Abstract Mathematics as Perceived by Pre-Service Mathematics Teachers: "It is Not Gonna Benefit Me in Teaching"

Seyda Uysal
The University of Southern Mississippi

The secondary-tertiary transition (STT) in mathematics presents university students with multiple cognitive-epistemological, didactical, sociocultural, and affective challenges. This qualitative study explores the perceptions of two pre-service mathematics teachers regarding the relevance of abstract mathematics to their future teaching practices. Using three interviews and eleven reflections, the investigation yields in-depth insights. In addition to highlighting the affective aspects of the STT in mathematics, the study addressed certain difficulties pre-service teachers encountered while dealing with advanced mathematics courses.

Keywords: affect, perception of abstract mathematics, pre-service mathematics teachers, secondary-tertiary transition

The secondary-tertiary transition (STT) in mathematics represents a significant milestone in the academic journey of pre-service mathematics teachers as they progress through university. While these individuals possess a deep passion for mathematics and aspire to become educators, they often encounter formidable challenges as they navigate the shift from concrete, school-level (i.e., empirical) mathematics to the more abstract and theoretical realms of university mathematics. This transition is of paramount importance, as it not only influences their own understanding of the subject but also shapes their ability to effectively teach complex mathematical concepts to future students. This research study seeks to explore the multifaceted challenges that pre-service mathematics teachers encounter during this pivotal transition to abstract mathematics in university settings. By examining these challenges in-depth, I aim to further understand the pertinent issues and identify potential strategies to better prepare pre-service mathematics teachers for the unique demands of abstract mathematics, ultimately enhancing the quality of mathematics instruction in the classroom.

Theoretical Perspectives

The secondary-tertiary transition (STT) in mathematics education has been extensively explored in the literature, with scholars identifying various categories of challenges. De Guzman et al. (1998) categorized these challenges into three main areas: cognitive-epistemological, didactical, and sociocultural, with recent studies adding affective factors as a fourth component (Di Martino & Gregorio, 2019). The nature of mathematics changes as students transition to university, becoming more formal and abstract (Tall, 2008). This shift requires advanced thinking and can lead to cognitive and affective challenges when there is a lack of alignment between the mathematical expectations of secondary and tertiary institutions (Pepin, 2014). Affect in mathematics learning, including beliefs, attitudes, and emotions, plays a pivotal role in shaping students' engagement and performance in the subject (McLeod, 1998). Understanding and addressing the affective aspects of mathematics education is crucial for fostering a positive and productive learning environment.

The Study

The current study focuses on exploring the secondary-tertiary transition experiences in university mathematics from affective perspectives. Five women and/or racially ethnically minoritized women participated in this research who were majoring in mathematics at a large university in the southeastern United States. The semi-structured interviews and reflection videos were used as data collection method focusing on students' experiences in demographic information, secondary school mathematics, decision to major in mathematics, current experiences in the major, vision of mathematics, perceived competence in mathematics, approaches to and assessments of learning mathematics, and mode of belonging.

In this study, I focus on Nia and Bethany, both pre-service mathematics teachers who frequently brought up the disparities between university-level mathematics and secondary school mathematics. This emphasis on their experiences arises from the study's primary focus on the differences between secondary school and university mathematics, which is recognized as a "knowledge gap," or "discontinuities" between two institutional settings (Gueudet, 2008). It is noteworthy to highlight that both Nia and Bethany were first-generation college students who also openly discussed their financial challenges in pursuing a college education. Nia identified as a biracial female with Black and White heritage, while Bethany was a White female student. Nia and Bethany participated in three monthly interviews and recorded eleven weekly reflections in the semester of Spring 2022. Following stages of open, axial, and selective coding, the grounded theory method guided data analysis and enabled the identification of themes that emerged from student responses (Charmaz, 2006).

