What Mathematicians' Emotions Can Teach Us About Groupwork and Emotion Regulation

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Two mathematics professors at a large university were interviewed about how their emotions impact their research work. Joy and frustration were the emotions they reported experiencing most frequently. Both reported the benefits of collaboration and the importance of regulating both positive and negative emotions. Implications on collaborative learning in the classroom (e.g. groupwork, pair work) and implications for emotions in the classroom are discussed.

Keywords: emotions, collaborative learning, emotion regulation

This paper brings together research on emotions in mathematics education, collaborative learning, and the professional work of mathematicians.

Literature Review

Emotions and Mathematics

There has been an increased emphasis on research on emotions and affect in mathematics education (McLeod, 1992; Middleton et al., 2017). In recent years, there have been more publications in mathematics education journals focused on emotions and affect, and the number of publications appears to be continuing to increase (Schukajlow et al., 2023). Both negative emotions (D'Mello et al., 2014; Schukajlow et al., 2021) and positive emotions (Camacho-Morles et al., 2021) have been shown to be beneficial to student learning. However, both negative and positive emotions have been shown to be detrimental to student learning (Barenes, 2021; Villavicencio et al., 2016). Villavicencio et al. (2016) found that pride had a negative association with final course grades among engineering students in the Philippines. Barnes (2021) found that joy was associated with decreased focus and self regulation.

Collaborative Learning

Research on collaborative learning tends to focus on social components or the connections between collaborative learning and mathematical ideas (e.g. Langer-Osuna et al., 2020; Dekker et al., 2006; Fujita et al., 2019; Sjöblom et al., 2021).

Additionally, research on collaborative learning in classrooms includes work on collaboration promoting students building off of each other's ideas which in turn promotes individual learning (Francisco, 2013; Schindler & Bakker, 2020).

Some work has been done demonstrating emotional benefits of collaborative learning for students. Group work can have a positive impact on student problem posing (Schindler & Bakker, 2020). Students also report higher levels of perceived control (Pekrun, 2006) when working in pair or small groups which is associated with higher reported positive emotions (Bieg et al., 2017).

Mathematicians as Research Participants

Research on mathematicians has focused mainly on the cognitive aspects of their work. Researchers have explored mathematicians' thinking and problem solving abilities (Burton, 1998; Carlson & Bloom, 2005). Other work has shown that mathematicians enjoy collaboration with others, and that part of this enjoyment is through getting the intellectual input from

colleagues on their work (Burton, 1998). Mathematicians' negative feelings of being stuck on solving a problem and potential implications on students' negative emotions towards mathematics have also been discussed in the literature with regards to choosing particular problems to do (Misfeldt & Johansen, 2015).

However, research focused specifically on mathematician's emotions appears to be a new area of study. The previously mentioned literature is focused on the cognitive side of mathematics rather than emotional components of doing and learning mathematics.

Theoretical Perspective

Two areas of psychological research provide a background for this study: the Broaden and Build Theory of Positive Emotions and The Cognitive Theory of Emotions. In relation to the mathematicians in this study, these theories imply that a) positive emotions provide two distinct benefits and b) emotions are the result of cognitive appraisals of events.

Broaden and Build Theory of Positive Emotions

The Broaden and Build Theory describes two distinct benefits of positive emotions beyond just feeling good (Fredrickson, 2001). First, positive emotions broaden awareness. When people experience positive emotions, they are more aware of "the bigger picture" and notice more than when they are feeling negative emotions. Second, this broadened state of awareness created by positive emotions allows individuals to build more resources for themselves. The explanatory power of this theory has been demonstrated by multiple authors (e.g. Fredrickson, 2004; Garland et al., 2010; Vacharkulksemsuk & Fredrickson, 2013).

For this study, this creates the perspective that positive emotions could give mathematicians benefits beyond just feeling good. Positive emotions could allow them to potentially work more creatively or be interested in solving new problems.

