Exploring How Use of a Shared Google Doc for Collective Proof Construction Supports Students' Mathematical Progress

Diana Salter
Portland State University

Tenchita Alzaga Elizondo
University of Texas Rio Grande Valley

While the pandemic brought much hardship for mathematics education, it also brought opportunity for ingenuity and transformation. This study builds on results from prior work that found Google Docs to be rich tools that supported students in effectively engaging in collaborative proof activity in a remote introduction-to-proofs course. To study if Google Docs would have similar benefits for students' collective proof activity in a face-to-face setting, we investigate how two students in a face-to-face course leveraged a shared Google Doc to support their collaborative work on a proof construction task. We found that the students used the shared Google Doc to support their collective proof construction in similar ways as students in the remote class. In addition, we observed that the Google Doc was an inherent component of students engaging in an active proof writing process.

Keywords: Proof Construction, Technology, Google Docs, Collaborative Activity

For mathematics education the COVID-19 pandemic brought drastic changes in how courses were structured and how interactions between students and teachers took place (Tate & Warschauer, 2022). While these changes brought many challenges both educationally and emotionally (Engelbrecht et al., 2023), they also brought the opportunity for ingenuity and for reimagining what mathematics classrooms could look like. Specifically, in many ways the pandemic forced educators to find new means by which to use technology to engage students with mathematics and with each other, potentially, enriching students' learning experiences. Literature on educational technology has highlighted how we can create rich learning environments with the aid of technological tools (e.g., Clark et al., 2007; Öner, 2008; Stahl, 2006). As we transition (or have transitioned) back to face-to-face (F2F) courses, it becomes imperative to reflect back on what we have learned as educators from our experience teaching remotely. Specifically, we should consider what aspects of our remote mathematics courses can be (and perhaps should be) transferred to F2F. Studying remote instruction in an introduction to proof course, Alzaga Elizondo (2022) found valuable implications of using Google Docs for students' collective work in small groups. Most notably, she found that by coordinating the use of Google Docs with Zoom, students were able to actively engage with one another's thinking and produce truly collaborative work. Subsequently, we wondered if and how shared Google Docs would provide similar support to students in a F2F version of this course. In this paper, we present a case study of two students working on a task to develop a proof in a F2F class with access to a shared Google Doc. We explore the connection between the students' use of the Google Doc to support their collective proof activity and their proof construction progress.

Theoretical Grounding

To study the role that the Google Docs played in students' collective proof construction, this study builds on the work done by Alzaga Elizondo (2022). Alzaga Elizondo studied students' small group activity in a synchronous online Introduction to Proofs course where she investigated how students operationalized technological tools (e.g., Zoom, Google Docs) in the remote environment to engage in various collective proving activities (i.e., proof construction, defining, conjecturing). To analyze the students' tool use, Alzaga Elizondo leveraged the theory of Instrumental Genesis (Rabardel, 2002) which describes a reciprocal relationship between artifacts (or tools) and their users and the impact of that relationship on cognition. Any given tool can be transformed into an instrument by assigning it a *utilization scheme* (designated methods for using the tool) for a specific *goal directed activity* (Béguin & Rabardel, 2000; Carvalho et al., 2019). In her study, Alzaga Elizondo identified several collective instruments that students developed by coordinating the different technological tools available to them. These were collective in the sense that students coordinated their utilization schemes across different tools to develop collective schemes that were closely tied to their mathematical activity. For instance, students developed an instrument to *illustrate a new idea by writing a mathematical statement* where they would coordinate the use of Zoom and Google Docs to verbally communicate and simultaneously visually illustrate a new idea. Simpler instruments were created to support the development of more complex ones, these are identified in Table 1 with the supportive instruments italicized. We conjecture that students may develop similar instruments in a F2F class where, in this setting, students coordinate their use of the Google Doc with their verbal face-to-face interactions rather than through Zoom.

