on executing procedures, providing brief responses to dichotomous questions with binary answers (e.g., yes/no or right/wrong), and seeking confirmation of ideas or the correctness of a solution. Lastly, agentive actions in the *nominal interactivity* category involve limited interactions with others and material resources, such as *note-taking* and *launching tasks*. Agentive actions in these latter two categories are considered low cognitive demand because they entail memorizing or carrying out procedures without making connections to facts, procedures, and ideas (Stein et al., 2000).

In addition to yielding the emergence of these codes and categories, the analysis also revealed periods of "integration" (assimilating norms and expectations) and "expansion" (growth in participation) in the students' participation, which I was able to discern using sequences of Venn diagrams that show the trajectory of students' participation over time. Given the space constraints, this phenomenon will be presented in-depth in my presentation should this proposal be accepted. In order to illustrate how participation codes and interactivity categories are used to address the research question, my analysis of Boris's case is presented next. I chose to present Boris's case because it accentuates the unique opportunities to enact different forms of agentive participation in each of the two spaces.

Categories of Interactivity	Level 1 Codes	Cohorts A & B		
		Class A	**Class B**	**Cohorts A & B Workshop**
High	Sharing	(Voluntary [Answer] [Idea] [Resources] [Work]) (Upon request [Answer] [Idea] [Resources] [Work]) (Solicit [Answer] [Resources] [Work])		(Voluntary [Answer^{A}] [Idea^{A}] [Resources^{A}] [Work^{A}]) (Upon request [Answer^{A}] [Idea^{A}] [Resources^{A}] [Work^{A}]) (Solicit [Answer^{A}] [Idea] [Work^{A}]) (Offer [Work] [Idea])
	Inquiring	(Conceptual) (Procedure)	(Procedure)	(Conceptual^{A}) (Other mathematical) (Procedure^{A})
	Scaffolding			(Scaffolding)
	Explaining	(Concept [Representation]) (Mistake [Peer's]) (Procedure) (Struggle) (Task) (Technicality)	(Mistake [Instructor's]) (Procedure) (Reasoning)	(Concept [Definition] [Representation^{A}]) (Mistake [Facilitator's] [Peer's^{A}] [Self]) (Notation) (Procedure^{AB}) (Provide Example) (Reasoning^{B} [Realistic]) (Struggle^{A}) (Task^{A}) (Technicality^{A})
Moderate	Independent work	(Student initiated [Task] [Review] [Homework]) (Instructor initiated)	(Student initiated [Task] [Review] [Homework]) (Instructor initiated)	(Student initiated [Task^{AB}]) (Facilitator initiated)
	Seeking	(Confirmation) (Clarification [About something] [For someone]) (Help) (Resources) (Time)	(Confirmation) (Clarification [About something]) (Help)	(Confirmation^{AB}) (Help^{AB}) (Resources^{A}) (Time^{A}) (Clarification [About something^{AB}] [For someone^{A}])
	Responding	(Agree/Disagree) (Answer) (Confirm) (Respond to help request) (Private) (Uncertain) (Unfamiliar)	(Agree/Disagree) (Answer) (Confirm) (Private) (Unfamiliar)	(Agree/Disagree^{AB}) (Answer^{AB}) (Confirm^{AB}) (Respond to help request ^{A}) (Uncertain^{A}) (Unfamiliar^{AB})
	Check-in	(Peer) (Self)	(Self)	(Peer^{AB}) (Self^{AB})
	Check (and revise)	(Compare) (Other's) (Self)	(Self)	(Compare^{A}) (Other's^{A}) (Self^{AB})
	Accessing resources	(Lesson) (Notes) (Online resources)	(Homework) (Notes) (Online resources) (Textbook)	(Homework^{B}) (Notes^{AB}) (Online resources^{AB})
Nominal	Agency request unfulfilled	(Public) (Private)	(Public)	(Private^{A})
	Refraining	(Refraining)	(Refraining)	
	(Re)launches task	(Read aloud)	(Read aloud)	(Read aloud^{AB}) (Recite info) (Invitation to work on problem)
	Emoting	(Affirmation) (Confusion) (Frustration) (Success)	(Affirmation) (Confusion) (Frustration) (Success)	(Affirmation^{AB}) (Confusion^{AB}) (Frustration^{AB}) (Relief) (Success^{AB})
	Note-taking	(Note-taking)	(Note-taking)	(Note-taking^{AB})
	General coursework	(Give) (Seeking)	(Seeking)	(Give^{AB}) (Seeking^{AB})
	Non-participation	(Non-participation)	(Non-participation)	(Non-participation^{AB})

(Lvl 2 code [Lvl 3 code] [Lvl 3 code])
(Lvl 2 code [Lvl 3 code^{ClassA}] [Lvl 3 code$^{Class\ B}$]) = Occurred in both spaces

Figure 1. A table of participation actions in class and workshop across both cohorts.