Cognitive Theory of Emotions

The Cognitive Theory of Emotions posits that cognitive appraisals of a situation are what triggers an emotional response to that situation (Ortony, Clore, & Collins, 2022). For example, a student taking a test realizes they do not know how to answer the questions. The student wants to do well on the test, but now knows that they will fail. Thus, the student feels anxious because of their appraisals that a) the test is important and b) they are unable to succeed. It is not the test itself which makes the student anxious. It is the student's interpretation of the situation that leads to the emotions.

Research Questions

The focus of this research was investigating the emotions experienced by mathematicians, and their interpretations of significance of these emotions. The guiding research questions were: (1) How can the emotions experienced by mathematicians be characterized?, (2) In what ways do mathematicians feel that emotions impact their work?, (3) In what ways does collaboration with other mathematicians relate to these emotions?

Methods and Analysis

Two active research mathematicians at a large research university in the Southwestern United States participated in interviews over Zoom. Professors were recruited via an email sent to all full time faculty members of the mathematics department. The participants will be referred to as Professor 1 and Professor 2.

Both participants were full research professors at the university, and they both described themselves as theoretical mathematicians and both were male.

The interviews lasted between 30 and 40 minutes. Participants were asked about their current mathematical research interests and the role that their emotions play in their work. They were asked to describe positive and negative emotions felt while doing their research. The interviews were loosely structured around the impact of the participants' emotions on their research, as well as emotions related to collaboration with other researchers. Interviews were recorded over Zoom then transcribed.

Due to the limited research on mathematicians' emotional experiences, the interviews were coded using open coding to identify common themes of mathematicians' emotions (Strauss & Corbin, 1990). After completing the interviews, field notes were taken by the researcher on any details from the interviews which felt relevant in the moment. Then, the interviews were transcribed. The transcribed interviews were read with the goal of identifying specific emotions experienced and how the participants felt the emotions impact their work. Preliminary categories of emotions were grouped together. Afterwards, the interviews were reread multiple times to compare the categories found with the interviews.

Results

Joy and frustration were the most common emotions reported. Without being prompted, both participants discussed the importance of emotion regulation of both joy and frustration in order to do their work effectively. Both participants also discussed the impact of collaboration on their emotions. These results are discussed in more detail below.

Joy: A Common Positive Emotion

Joy was the positive emotion that both participants described as feeling most often while doing their research work.

Collaboration Enhances Joy Both participants felt that working with others increased their joy in research. Professor 1 said, "I think it [collaboration] enhances the joy. So when you prove something, you're happy to prove it. But, you're even more happy if you could show it to someone." Being able to share their positive feelings with others amplifies the joy that these mathematicians feel from their work. Professor 2 said, "If I'm working with a co-author, then it's nice to have it [an idea] affirmed by the co-author. And then, when that happens, that feels really good."

Emotional Regulation of Joy However, these mathematicians did not feel that joy and positive emotions should be fully embraced. When discussing joy, both participants felt that joy should be treated with some caution and distrust while doing their work. After a probing question on Aha! moments of sudden realization (Liljedahl, 2005), Professor 2 said, "I always have to temper my enthusiasm… you get a stroke of brilliance…want to come back and write it down… Oh no, that's not gonna work. So I'm careful to not get too excited until I write it down, and see if it all works."

Professor 1 shared a story of another mathematician who would just lay down in bed and enjoy the feeling of believing he had figured something out because, in Professor 1's words, "9 times out of 10 - what you've done is wrong".

Frustration: A Common Negative Emotion

Both professors described frustration as their most commonly experienced negative emotion. Again, both spoke of the importance of regulating that frustration.

Collaboration Helps Regulate Frustration Both professors discussed collaboration as a method of dealing with the inevitable frustrations of research. Co-authors were mentioned by both participants as a useful tool to offset feelings of frustration.

Professor 1 said, "It [collaboration] may actually spread it [frustration] out so that it's more tolerable. It's more tolerable to be frustrated if you're frustrated at the same thing that somebody else is frustrated at." After a slight pause, he continued and said, "Isn't that the reason for help groups?" in reference to group therapy. Professor 1 felt strongly that collaborative work helped him deal with the frustration of being stuck on a problem. Being able to share the load of frustration with another made the frustration easier to bear.