Table 1. Collective Instruments

Instrument	Observed Student Action
Engage in collective argumentation by coordinating visual mediators and verbal communication	Discuss and debate ideas by using visuals on the Google Doc to illustrate verbally spoken idea.
Illustrate new idea by writing a mathematical statement	Write mathematical statement on Google Doc to illustrate an idea to group before group has agreed on new idea.
Analyze shared idea by referencing written text	Reference written text by highlighting (or placing cursor next to) text on the Google Doc and/or by using indexical terms like “this” or “that”.
Develop shared understanding by introducing diagrams and examples	Adding examples or diagrams to Google Doc for the goal of developing shared understanding among group.
Co-construct a group solution by refining shared text	Create final solution by refining written text on the Google Doc.
Execute solution by repurposing illustrated idea	Using existing text in the Google Doc for the final solution that was not originally written for that purpose.
Execute solution by validating illustrated idea	Keeping text in the Google Doc that was added to final solution for the purpose of illustrating a new idea.

Further, Alzaga Elizondo conjectured that use of these instruments supported students in engaging in a *process approach* to writing, which describes an iterative collective writing process in which students plan, draft, and revise repeatedly by incorporating feedback from peers and the instructor (Graham & Perin, 2007; Sun & Feng, 2009). These kinds of activities can reflect more meaningful engagement with proof-based tasks, as they promote exploration and testing which have been noted to reflect more authentic mathematical practices (Larsen & Zandieh, 2008; Melhuish et al., 2022; Öner, 2008). As such, students, regardless of course modality (i.e., face-to-face or remote) should be encouraged to engage in a similar writing process. The work presented here builds on Alzaga Elizondo's (2022) work by investigating how and/or if the collective instruments she identified appeared in one group's collective proof construction when using Google Docs in a F2F course. Further, and most notably, it investigates how the students' Google Doc use supported them in engaging in a process approach to collective proof construction.

Methods

Data Collection and Episode Selection

The data for this study is from a university introduction-to-proof course that was part of a larger NSF-funded project (DUE #1916490) that developed introduction-to-proof curricula and accompanying instructor support materials. The inquiry-oriented curriculum included tasks designed for small group collaborative work. In this 10-week F2F course each student had access to an iPad which enabled each group of students to share a Google Doc to work together on tasks during small group work. A stationary camera and iPad screen recordings captured one focal group's interactions each day (26 days total).

This study focuses on two students' collective work on one task during one day of this course. Jake and Roger (pseudonyms), worked together on proving the sequence $a_n = \lfloor 10/n \rfloor$ is eventually constant. The class had previously established a definition for an eventually constant sequence: *a sequence a_n is eventually constant if there exists a natural number n such that for every natural number $k \geq n$, $a_n = a_k$*. In other words, a sequence is eventually constant if at some point it becomes and stays constant. The Google Doc that the students used included the task and this definition at the top of the page. This episode was selected because the students worked together in their small group to complete the task and there was substantial discourse, both written using the Google Doc and verbal, between the students.

Data Analysis

To begin our analysis, we created a multimodal transcript (Hoffman, 2018) that captured both the students' written work on their Google Doc and their spoken interactions. Since the students' iPads recorded both their written and spoken work, time stamps added to our transcript provided an accurate record of the temporal relationship between spoken and written statements. Then, using the observed student actions in Table 1 as operationalizations of the collective instruments, the first author identified for each of the spoken and written communications in the multimodal transcript if and when a collective instrument from Table 1 was used and described any mathematical progress reflected in the communication. The second author reviewed the first author's identifications and descriptions and any disagreements were resolved through discussion. Through the analysis the authors found that students developed a sequence of proof drafts. Together, the authors then identified six points in the students' work on the Google Doc that represented completion of interim drafts of their final solution (see Figure 1). We identified a point in their Google Doc as a draft if it represented a significant mathematical change from the previous draft and seemed to be the culmination of the students' argumentation at the time. We then were able to describe how the students' instrumentation process impacted their progress through development of the draft. We present the findings from this analysis in the following section.

$k=11$

If $n \geq 11$, then

$a_n = \lfloor 10/n \rfloor$
$\leq \lfloor 10/11 \rfloor$
$= 0$

a.

