

How Students Reconcile Incongruous Mathematics Self-Efficacy

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Incongruous mathematics self-efficacy (IMSE) is when a student's self-efficacy does not match their performance. We present a subset of results from a larger qualitative study examining IMSE in collegiate intermediate algebra. We used qualitative interviews and surveys to longitudinally follow students for their entire semester in intermediate algebra and present the cases of three students with IMSE (over confidence). We show that participants' over confidence seemed to stem from them doubting their performance would be predictive of their future success; participants did eventually lower their self-efficacy in response to repeated low performance. Results have implications for college mathematics instructors and for the study of self-efficacy.

Keywords: Beliefs, College algebra, Incongruous mathematics self-efficacy, Mathematics self-efficacy

In this paper, we present a subset of the results from a larger study that sought to examine and describe instances of incongruous mathematics self-efficacy (when students' mathematics confidence does not match their mathematics performance). For this paper, we present the cases of three students who had incongruous mathematics self-efficacy in their college intermediate algebra class, and we describe how, throughout the course of the semester, the students reconciled their self-efficacy to their actual performance. Previous research on incongruous mathematics self-efficacy is limited and no previous research has examined how students reconcile their self-efficacy with their performance. This work has important implications for self-efficacy theory at the undergraduate level as well as for college instructors who teach introductory classes and want to help their students better calibrate their work and effort to be successful in their college mathematics classes.

Conceptual Framework and Literature Review

Mathematics self-efficacy (MSE) is defined as a person's beliefs about their ability to learn or perform mathematics (Bandura, 1986). MSE is individualized to the student. That is, a student might make the statement "I am good at math" and this could mean that they are good at algebraic computations or Calculus, depending on the level of mathematics they have taken. A person's self-efficacy is an individual judgement that only they can make; it is a belief about themselves constructed from their perceptions and experiences. These beliefs are subjective. In spite of this, MSE is a reliable predictor of performance and perseverance, suggesting that a large percent of the time students' MSE judgements are probably aligned with their abilities (Hackett, 1985; Hackett & Betz, 1982; Hackett & Betz, 1989; Hutchison et al., 2006; MacPhee et al., 2013; Marra et al., 2009; Multon et al., 1991). However, previous research has found that sometimes self-efficacy beliefs and external measures of ability do not match (Chen, 2006; Dassa & Nichols, 2019; Pajares & Miller, 1994; Schraw, 1995; Sheldrake, 2016; Stolp & Zabrocky, 2009). This has been called the "feeling-of-knowing accuracy" (Schraw, 1995, p. 401), a student's calibration (Pajares & Miller, 1994), or incongruous mathematics self-efficacy (IMSE) (Russell & Phelps-Gregory, 2022), the term we will use in this paper.

Students who have IMSE can either be “under confident” (their self-efficacy is lower than their external performance would suggest) or “over confident” (their self-efficacy is higher than their external performance would suggest). Confidence and self-efficacy are not the same (confidence is a broader domain general feeling and self-efficacy is a person’s specific subject-based beliefs in their performance) (Morony et al., 2013). However, for ease of communication, we will use these terms instead of under-efficacious and over-efficacious. IMSE can have multiple drawbacks on students’ learning. For example, over confidence might lead to students not studying or asking for help (Bandura, 1986).

Previous research on IMSE is lacking but does suggest that it exists. Several studies of incongruous self-efficacy (not in mathematics) have been conducted with prospective teachers. These studies have found that many prospective teachers demonstrate over confidence in their teaching efficacy, rating their teaching ability as higher than outside observers rate it (Dassa & Nichols, 2019; Wyatt, 2014). In terms of mathematics, previous research has shown evidence IMSE exists (Champion, 2010; Chen, 2006; Labuhn et al., 2010; Lopez & Lent, 1992; Russell & Phelps-Gregory, 2022; Sheldrake, 2016; Tellhed et al., 2017). However, this research is often limited in sample size and, in many cases, did not set out to identify IMSE. For example, Chen (2006) examined seventh graders’ MSE assessments and their teachers’ assessments of their abilities in mathematics problem solving. Chen found that students’ judgements were less aligned with their performance than their teachers’ judgements, suggesting at least some of the students had IMSE (but describing IMSE no further).