Preliminary Findings

I explore the specific beliefs held by Nia and Bethany about abstract mathematics, which arose as they reflected on their mathematical skills and the nature of advanced mathematics at the university level. These beliefs contribute to their skepticism about the relevance of advanced math to their future careers. Using several direct quotes, I delve deeper into Nia's and Bethany's views on abstract mathematics, which are rooted in their perceived ability to grasp abstract concepts in proof-based courses. The following excerpt illuminated Nia's first proof experiences in the Introduction to Advanced Mathematics (IAM) class:

> I didn't like that [IAM] course mainly because the proofs weren't really interesting to me. Because it was me memorizing how to do the proofs and really understanding how to prove something in math, which is something that I couldn't kind of wrap my brain around like, 'okay, where do you start?' It wasn't concrete. These are always like, 'do the proof this way' it was like well, 'maybe the proof is this way, maybe it's that way.' So, I have to assume a lot of information. It was pretty hard to keep up. I mean, I ended up with a B in there. But it was not my favorite math course. (Nia, Interview 1, February 2022)

Nia shared her experience in the Introduction to Advanced Mathematics (IAM) course, primarily expressing her dislike for it. She found the memorization of proofs uninteresting and struggled to grasp the abstract nature of mathematical proofs. This aligns with her preference for mathematics that involves practical application and processes. Nia's frustration stemmed from not being able to determine the correctness of her proofs, as she lacked clear expectations and the means to verify her work:

> When I don't know the expectations or when I am not able to check my work, that's when I am unsure, as I will be 'is this right thing?' is my problem with the proofs as well. Well, I took the [Introduction to] Advanced Math, and I couldn't determine for myself, like 'is this right, is this the answer?' I don't understand how I know if I'm writing this proof right and if it's an accepted answer. (Nia, Interview 1, February 2022)

Similarly, Bethany had her own perspective on proof-based classes. She believed that exam questions should closely resemble the material covered in class, and when they didn't, she considered the course difficult. Bethany didn't enjoy proofs, especially in courses like Abstract Algebra and Analysis, where the abstract nature of the subject made it challenging for her to connect with the content. She believed that her future teaching career wouldn't require knowledge of such abstract proofs and preferred mathematics with practical applications:

> I don't like the proofs as much. Well, I guess it depends on what it's about, because I think groups in Geometry were really fun because I could do it. It was really tangible, I could understand angles and shapes. But in Abstract Algebra and Analysis, I definitely struggled more because it was more abstract. I think for teaching, like, even in high school, I don't need to know proofs for abstract things. It's not gonna benefit me in teaching, really. So I don't think it'll be a problem in that context. (Bethany, Interview 2, March 2022)

Bethany's responses revealed a mix of emotions, including both satisfaction and aversion, when she contemplated abstract mathematics. She provided several explanations for her lack of enthusiasm towards mathematical proofs. Initially, Bethany linked this sentiment to the inherent abstractness of proofs, where the concepts lacked a tangible quality. In her first interview, she also conveyed that the majority of her university mathematics courses were formal and did not align with her aspirations in teaching. Bethany's perspective on mathematics in her initial and subsequent interviews remained consistent. Specifically, she conveyed a preference for mathematics with practical applications, believing it would have greater relevance to her future career in teaching. Bethany also mentioned her preference for what she termed a "straightforward course," where lectures were followed by problem-solving sessions, and exams focused on class exercises. In contrast, she perceived proof-based classes as less straightforward because they required students to think independently rather than follow step-by-step instructions:

> I guess the courses are just lectures, and then you do the problems. Like in Linear Algebra, you just write on the doc camera, doing the problems, and then you'd go home and do the same problems. The tests were about the exercise problems from class. So, I thought that was pretty straightforward, versus proof-based classes, they don't tell you how to do the problems. You have to figure it out. In Calc II, we also had some group projects which required more thinking on our part and not just doing what we were told to do. (Bethany, Interview 3, April 2022)

Bethany's perception of proof-based classes as challenging due to their requirement for independent thinking rather than following explicit instructions reflected her initial beliefs. These beliefs highlighted a mismatch between in Bethany's expectations of upper-division mathematics

courses and the actual classroom experience in secondary school teaching. Consequently, both Nia and Bethany viewed abstract mathematics as less relevant to their career aspirations. This misalignment led to their disengagement from proof-based courses. They did not see a connection between their mathematical abilities and the demands of these courses. Additionally, both students expressed little interest in pursuing careers that involve abstract mathematics, possibly due to the perceived gap in their mathematical background and competency required for such careers.

