Student Happiness in Mathematics: Antecedents of Positive Emotions

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Students often express negative emotions in mathematics classes. Positive emotions feel good, but they are important beyond just feeling good. Positive emotions can benefit learning, attitudes, and perceptions of mathematics. In this paper, we discuss why positive emotions are beneficial along with a literature review of common themes which are associated with positive emotions among students. For example, increases in students' perceptions of personal control and value in the classroom have been associated with increased positive emotions. Many practices already encouraged by mathematics education researchers (e.g. teaching conceptually, helping students find mathematics important) have the added benefit of potentially increasing positive emotions in students. Positive emotions have been shown to be associated with engagement and motivation in mathematics. By increasing positive emotions in the classroom, instructors can encourage the gradual growth of positive attitudes and beliefs about mathematics to students which are currently lacking among many students.

Keywords: positive emotions, affect, happiness

Happiness and wellbeing in education have become increasingly emphasized in psychological and educational research. The Programme for International Student Assessment (PISA) has been including measures for student affect and wellbeing in their assessments of students around the world since 2015 (Govorova et al., 2020). In a recent survey of 21,678 US high school students, 75% of students reported that they typically experience negative feelings at school, with the most frequently mentioned feelings including tired, stressed and bored (Moeller et al., 2020).

It is well documented that students have similar negative feelings towards mathematics. For example, students report finding mathematics boring, unimportant to their lives, and a generally negative experience (Brown et al., 2008; Hernandez-Martinez & Vos, 2018; Zazkis, 2015). Emotions in the classroom have been found to be connected to engagement and motivation with mathematics and attitudes towards mathematics both in the short and long term (Brown et al., 2008; Middleton et al., 2017; Middleton et al., 2023). Negative emotions have been associated with reduced motivation and negative attitudes (Brown et al., 2008; Middleton et al., 2023). While, positive emotions have been connected to increased motivation (Middleton et al., 2023). Given the prevalence and potential consequences of negative emotions about mathematics, more work is needed to consider how students' emotions about and during mathematics can be improved. We suggest that cultivating positive emotions in mathematics would offer new perspectives for this pervasive problem.

Robust evidence demonstrates the positive association between positive emotions and achievement (Camacho-Morles et al., 2021; Coffey, 2020; Villavicencio & Bernardo, 2016). For example, Coffey (2020) found that positive affect at at age 1.5 years directly predicted higher educational attainment at age 29. In a meta analysis of 68 studies, Camacho-Morles et al. (2021) found a positive relationship between enjoyment and academic performance. Thus, positive emotions are associated with lasting benefits for academic achievement.

The benefits of positive emotions in mathematics likely accrue over time. According to the Broaden and Build Theory, positive emotions are associated with two categories of benefits (Fredrickson, 2001). First, positive emotions *broaden* awareness. People experiencing positive emotions are more aware of the “big picture” and able to take in a wider range of information. For example, smiles have been associated with increased breadth of attention (Johnson et al., 2010). Second, when in a broadened state of awareness from positive emotions, people are able to *build* more resources for themselves. This building of rescources has been associated with greater cognitive flexibility in an experimental task among participants in a positive emotion condition (Wang et al., 2017). Further, genuine smiles (Duchenne smiles) have been shown to predict increased flexibility in attention on an orienting task (Johnson et al., 2010).

Given these potential benefits of positive emotions in mathematics, an important area of research also involves understanding the factors that may predict positive emotions. We now review existing literature exploring the correlates of positive emotions in educational settings, which may inform strategies for educators or researchers interested in promoting positive emotions in mathematics classrooms. We begin by discussing student characteristics on an individual level that are associated with positive emotions. Then, we zoom out and discuss predictors within the classroom environment which are associated with positive emotions.

Student Cognitive Appraisals of Control and Value

Control Value Theory (CVT) posits that students’ personal appraisals of a) *control* and b) *value* of the material are proximal antecedents of emotions related to achievement (Pekrun, 2006; Pekrun, 2021). Students’ control appraisals describe how much control or influence students perceive they have over events, and value appraisals describe how much importance or worth students give to a particular object of focus (e.g. an exam, mathematics in general; Pekrun, 2006; Pekrun, 2021). Control and value appraisals can operate independently; thus, students can experience a combination of high and low appraisals of control and value. For example, a student may feel that they are able to answer every question on their homework assignment easily (high control), but the same student may feel that the homework is boring and unimportant (low value). According to CVT, students experience more positive emotions when they have high levels of both control and value (Pekrun, 2006; Pekrun, 2021).