Regulation of Frustration in Other Ways - Taking Breaks Professor 2 spoke frequently of the importance of regulating frustration through taking breaks from research work. Specifically, he said "the other thing I've always done throughout my career is, I do a lot more than just research. Like, I teach classes. I get involved with some administrative work and I think for me that was always important. I didn't really like to have a job where 100% of the time is spent doing research". He also discussed that stepping back from a problem and looking at examples is a "first way of relieving the frustration" for him.

Professor 2 actively works to keep frustration at bay and prevent burnout in his work.

Another Benefit to Collaboration: A New Perspective

Both professors described the benefits of being able to use a colleague's perspective to benefit their work.The perspective of others helps them get through when they are stuck and having trouble making progress.

Professor 1 describes it as, "You try to prove things, and you know at least 9 tenths of the time. You can't do it. So yeah, you have to keep trying, and that's another reason why it's worthwhile to work with other people. Because you have to have lots of ideas to try it. I mean if you can't do something… You never know whether that's because you're being stupid, or because it's not true or really hard.".

Professor 2 says, " We can share what we're stuck on with the other. And you know, maybe the coauthor oftentimes sees something that you don't see And then that's a a way to make a forward progress".

Discussion

Both collaboration and emotion regulation appear to play an important role in the work of mathematicians. Limitations and future directions will be discussed.

Limitations

Emotions are a vulnerable topic to discuss, thus the topic itself creates many limitations. Validity of self-reported data on emotions depends on many factors. First, participants may not be fully honest with their self-reported emotions (Rasinski et al., 2005). Also, social pressure exists against reporting negative emotions (Tourangeu & Yan, 2007). Social pressure may have presented itself in this study through participants focusing on overcoming and regulating the negative emotion of frustration while not discussing the actual experience of frustration in much detail. In other words, social pressure against negative emotions may have led the participants to focus the discussion on ending negative emotions rather than discussing the experience of negative emotions. An additional limitation is the possibility that participants may not have the emotional intelligence (Salovey & Mayer, 1990) to recall and name their emotions.

All of the limitations discussed above are out of the control of the researcher, but should be considered when interpreting the results.

Future Directions

Emotions and Collaboration in the Classroom As discussed in the literature review above, most research on collaborative learning focuses on cognitive rather than emotional factors in the classroom. Considering the results of this study, future research could examine how students can provide emotional support for each other while working through problems.

Collaboration is an important part of their work for these mathematicians. Yet group and pair work in the classroom is not frequently used, while lecture style direct instruction is likely the most common form of instruction (Bieg et al., 2017; Givvin et al., 2005). Both positive and negative emotions were shown to be an important part of mathematicians' work, and collaboration has a positive effect on both the positive and negative emotions. Confusion, an emotion similar to frustration, has been shown to be beneficial for learning in adults if the confusion is resolved (D'Mello et al., 2014). Thus, lecture style teaching in which confusion and frustration are avoided may be detrimental to student learning. Science education research has seen a push to engage students in ways that mimic the feelings of scientists (Jaber & Hammer, 2016). Perhaps, a similar movement can occur in mathematics education.

Emotion Regulation for Students With the discussed importance of emotion regulation of both positive and negative emotions, future research can examine ways of promoting this regulation among students. Some existing work provides promising results from an intervention on helping students cope with confusion in mathematics at the elementary level (Di Leo & Muis, 2020). This work can be extended to help students regulate other negative emotions. Students tend to report more negative emotions related to mathematics as they progress through the school system (Brown et al., 2008; Frenzel et al., 2009; Zazkis, 2015). Thus, it is crucial to help students to regulate these increasing levels of negative emotions at the undergraduate level. Other interventions can also be developed to help students regulate positive emotions. A study with elementary students has shown that positive emotions can be a barrier to perseverance in mathematical reasoning (Barnes, 2021), so similar interventions promoting regulation of positive emotions could also provide fruitful and beneficial results.

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