$k=11$

If $n \geq 11$, then

$a_n = \lfloor 10/n \rfloor$

$10/n$, when $n \geq 11$, $0 \leq 10/n \leq 1$. So

$\lfloor 10/n \rfloor = 0$

b.

If $n \geq 11$, then

$a_n = \lfloor 10/n \rfloor$

$10/n$, when $n \geq 11$, $0 \leq 10/n \leq 1$. So

$\lfloor 10/n \rfloor = 0$ for $n \geq 11$.

c.

Let $n=11$. When $k \geq n$, $0 \leq 10/k \leq 1$. So

$a_k = \lfloor 10/k \rfloor = 0 = a_n$.

d.

Let $n=11$.

Let k be any natural number such that $k \geq n$. Notice that $0 \leq 10/k \leq 1$. So

$a_k = \lfloor 10/k \rfloor = 0 = \lfloor 10/11 \rfloor = a_n$.

e.

Let $n=11$. Then $a_n = \lfloor 10/11 \rfloor = 0$.

Let k be any natural number such that $k \geq n$. Notice that $0 \leq 10/k \leq 1$. So

a_k	$= \lfloor 10/k \rfloor$	(Formula for a_k)
	$= 0$	($0 \leq 10/k \leq 1$ so rounding down we get 0)
	$= a_n$	(As we saw above.)

f.

Figure 1. Students' drafts of their proof that the sequence $a_n = \lfloor 10/n \rfloor$ is Eventually Constant

Results

In the episode analyzed below we observed students using the Google Doc as a tool to engage in collaborative proof activity. Here we report the students' progress through the development of each draft, highlighting their use of collective instruments to achieve each draft.

Development of Draft 1

To start off their work, Jake suggested that they write a "few terms of the sequence and just see what it converges to". Without verbal confirmation, Roger responds by writing the first few values of the sequence in the Google Doc, writing "$a_n = 10$, 10/2=5, 3, 2, 2, 1, 1, 1, 1, 1, 0, 0, 0,...". Simultaneously, Roger verbalized his reasoning: "10 over 8, 1, 10 over 9, 1, 10 over 10, 1, and then 10 over 11, 0. I think at that point." Here Roger used the Google Doc to *develop a shared understanding by introducing examples* (i.e., the first terms of the sequence). As Roger typed, we saw evidence of Jake following along, verbally stating "and after *that* it becomes zero" at the same moment Roger says "I think at that point". In doing this, Jake is using the Google Doc to *analyze the shared idea* (the sequence terms) *by referencing text* when saying "and after *that*" where "that" referred to the 11th term in the sequence. At this point, Jake suggested that they construct a direct proof that follows the structure of the definition. Roger proceeds to *illustrate an idea by writing mathematical statements* on the Google Doc, writing

"$k = 11$, If $n \geq 11$, then $a_n = \lfloor 10/n \rfloor = \lfloor 10/n \rfloor$". As Roger is writing, Jake follows along and suggests they use inequalities instead which prompts refinement from Roger. Together, they produce the first draft of their proof (see Figure 1a).

Development of Draft 2

After a brief pause, Roger suggests "Maybe we should say something different. So like 10 over *n* when *n* is greater than 11." Simultaneously he adds to the Google Doc "10/*n*, when $n \geq 11$, 10/*n*" before the line "$\leq \lfloor 10/11 \rfloor$". This suggests Roger recognizes the inequality in Draft 1 is not correct and chooses to *illustrate a new idea by writing a mathematical statement* in the Google Doc to fix this error. After Roger writes this line in the Google Doc Jake responds by saying "is less than 1" (in other words, proposing "< 1" should follow the "$10/n$" Roger has just written). Here, Jake is not only suggesting the completion of this line, he is also communicating to Roger that he concurs with what Roger has written so far. As such, Jake is using the Google Doc to *execute a solution by validating an illustrated idea* (the line Roger has just added). In other words, Jake agrees with Roger's new addition and thinks it should be kept as part of their proof. Robert acknowledges Jake's idea and proposes a variation, saying "well yeah right sort of bound it like this. It's between zero and one." As he says this, Roger replaces "$10/n$" with "$0 \leq 10/n \leq 1$," a compound inequality. Again, using the Google Doc to *illustrate a new idea by writing a mathematical statement* that he only partially communicates verbally. Jake then again *executes a solution by validating an illustrated idea*, concurring with Roger's compound inequality by saying "Oh true. Yeah, it's not just less than one, it's [...] positive." With this validation, Roger then changes "$\lfloor 10/11 \rfloor = 0$" to "$\lfloor 10/n \rfloor = 0$." This change moves the last two lines of the proof closer to showing how the sequence formula generates 0 when $n \geq 11$ and results in Draft 2 (see Figure 1b).