Control Appraisals

Higher perceptions of control are generally associated with more positive emotions from students in math classes (Bieg et al., 2017; Bieleke et al., 2023; Buff, 2014; Buff et al., 2017). However, students on average report low feelings of control in the classroom (Bieg et al., 2017). These reported low feelings of control may be related to the many aspects of the classroom that are decided by teachers or administrators (e.g. assignments, exams; Bieg et al., 2017; Boehme et al., 2017). Research has shown that changes in perceived control are positively associated with changes in enjoyment of learning (Buff et al., 2017). Thus, perhaps interventions to increase student perceived control may have benefits by increasing positive emotions. For example, Bieg and colleagues (2017) demonstrated that working in groups and working individually increases perceived control when compared to learning from direct instruction.

However, it is important to note that too much control can lead to negative feelings for students (Buff et al., 2011; Pekrun, 2006, 2021). For example, if a student has very high control over what is happening in the classroom, they may feel bored and unengaged in mathematics (Pekrun, 2006). Buff and colleagues (2011) demonstrated that the relationship between control and positive emotions is nonlinear with the benefits of control tapering off at the highest levels.

Value Appraisals

Similar to control, it has been demonstrated that changes in perceived value are positively predictive of changes in enjoyment of learning (Buff, 2014; Buff et al., 2017). Math students generally report low values for math and feel that math is unimportant to their lives and boring (Brown et al., 2008; Hernandez-Martinez & Vos, 2018). Low value for math is associated with lower levels of motivation as well which contributes to negative emotions (Brown et al., 2008). Thus, interventions to increase student value could be beneficial to increase positive emotions for students.

However, just as with feelings of control, increasing value appraisals too much may be damaging to some students which has been demonstrated empirically (Boehem et al., 2017; Lauermann et al., 2017). High levels of value among families was associated with greater math anxiety in one study (Boehme et al., 2017). This view is also supported theoretically by CVT. For example, a student who believes succeeding in math is necessary for success in their future career has a high value of math, and anything that gets in the way of the success that they feel is necessary could lead to anxiety.

Control and Value Appraisals Occur Together

As discussed above, appraisals of control and value should not be increased for every student. Potential intervention studies are made more nuanced by the fact that control and value appraisals often co-occur. Thus, control and value appraisals become a type of optimization problem. For each student, there is a different optimal level of control and value which leads to the most success and positive emotions in the class. For example, Boehme et al. (2017) found that high family values of math were associated with test anxiety, yet high family values were associated with lower test anxiety through increased feelings of control among students. This finding suggests that increasing only value may worsen math anxiety; however, when an increase in value occurs with an increase in control, students may experience less math anxiety. Similarly, students with high math anxiety may benefit from an increase in control in the classroom while gifted students may benefit from classroom practices designed to decrease feelings of control as this may reduce feelings of boredom and increase positive emotions (Pekrun et al., 2010).

Classroom Environment Predictors of Positive Emotions

Research has also shown many factors within the classroom environment that can contribute to students experiencing positive emotions.

Learner-Oriented Instruction

General education research has demonstrated the benefits of learner-oriented teaching practices (e.g. group work, project based learning; Bruder & Prescott, 2013; Cevikbas & Kaiser, 2020; Laursen et al., 2014). However, direct instruction in a lecture format is still one of the most common teaching practices (Bieg et al., 2017; Jiang et al., 2021; Stigler & Hiebert, 2009). Direct instruction has also been shown to be associated with negative emotions in students (Bieg et al., 2017; Jiang et al., 2021). However, learner-oriented instruction, like working in pairs or small groups) has been shown to give students more feelings of control (e.g. control over discussions, control over pace of the material; Bieg et al., 2017).

Bieg et al. (2017) used experience-sampling methodology to measure students' emotions during math class. Students received notifications sent to an iPod Touch with surveys to complete during class. The survey asked them to report the current type of instruction (e.g. lecture, small group work), feelings of control in the moment, and current emotions (enjoyment, pride, anger, anxiety, and boredom). Results revealed that working in groups or working individually was associated with greater student enjoyment and perceived student control.