Boris's Participation Profile

The instructor of Boris's class tended to teach through lectures and demonstrations of solving problems on the board. It was rare for the instructor to engage students by asking questions or providing problems for individual, in-class practice. Occasionally, however, the instructor implemented what he called a "homework active learning activity," where he would guide the class in solving a selected homework problem by having students take turns responding to his guiding questions. During this homework activity, the instructor would pose an assortment of moderate and higher interactivity questions to lead students through the problem-solving process.

Across 22 in-person class observations, when the instructor offered students an explicit opportunity to participate, Boris *refrained* from participating 136 times, averaging around six times per observation. [Note: Words in italics are participation codes.] He only *responded* to the instructor when the instructor directly asked him questions during a homework activity. In general, Boris was an *independent* and *resourceful* student in class and an *explainer* and problem-solving leader in workshop. Even though a comparison of his class and workshop word clouds (Figure 2 and Figure 3) shows that Boris *worked independently* in both spaces, his characteristic *independent work* was magnified in class. The *independent work* cluster supports this observation as it is the largest cluster in his word cloud, taking up nearly half the space. On the other hand, the *independent work* cluster in Boris's workshop word cloud is only the fifth largest cluster, which depicts his tendency to initiate *independent work* moments in class by working on homework assignments or on the instructor's examples on the board. Along with *working independently* on problems in class, Boris often *accessed resources* (e.g., textbook, notes, and other online resources) to support his sense making and problem solving. Hence, the *accessing resources* cluster is the second largest cluster in his class word cloud. Considering his active participation in workshop and the limited opportunities to enact agency or work on practice problems in class, it can be inferred that Boris was inclined to agentively pursue learning on his own, even when these opportunities were not explicitly presented to him.

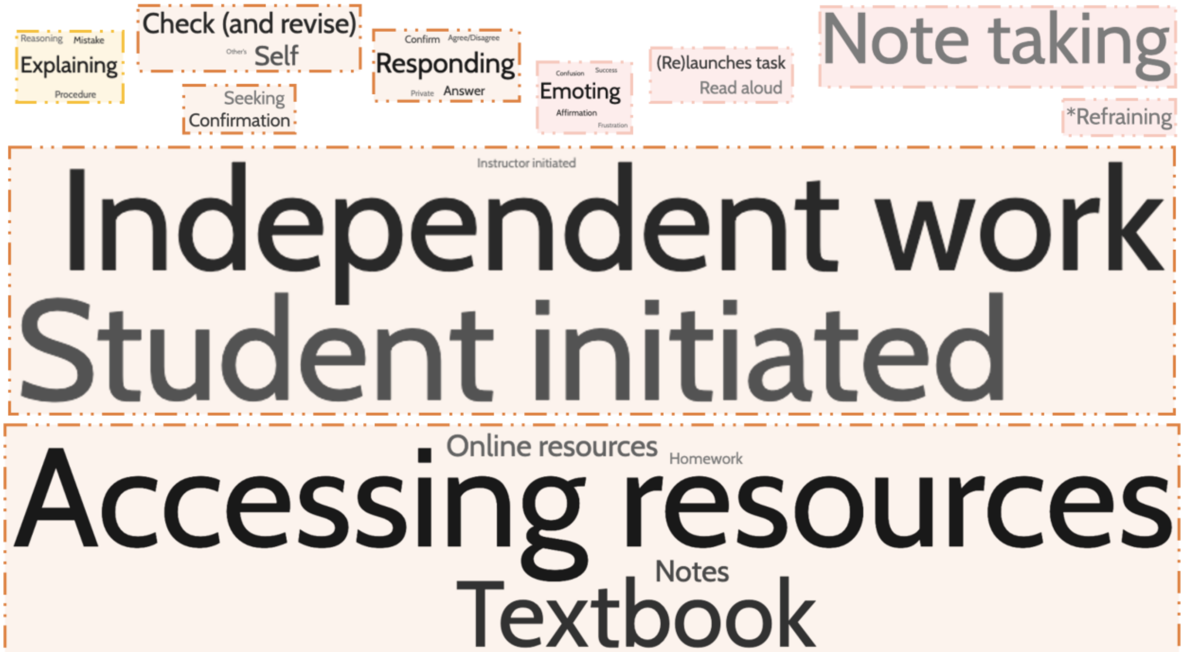

Figure 2. Boris's class word cloud.

