Student Thinking and Challenges when Representing Distance in the Cartesian Plane

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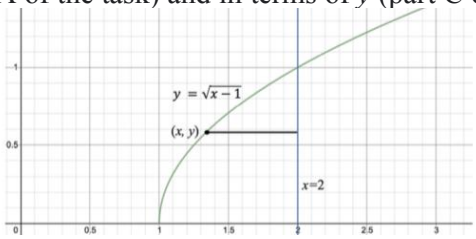
The purpose of this poster presentation is to highlight the variations in student thinking when working with difference expressions in the Cartesian plane — a fundamental skill crucial for grasping numerous Calculus concepts illustrated graphically, such as area, volume, and difference quotients. Previous research has shown students often struggle to conceptualize and express distances in the Cartesian plane (Parr et al., 2021). We view two key understandings, a magnitude interpretation, and the Cartesian connection, as prerequisites to this ability. By magnitude interpretation, we mean interpreting a difference expression as the magnitude of the distance between two corresponding positions in a single dimension (Parr, 2023). The Cartesian connection is the connection between pairs of values of x and y that satisfy an equation and the ordered pairs of points on the graph of that equation (Moschkovich et al., 1993).

The goal of this study was to investigate the prevalence and nature of challenges that students face when engaging in tasks to represent distances in the Cartesian plane. To investigate this question, we surveyed n=169 undergraduate math students at a private liberal arts college located in the southern United States between spring 2022 and fall 2023. On the survey, students were asked to represent distances within the Cartesian plane using expressions in terms of x (part A of the task) and in terms of y (part C of the task).



a. Represent the horizontal segment's length in terms of x (Your expression should have "x" in it).

b. In a sentence or two, explain how you came up with your expression.

c. Represent the horizontal segment's length in terms of y (Your expression should have "y" in it).

d. In a sentence or two, explain how you came up with your expression.

Figure 1. Survey item to represent the length of the segment in black in terms of x and in terms of y.

We found that only 16 students (9.47%) correctly responded to both A and C, 22 students (13.02%) correctly answered A but not C, five students (2.96%) correctly answered C but not A, and the remaining 126 students (74.56%) answered both questions incorrectly. We further analyzed the structure of student responses to see if they contained a difference expression, which could indicate the use of a magnitude interpretation. We also analyzed responses for evidence of issues related to the Cartesian connection, including when and how to manipulate the given equation. Our findings highlight a need for instruction on the skills foundational to understanding distances in the Cartesian plane within Calculus education.

References

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