Collaborative work through learner-oriented instruction also offers the benefits of encouraging positive social interaction among students. Working in groups and having positive interactions with others is associated with positive emotions in math classes (Satyam, 2020). Also, giving students the opportunity to work in groups allows students to experience mathematics in a way similar to mathematicians, who also collaborate frequently (Burton, 1998). This approach is consistent with current approaches to science education, which has been dedicating more attention to cultivating experiences for students to make them feel like scientists (Jaber & Hammer, 2016). Burton (1998) reports that mathematicians report feeling increased joy and creativity through their collaborative work. Thus, group work could have a twofold effect of increasing positive emotions and allowing students the opportunity to work in ways similar to mathematicians and experts.

Learning Materials and Curriculum

Just as instruction type can influence student emotions, classroom activities and curriculum can influence student engagement which in turn can influence student emotions (Middleton et al., 2017). For example, giving students opportunities to work on challenging problems has been associated with more positive emotions and fewer negative emotions (Dettmers et al., 2011; Greensfeld & Deutsch, 2016; Greensfeld & Deutsch, 2022; Liljedahl, 2005; Satyam, 2020; Schukajlow & Rakoczy, 2016). For example, Dettmers et al. (2011) found that when students perceived homework assignments as challenging with well-selected problems, students put more effort into their homework and experienced fewer negative emotions.

Satyam (2020) describes "satisfying moments" as moments that create significant positive emotions for an individual. Eleven undergraduate students in a transition-to-proof class were interviewed to determine moments of satisfaction. The most common satisfying moments included overcoming a challenge. Thus, overcoming difficulties may create positive moments for students.

Alternative forms of curriculum and assessment can also decrease the common negative emotion of test related anxiety (Krispenz et al., 2019; Walen & Williams, 2002). Walden and Williams (2002) found that their participants had anxiety specific to timed assessments. These undergraduate students enjoyed mathematics, but the time limit of exams created anxiety. Further, using timed assessments likely deters students from using a problem solving approach as procedural methods will likely lead to faster solutions (Walden & Williams, 2002). So, even if teachers are teaching conceptual lessons and using well developed materials with challenging problems, timed assessments may undermine the work that went into those lessons and create more negative emotions for students as well.

Conclusion

Many practices already encouraged by mathematics education researchers (e.g. teaching conceptually, helping students find mathematics important) have the added benefit of potentially increasing positive emotions in students. By increasing positive emotions in the classroom, instructors can encourage the gradual growth of positive attitudes and beliefs about mathematics to students which are currently lacking among many students (Brown et al., 2008; Zazkis, 2015).

Although we have focused on positive emotions for this paper, it is important to not neglect the prevalence of negative emotions in the classroom as well (Deshler et al., 2019; Di Leo et al., 2019; Di Leo et al., 2020; D'Mello et al., 2014; Zazkis, 2015). Research has demonstrated that negative emotions can also be beneficial for students' learning (D'Mello et al., 2014). For example, although being confused is an uncomfortable feeling, and thus a negative emotional state, confusion has been shown to be beneficial to learning if the confusion is resolved (D'Mello et al., 2014). In D'Mello et al. (2014), the participants who experienced confusion were more successful in remembering information presented to them than participants who learned without experiencing confusion. Anxiety has also been found to have a positive relationship with student performance when students have high prior knowledge of mathematics (Schukajlow et al., 2021).

Just as negative emotions can be beneficial, positive emotions can also be detrimental. Villavicencio et al. (2016) found that pride among engineering students in the Phillipenes had a negative association with final course grades. Studies have also shown that positive emotions such as excitement and enjoyment can be barriers to perseverance in mathematical reasoning (Barnes, 2021; Ben-Eliyahu & Linnenbrink-Garcia, 2015).

Thus, there are benefits and detriments to both positive and negative emotions. However, on average, positive emotions have been shown to have more beneficial effects in mathematics learning than negative emotions (Bieg et al., 2017; Greensfeld & Deutsch, 2022; Greensfeld & Deutsch, 2016; Jiang et al., 2021; Liljedahl, 2005; Middleton et al., 2023; Satyam, 2020; Schukajlow & Rakocy, 2016). For example, Middleton et al (2023) found that positive emotions were associated with more motivational outcomes than negative emotions. Thus, in this study, perhaps an intervention to increase positive emotions would provide more benefit than an intervention to decrease negative emotions. With an increased focus on increasing positive emotions in the classroom, mathematics may become a more enjoyable place for students filled with creativity and curiosity.

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