Development of Draft 3:

Roger's addition to the last line in Draft 2, "$\lfloor 10/n \rfloor = 0$," prompts Jake to respond verbally "So in that case a sub n equals zero for *n* greater than or equal to 11." Jake is observing that the sequence formula does equal zero, as Roger has written in the Google Doc, but only when $n \geq 11$. Roger then *illustrates this new idea by writing the mathematical statement* "for $n \geq 11$" directly following "$\lfloor 10/n \rfloor = 0$." Roger *validates this illustrated idea* verbally when he follows up by saying "Yeah". This addition directly connects $\lfloor 10/n \rfloor = 0$ to the index value where the sequence becomes constant. This new addition prompts Roger to say "I guess we don't need *k*" and delete "$k = 11$" from the first line. Presumably, Roger suggests deleting *k* because it is now an unused variable in their proof (see Figure 1c).

Development of Draft 4

Prompted by Roger removing the variable *k* from the proof, Jake makes the case that this proof does require two variables. He says, "Yeah. I guess maybe just to make it match the definition as much as possible, what if you said at the beginning let *n* be 11. And then we used *k* after that [...] like let *n* be 11 and *k* is greater than *n*." Roger takes Jake's suggestion and adds "$Let\ n = 11.\ When\ k > n$" to the beginning of the proof, *illustrating a new idea* (Jake's) *by writing the mathematical statement* at the beginning of their proof draft. Further, Roger *executes the solution by* (implicitly) *validating the illustrated idea* when he keeps the new text and *illustrates his own new idea* by refining the rest of the proof to be consistent with this change saying, "I think I can actually cut some of this out. Let's see." He proceeds by deleting the first few lines of their proof and, changing “$0 \leq 10/n \leq 1$” to “$0 \leq 10/k \leq 1$,” and changes “$\lfloor 10/n \rfloor = 0$” to “$a_k = \lfloor 10/k \rfloor = 0$” to align with the new use of “*k*” leading to Draft 4 (see Figure 1d.). Jake follows Roger’s written work and eventually *validates the illustrated idea* by verbally rephrasing the argument being presented in their current draft. With the addition of these two variables in Draft 4 the students refined their proof to better match the structure of the definition of an eventually constant sequence.

Development of Draft 5

A few minutes later, with a hint from their instructor, the students have added “Let *k* be any natural number” to the beginning line of their proof. The addition of this line in the Google Doc prompts Jake to say “might need to combine *these* two statements here. I'm going to take out the “when” and say “and” or “such that”. At this point, Jake is *analyzing a shared idea by referencing* lines in the proof when he says “*these* two statements here.” He follows up by *illustrating his idea,* replacing “When” with “such that” so the text that defines the variable *k* now reads “Let *k* be any natural number such that $k \geq n$.” This prompts Roger to follow a similar pattern when he says “I think *this* should be a new sentence. Notice that.” Like Jake, Roger begins by *analyzing their shared text by referencing* a line in their proof when he says “*this* should be” and then *illustrates his new idea* when he then adds “Notice that” before "$0 \leq 10/k \leq 1$” which separates this statement from the definition of the variable and results in Draft 5 (see Figure 1e.). While minimal, the refinement in this draft supported the readability and logic flow of the proof.