Why might IMSE develop and how might students hold these beliefs in spite of their performance? Previous research has confirmed Bandura’s (1986) theory that MSE develops based on four sources: (1) mastery experiences (performance on assessments and in mathematics courses), (2) vicarious experiences (judgements based on comparison with peers or colleagues), (3) social persuasions (verbal encouragement or discouragement from parents or teachers), and (4) affective or physiological state (a physical response, like crying during a mathematics test) (see, e.g., Hackett and Betz, 1982; Lopez and Lent, 1992; Zeldin et al., 2008). It is likely that these four sources also influence the development of IMSE though little research has examined this. One study, Sheldrake (2016), found that students with congruous self-efficacy appeared to have mostly based their self-efficacy on past performance. In contrast, Sheldrake found that under confident students appeared to have based their MSE on perceived peer comparison, perceived teacher encouragement, and interests in mathematics. The over confident students appeared to base their beliefs on their high perceived belief in the utility of mathematics. This work suggests that students may use different sources to form their MSE and which source they use may determine if they develop IMSE or not. However, further work is needed to examine this relationship. In addition, no work has examined how students continue to hold these beliefs in spite of their performance (or adjust their beliefs in response to continued performance measures that run contrary with their beliefs). In this paper, we sought to examine this. Our research question was:

How do undergraduate intermediate algebra students who have IMSE (over confidence) reconcile their self-efficacy beliefs with their performance during the semester?

Methods

To identify students with IMSE, we conducted a larger study. Participants in the larger study were undergraduate students enrolled in one of many intermediate algebra courses at a Midwestern public university. Students in this course who consented were given a survey to

identify their MSE and then students with high and low MSE were invited to participate in two interviews and to complete two additional surveys, administered at various points throughout the semester. Fifteen participants were invited to interviews in the larger study, 11 identified as women and 4 identified as male, non-binary, or transgender. To protect the identities of the participants, we have given them all pseudonyms (using female or non-binary pseudonyms), and we will use non-binary pronouns (they/them) for all participants.

Interview 1 included questions designed to better understand students' self-efficacy generally and their self-efficacy for intermediate algebra in particular. In it, we asked the students questions such as "How would you describe yourself as a math student?" and "On your [first] survey you responded by saying [paraphrase], can you tell me why you feel that way?" Survey 2 was an online survey administered immediately after the first exam, before students received their grades for the exam. In it, participants were asked about how they thought they had performed on the first exam and how they expected to perform in the course overall. Interview 2 occurred after students had received their exam grades and before the next exam; we again asked participants about their MSE beliefs as related to their experience with the first exam and about their MSE for the course overall. There was overlap in what the interviews covered because they were designed to examine changes in participants' MSE throughout the semester. Finally, survey 3 was administered electronically at the end of the semester once the students had received their final grades. This survey included questions about the participants' final grades as well as how they felt about their MSE after the course was over and how it compared to their experiences during the semester. The final survey also asked students questions about how their second and third exams went, including questions for students to describe how they felt leading up to, during, and after each exam compared to the grade they received. Having these final pieces of data enabled us to compare students' MSE beliefs longitudinally throughout the course and to compare how those beliefs relate to their final grade.

The interviews were transcribed and coded for analysis. The codes were based on a pilot study with additional codes developed inductively based on the data (Lichtman, 2012). To ensure trustworthiness, we used peer debriefing, member checks, and a coding reliability check with a third (non-author) researcher (Campbell et al., 2013; Lincoln and Guba, 1985). Once coding was complete, we developed emergent themes based on the coding. We defined cases of IMSE as when a participant's predicted grade was at least one full letter grade (10 percentage points) apart from their actual grade. That is, a student who predicted they would receive a B- but received a C- or lower would be identified as having IMSE. For this paper, we focus specifically on three participants who demonstrated IMSE for the course. The three participants were over confident, expecting to perform better in the course than they actually performed. We will describe how, over the course of the semester, these three participants reconciled their MSE with their ongoing lower than expected performance with the larger goal of better understanding IMSE and how MSE develops and changes as a result of repeated performance measures.

Results

Michelle, Lisa, and Adrian were all over confident in their ability to succeed in intermediate algebra. Michelle predicted (at the start of the semester) they would receive an A in the course but their final grade was a B. Lisa predicted they would receive an A- but received a C. And Adrian predicted they would receive a low B but failed the course with an F. We will examine each case individually with the goal of examining how each participant reconciled their MSE with their performance in the course. Across all three cases, we will see that participants did

lower their MSE during the semester, though not always in line with their actual performance. And participants did use their algebra performance to judge their MSE; however, they often had reasons to doubt their performance would be predictive of their future success which may explain why they maintained their over confidence.