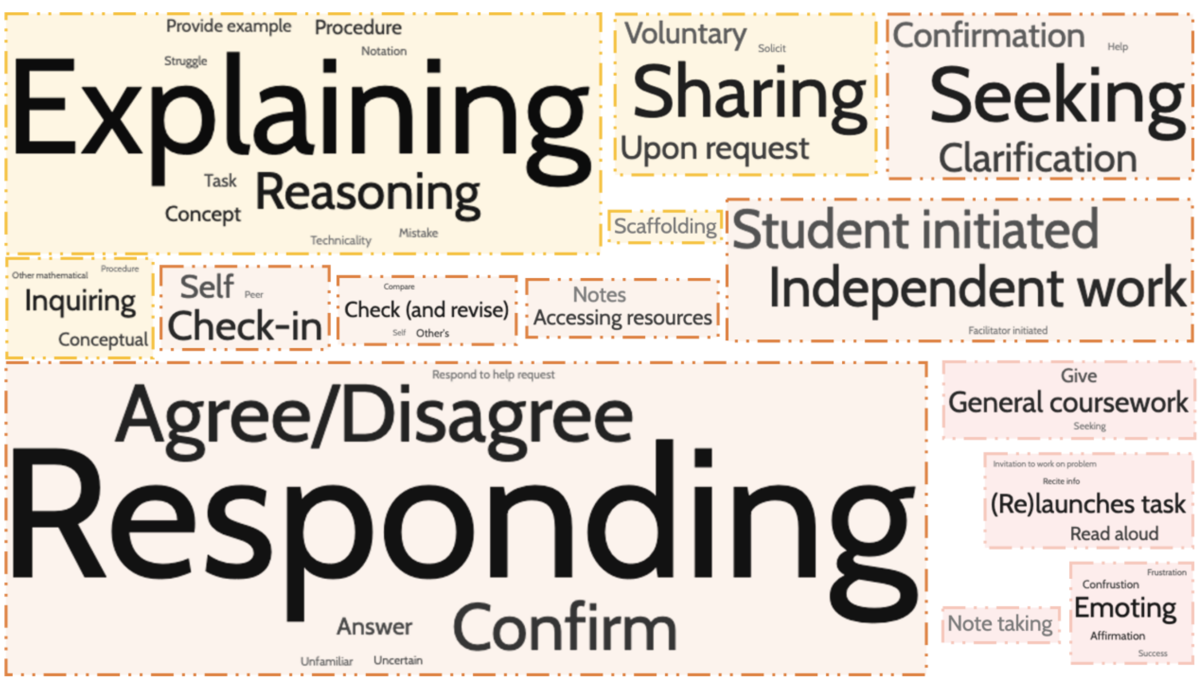

Figure 3. Boris's workshop word cloud.

Rather than choosing to *work independently*, as was his tendency in class, Boris tended to take on the highly interactive roles of a *responder*, *explainer*, and problem-solving leader in workshop. As a *responder*, his responses tended to articulate his *confirmation* and *agreement* with ideas shared by others. Compared to the rare occasions he enacted *explaining* in class, Boris had opportunities to enact *explaining* more frequently and extensively in workshop. This is evident from the *explaining* cluster as the second largest cluster in his workshop word cloud. Specifically, his workshop word cloud indicates his tendency to *explain concepts* and *procedures* and provide *reasonings* and *examples* in his explanations. Overall, Boris's role as a problem-solving leader in workshop was portrayed through his *explaining* actions and the occasional acts of *scaffolding* his cohort peer's problem solving.

Boris's case illustrates the differential forms of participation he enacted in class and in workshop. A review of his participation profile highlights these rather distinctive opportunities to enact agentive participation in each of the two spaces. In class, Boris tended to enact low-demand independent participation actions in moderate and nominal interactivity categories (i.e., *independent work*, *accessing resources*, and *note-taking*). In contrast, in workshop, Boris was more inclined to enact high demand participation actions in the high and moderate interactivity categories (i.e., *explaining*, *sharing*, and *seeking*).

As depicted in Figure 1, cohort A's participants had more opportunities to enact higher interactivity moves in class (i.e., explaining, inquiring, and sharing) than participants in cohort B, of which Boris was a member. Nonetheless, class and workshop were found to complement each other to offer all the participants a broad range of agentive actions. Given the value of highly interactive participation actions in particular, it is critical for students to have more of these in order to better support and enhance their learning.

Discussion and Conclusion

Reflecting on the MAA's seven recommendations for establishing a successful calculus program, the findings from this study can be used to inform calculus instruction by illustrating opportunities for high and moderate interactivity participation actions that can be enacted through student-centered pedagogies and active learning strategies (recommendation 4) in coursework or in complementary instructional workshop. Additionally, the participation codes observed in this study give a vision to calculus instructors of the kinds of interactive participation that are characteristic of challenging and engaging courses (recommendation 3). In turn, these findings can inform and guide the design and implementation of parallel spaces of coursework and complementary instruction, particularly when the realities of coursework alone impose constraints that do not allow for adequate opportunities for high and moderately interactive participation.

To summarize, this multiple-case study sought to address the research question, *How do undergraduate students experience their calculus learning in the parallel spaces of coursework and inquiry-oriented complementary instruction?* This study found a range of agentive participation actions that were further categorized into *high*, *moderate*, and *nominal interactivity* categories based on the quality of their interactions with others, tasks, or material resources. All in all, these findings would be of value to postsecondary calculus educators and program directors who are committed to offering students the kinds of participatory experiences that are productive for their learning of calculus. That way, they can be more mindful in planning, structuring, and designing their calculus programs so as to dismantle the persistent barriers imposed by calculus on access to postsecondary STEM fields.

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