Development of Draft 6

The students make two changes between Draft 5 and their final proof, Draft 6. Roger says, “[The instructor] might want *these* on new lines. He kind of likes writing justifications.” Here Roger *analyzes their shared text by referencing* “these” lines in their draft. He then breaks the last line of Draft 5, “$a_k = \lfloor 10/k \rfloor = 0 = \lfloor 10/11 \rfloor = a_n$,” into individual lines so that he can *illustrate his new idea by* adding justifications for each of those steps (see Figure 1f.). The second change they make is led by Jake when he says, “Maybe we should put in a part where it says then a sub n equals [mumbles to self].” As he’s talking, he *illustrates his new idea by* adding “Then $a_n = \lfloor 10/11 \rfloor = 0$” following “Let $n = 11$” in the first line of the proof. This addition prompts Roger to delete “$= \lfloor 10/11 \rfloor$” from the second last line in the proof. This suggests Roger *validates* Jake’s *illustrated idea* as Roger recognizes they do not need this at the end of the proof if they have already established it at the beginning. This change brings them to Draft 6, their final proof.

Discussion

These results show how a group of students used a Google Doc as a collective instrument in a F2F course to support their process of collectively developing a proof. By coordinating use of the Google Doc with their verbal F2F interactions, we saw similar collective instruments developed as Alzaga Elizondo (2022) found in a remote class, including *illustrating a new idea by writing a mathematical statement* in the Google Doc, *analyzing a shared idea by referencing written tex*t in the Google Doc, and *executing a solution by validating an idea illustrated* in the Google Doc. This suggests the productive use of Google Docs for collective activity extends beyond the online setting.

Further, throughout the students' work, we found that as a result of sharing and debating different ideas the students iteratively refined their proof, developing several drafts of this proof throughout their collective work. In other words, we found students engaging in a *process approach* to proof writing. Most notably, we found that the students' use of the Google Doc not only supported this refining activity, it was inherently tied to it. The results presented above, highlight *how* students used the Google Docs to engage in this process approach. In particular, there were multiple instances of a mathematical statement written in the Google Doc that served as a springboard for subsequent mathematical ideas. Frequently, when a student *illustrated a new idea by writing a mathematical statement*, it was immediately followed by a student *analyzing this shared idea by referencing this text in the Google Doc* with a verbal remark that reflected building on the referenced text. Often this cycle continued with the analysis comment leading to *executing a solution by validating an illustrated idea.* The validation was sometimes in the form of verbal agreement, but frequently the validation was implicit in that one student would just add to another's idea suggesting they accepted it. As an example, while developing Draft 4, Roger removing k from the Google Doc was the springboard that prompted Jake to suggest changing the variable definitions. Roger then executed and validated Jake's idea when he added k back into the Google Doc. This, in turn, served as a springboard when it prompted Roger to delete lines that were no longer needed.

We conjecture there may be several reasons the students' use of the Google Doc instruments supported their progress through this proof construction task. The Google Doc provides a written format and, unlike spoken ideas, mathematical statements placed in a Google Doc do not disappear into thin air. Relative to spoken ideas, Google Doc ideas give students something visual to consider, acting as a visual mediator (Sfard, 2008) for the students' communication. In addition, ideas in the Google Doc provide the writer with a way to express a thought they may not be able to adequately verbalize. As a written format, the Google Doc plays two roles. It is both the place where students record their interim written ideas as they work and the place where their final solution lives. As a result, when a student is considering an idea presented in a Google Doc, the format of the presentation bears some resemblance to a solution. The format itself suggests progress toward a solution. At the same time, an idea in the Google Doc is easily editable by all group members. This combination of a format that moves toward a solution and is easy to edit may well invite student participation and inspire ideas.

This case study explored how a shared Google Doc supported a small group of students working on developing a proof in a F2F class. Future research could explore the role Google Docs might play when students were faced with other types of mathematics tasks besides developing a proof. Also, while we highlight certain advantages of the Google Doc, we wonder how students' activity might compare if they were to use other non-technological tools like a whiteboard for instance which is also easily editable and shared by all members of the group. Overall, this study presents a case for using Google Docs in a face-to-face proof course as it can promote a more active writing process that naturally ties the students' collective discourse and sharing of ideas with a truly collaborative final solution.

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