Michelle

When describing why they felt confident about their math work, Michelle said it was because they felt the material was mostly review. They said, “[I feel] pretty confident, especially in the class I am in now because the stuff we’re doing, I have already done.” When asked about understanding the material Michelle said, “Just from my academic background, I feel that I’ll be able to get through the course fairly easy because I’ve done math similar to this before.” In spite of this, by the end of the course, Michelle’s performance was lower than they initially predicted.

We can see their grade predictions throughout the course in Table 1. They initially predicted they would receive an A, changed to an A or B after exam 1, to a B or A (with the B first) in interview 2, and ended the semester with a B. In interviews, Michelle remained confident even after exam 1 because they attributed their low score on exam 1 to them forgetting their calculator. That is, they said, “I didn’t get as good as the grade I would have gotten I believe with a calculator.” They also said, “I thought I would have done way better and like looking at the exam doing it... it’s just little tedious mistakes, which I believe from not having a calculator kind of screwed me over.” However, as the course went on, Michelle continued to perform lower than they expected. After the first exam, when asked what they would do differently, Michelle said “I would bring a calculator, and maybe go slower... maybe some of the mistakes I could have prevented but I honestly think most of them were from the calculator.” After the third exam they stated, “I was nervous about this exam and ended up making dumb mistakes. So, I was disappointed with this score (71%) because it was low, because of little mistakes and not actually because I don’t know the concepts.”

Table 1. Over Confident Grade Predictions Over the Semester.

Participant	First Prediction (before exam 1)	Survey 2 Prediction (before grade for exam 1)	Interview 2 Prediction (after exam 1 grade)	Final Grade
Lisa	A-	A	B or C	C
Michelle	A	A or B	B or A	B
Adrian	Low B	High C, Low B	C	F

In general, Michelle initially seemed to be over confident because they believed the material was review. However, they then seemed to maintain their overconfidence because they blamed their lower performance on what they termed “little mistakes” and forgetting a calculator. This type of reasoning, blaming something external like a calculator, is often called an external attribution (Kelley & Michela, 1980). In the end, Michelle did reconcile their performance with their MSE by slowly lowering their MSE (suggesting a B was possible by Interview 2) but they never entirely gave up their prediction of receiving an A. That is, Michelle seems to be a case of remaining slightly over confident throughout the semester, with some reconciliation happening around Interview 2.

Lisa

Like Michelle, Lisa also believed the math in the intermediate algebra course was review. Lisa said “I’ve noticed, especially in [intermediate algebra], I feel like it’s the same stuff I learned in Algebra 2 [High School]. So, I feel like I’m reviewing a lot of the time.” And “It kind of just feels like I’ve been reviewing, I mean I know it’s only like four weeks in... So, I don’t really know what to expect in the future, but as of right now, I feel like it’s all review.” Based on this initial belief, Lisa predicted they would receive an A- in the course. Table 1 shows their grade predictions throughout the course of the semester. We see that after exam 1 (before they received their score) their MSE had increased, and they believed they would receive an A. However, by Interview 2, they had lowered their MSE closer to their actual final grade. In Interview 1, they said they felt “pretty confident” in the course but, when asked how they felt in the second interview, they said “not very confident.” Despite this, Lisa continued to be somewhat surprised by their exam scores throughout the semester; for example, regarding the third exam Lisa said, “I thought I’d do better on this exam.”

Lisa’s initial over confidence seemed to stem from their belief that the course was review and this over confidence seemed initially reinforced by the exam. In addition, like Michelle, Lisa also demonstrated an external attribution for her lower performance, and this seemed to help maintain her over confidence. In particular, Lisa partially blamed the instructor for their lower-than-expected scores. Lisa said, “[My professor] doesn’t teach in a way that I have normally been taught to do math and this unit is something that I’ve never done before like rational expressions and equations. So, I’ve been struggling.” Lisa also stated of their instructor “I don’t really think my teacher has covered enough of [the material] for me to understand.” As the semester went on, Lisa also felt the material became more difficult, which may be why they lowered their MSE. These lower-than-expected scores were motivating to them, causing them to study harder which helped them sustain higher MSE (believing the extra studying would raise their performance). Regarding the final exam they stated, “I definitely studied a lot more leading up to the final after I received my previous test grades.” Despite this, Lisa received a C in the course and, in the final survey, expressed not understanding the material as well as they thought.

In the end, Lisa did not reconcile their performance with their MSE until Interview 2, where they predicted a C was possible and described themselves as not very confident. Lisa seems to be a case of being over confident in the beginning but, as the difficulty of the material increased and they continued to perform poorly, reconciling their MSE and performance by lowering their MSE. In addition, we also see that external attributions for poor performance helped Lisa maintain their over confidence.