Significance and Implications

When analyzing Nia's and Bethany's cases, several noteworthy patterns surfaced in their experiences with university-level mathematics, with a specific focus on abstract mathematics. Both Bethany and Nia raised doubts about the relevance of abstract mathematics, as they believe it is disconnected from their future teaching practices (Wasserman et al., 2019). Some of their concerns stemmed from instructional practices encountered in university, which predominantly adhered to traditional lecture formats. Additionally, their doubts were further fueled by the perceived difficulty in discerning the validity of mathematical proofs, shedding light on challenges related to mathematical competence. Furthermore, Nia and Bethany commented on the abstract nature of mathematics at the university level, contrasting it with the more tangible mathematics of secondary education, which also influenced their attitudes toward learning abstract mathematics. Schoenfeld's assertion that "formal mathematics has little or nothing to do with real thinking or problem solving" (1985, p. 43) seemed to resonate with the beliefs held by Bethany and Nia about the nature of mathematics. Their reactions to the changing nature of mathematics towards upper-division courses corroborated the findings of Geisler and Rolka (2021), which highlighted that students anticipate university-level mathematics to align with the practical and applied aspects of the discipline. This observation underscores the disparities in students' perceptions of mathematics, particularly in the context of secondary school mathematics and university-level mathematics courses (Rach & Heinze, 2017).

Solomon and Croft (2016) proposed that when undergraduate students feel a challenge to their sense of ownership of mathematics, they tend to disengage from the subject. This disengagement often stems from mathematics either not meeting their high expectations for success or failing to reveal its inner workings while imposing strict adherence to its rules. The experiences of Nia and Bethany, who both excelled in high school mathematics and currently perceive themselves as rule followers in college mathematics, align with these ideas.
High school preparation significantly impacts students' experiences in university mathematics courses. Nia and Bethany, who are first-generation students appeared to be less prepared and struggling with advanced mathematics courses. Both students highlighted their difficulties in keeping up with their peers, attributing it to their limited background in advanced mathematics or the instructional style and expressing a need for additional resources to grasp the course materials effectively. Future research should investigate the alignment between pre-service mathematics teachers' teaching goals and the content of the advanced mathematics courses, with the purpose of tailoring these classes to their specific needs. Furthermore, future research should investigate the relationship between the experiences of first-generation college students and their views on abstract mathematics. Investigating the requisite support systems necessary for their success and their development into proficient mathematics educators should also be a priority.

References

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative research*. SAGE.

De Guzman, M., Hodgson, B., Robert, A., & Villani, V. (1998). Difficulties in the passage from secondary to tertiary education. *Proceedings of the International Congress of Mathematicians* (Vol. 3, pp. 747–762). Documenta Mathematica.

Di Martino, P., & Gregorio, F. (2019). The mathematical crisis in secondary – tertiary transition. *International Journal of Science and Mathematics Education*, *17*(4), 825–843. https://doi.org/https://doi.org/10.1007/s10763-018-9894-y

Geisler, S., & Rolka, K. (2021). "That wasn't the math I wanted to do!"—Students' beliefs during the transition from school to university mathematics. *International Journal of Science and Mathematics Education*, *19*, 599–618. https://doi.org/10.1007/s10763-020-10072-y

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics*, *67*(3), 237–254.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In Grows, D. A. (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). Macmillan Publishing Company.

Pepin, B. (2014). Using the construct of the didactic contract to understand student transition into university mathematics education. *Policy Futures in Education*, *12*(5), 646–657. https://doi.org/10.2304/pfie.2014.12.5.646

Rach, S., & Heinze, A. (2017). The transition from school to university in mathematics: Which influence do school-related variables have? *International Journal of Science and Mathematics Education*, *15*(7), 1343–1363. https://doi.org/10.1007/s10763-016-9744-8

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Solomon, Y., & Croft, T. (2016). Understanding undergraduate disengagement from mathematics: Addressing alienation. *International Journal of Educational Research*, *79*, 267–276. https://doi.org/10.1016/j.ijer.2015.10.006

Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24. https://doi.org/10.1007/BF03217474

Wasserman, N. H., Weber, K., Fukawa-Connelly, T., & McGuffey, W. (2019). Designing advanced mathematics courses to influence secondary teaching: Fostering mathematics teachers' "attention to scope". *Journal of Mathematics Teacher Education*, *22*(4), 379–406. https://doi.org/10.1007/s10857-019-09431-6