Creating Metaphors for Linear Algebra Concepts: Developing Cognitive, Behavioral, and Affective Domains

COLORADO STATE UNIVERSITY

Dr. Hortensia Soto
Colorado State University

Dr. Vladislav Kokushkin
Colorado State University

Dr. Jessi Lajos
Utah State University

UtahState University

Issues with Assessments Practices

Traditional Assessments:

- Privilege symbolism and formalism (Nemirovsky & Bunn, 2022)
- Characterize mathematics as mechanisms (Nemirovsky et al., 2023)
- Viewed as "easy" to implement, non-disruptive, and "real math" (Munter et al., 2014)
- Avoid mathematical aesthetics (Sinclair et al., 2006)

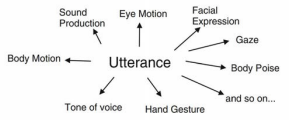
Current recommendations and ways forward:

Assessments should engage students: (Meta)Cognitively, Affectively, and Behaviourally (MAA Instructional Practices Guide).

Targeting Three Domains via Intentional Design of an Embodied Assessment

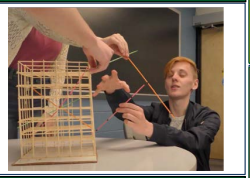
Embodied Cognition

Embodied design applied to "tools whose operatory function is **engineered** specifically so as to . . . **cultivate** . . . the development of particular **sensorimotor schemes** as a condition for masterful control of the environment in accord with **task demands**" and thereby "come to **ground mathematical concepts** we want these students to **learn**" (Abrahamson & Bakker, 2016, p.5, Piaget, 1954)



(Nemirovsky & Ferrara, 2009)

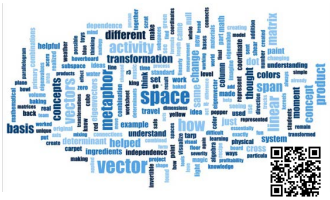
Formative Assessments (Play)



Summative Metaphor Creation

Two-part summative assessment:

1. **Written part:** Create and reflect on your own metaphors about various LA concept pairs (e.g., *Explain how you arrived at this metaphor and how it fits the formal concept.*
1. **Collaborative focus group:** Sharing and co-constructing new improved metaphors with peers.



Research Questions

1. What metaphors do students create for linear algebra concepts? How do they manifest?
2. What cognitive, behavioral, and affective behaviors emerge as students share their metaphors for linear algebra concepts?

Results: Students' Engagement During Summative Assessment

(Meta)Cognitive

Metaphor Resources:

- **Personal Experiences**
  - ***Hobbies***
  - ***Childhood experiences***
  - ***Professional Interests/Major:***
  - ***In-the-Moment***
- **Classroom Experiences**
- **Linguistics**

Evaluating the "Soundness" of Metaphors

A. **Incomplete model of real-world**
B. **Incomplete representation of linear algebra concept**

Extending Metaphor across Multiple LA Concepts: Popcorn Popper

kernels represent the **vectors**. Heat is the **matrix**, it's the one that's changing the vectors. The ones that pop and go into the bowl underneath the popcorn machine, those are in the **column space**, because they get **transformed** into a new location.	And the kernels that pop or just like fly out [throws hand over shoulder] of the machine and land on the floor, those are in the **null space**, because they get sent out of the system entirely.	Kernels that stay in the popcorn machine are like "**eigenvectors** because they didn't' really move" or "pop out".	The degree to which they [popcorn that don't move] got semi-popped is like the **eigenvalue** of the situation because it tells you how much it changed even though it's still like where it was before."

Affective

Behavioral

Slinky Metaphor for Invertible

*You can actually **see it** go back …* repeats gesture and then says *"to itself"*

*In some of my science classes we used to … how **sound moves***

If we could physically see it (grab both ends with eyes focused on her slinky and Billy sweeps his hand)

It always goes back to itself *(brings hands together)*

Findings illustrate the notion that "***Learning – the creation of the new – comes about from situations that were previously unimagined, impossible, unusual, & unexpected … expressed through diagram/gesture [and verbiage] interplay"*** (de Freitas & Sinclair, 2014, p. 109).

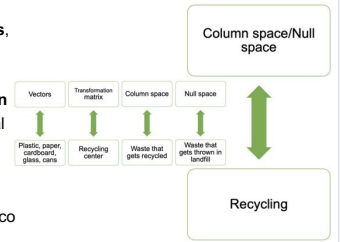
Framework for Engagement

(Anderson & Krathwohl, 2016; Bloom et al., 1956)

The Knowledge Dimension	The Cognitive Dimension					
	Remembering	Understanding	Applying	Analyzing	Evaluating	Creating
Factual knowledge	Listing	Summarizing	Classifying	Ordering	Ranking	Combining
Conceptual Knowledge	Describing	Interpreting	Experimenting	Explaining	Assessing	Planning
Procedural Knowledge	Tabulating	Predicting	Calculating	Differentiating	Concluding	Composing
Meta-cognitive knowledge	Using Appropriately	Executing	Constructing	Achieving	Taking action	Actualizing

Conceptual Metaphors

Appear as **cognitive links**, or mappings, between a **source domain** (abstract/formal concept) and a **target domain** (real-world/physical/concrete).



(Lakoff & Núñez, 1997)

Discussion

1. **Embodiment espouses learning from concrete to abstract** (Tall, 2013)
   - This activity moves students from abstract back to concrete (make full circle)
   - Forced to think deeply about the abstract
2. **Metaphor Assessment Privileges**
   - Blending Creativity & Metacognitive
   - Togetherness
   - Aesthetics of Mathematics (surprise, joy, frustration)
3. **Learning is inventiveness** (de Freitas & Sinclair, 2014)

References

De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom.* Cambridge University Press.
Lakoff, G., & Núñez (1997). The metaphorical structure of mathematics: Sketching out cognitive foundations for a mind-based mathematics. In L. English "Ed.), *Mathematical reasoning: Analogies, metaphors, and images.* Mahwah, NJ: Lawrence Erlbaum Associates.
Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics, 70*, 159-174.

Contact: hortensia.soto@colostate.edu