Adrian

Adrian demonstrates a different reason for over confidence than Michelle and Lisa. Adrian had actually taken the course before, referencing that this was their third or fourth time through this same material. From the beginning, they expressed a lot of doubt in their abilities to succeed in math courses in general; they specifically said that they were not confident, yet were hopeful, that they would pass intermediate algebra at the college level. Adrian initially predicted they would receive a low B in the course.

As shown in Table 1, between Interview 1 and Interview 2, Adrian’s MSE did fall but did not align with their final course performance. In fact, Adrian’s MSE never fully aligned with their performance. There seemed to be several reasons for this. First, Adrian responded to lower-than-expected performance by working more and getting help. They said, “I attempted to study more,

by working out more problems and getting help from others in the class.” Overall, Adrian seemed to believe working out more problems and working with classmates would result in better performance though, sadly, it did not. Adrian also believed that their homework and quiz scores could make up for poor exam performance, leading them to believe they could still perform well in the course. They said, “I know that I absolutely will not ace any exams, but I can probably still pass [the course] with the quizzes and homework.” Unfortunately, this is not true with the grade calculations of the course.

Finally, one other reason for Adrian’s high MSE seemed to be that, during the course, they were diagnosed with ADHD for the first time. In the second interview, Adrian seemed more hopeful that the course would go well because they finally felt like they could learn to navigate having ADHD and then do better in the class. Adrian said, “Currently I’m trying to find a medication that works for my ADHD and getting accommodations... So that may help once I find the right medication [and] actually get the accommodations.” Adrian seemed to attribute their past low performance externally, to ADHD, and seemed to believe that fixing this ADHD would help them succeed. However, in the end, Adrian was not able to pass the course.

Across the Cases

Looking across the three cases, two themes emerge. First, all three students did lower their MSE during the semester. That is, their continued lower than expected performance did lead to lower confidence in their ability to be successful in intermediate algebra. However, this reconciliation was slow (happening around interview 2 and not always ever fully aligning with their final grade). Second, the reason this reconciliation was slow varied by participant but included students externally attributing their poor performance (it was the teacher, the lack of a calculator, or ADHD) as well as believing that changes (studying more or getting ADHD treatment) would raise their future performance. Overall, we see that all participants doubted that their initial exam scores were predictive of future performance (allowing them to maintain over confidence). Their MSE only started to lower once they had repeated lower performance.

Discussion

Previous research has shown IMSE can exist though this research often lacked detail, was not at the collegiate level, or was not in mathematics (Chen, 2006; Dassa & Nichols, 2019; Wyatt, 2014). Our work shows that IMSE does exist in collegiate mathematics and describes qualitatively what that IMSE can look like. In addition, no previous research had examined how IMSE was reconciled with performance. Our study shows that over confident students do slowly reconcile their IMSE with their performance by lowering their MSE. However, this can be slow because students externally attribute their low scores and have reasons to doubt their performance scores will be predictive (because they are changing their behavior).

Recall that Sheldrake (2016) found that students with IMSE may base their MSE on sources other than mastery experiences (performance). Sheldrake found, in particular, that over confident students based their MSE beliefs on the perceived utility of mathematics. Unlike in Sheldrake’s study, our participants never mentioned the utility of mathematics as a reason for their MSE; however, given they were taking a mathematics course required for graduation, they were presumably aware of the utility for their degree. Also in contrast with Sheldrake, we did not find that participants were basing their MSE on sources other than performance; that is, they were not basing their MSE on social comparison, vicarious experience, or affective/ physiological state. Instead, we found that participants with IMSE were using performance as a source of MSE but

had reasons to doubt their performance measures were predictive of the future because they externally attributed their low performance and changed their behavior. However, these external attributions and behavior changes did not lead to them being more successful.

Limitations and Future Work

The first limitation of this study is the small sample size. Having only fifteen participants, all from the same university, limits our ability to extrapolate results. The choice of a small sample size allowed us to collect detailed qualitative data at five points in time throughout the semester. However, while we were able to identify three cases of over confidence, we were unable to identify cases of under confidence. It is unclear if this is because this is rare or if participants with under confidence simply did not participate in the study. This could be a topic for future research where larger sample sizes might help better identify such participants.

An additional limitation is the self-reporting nature of the surveys. Self-reporting is necessary to capture the MSE beliefs of students at a point in time and is common in previous research on MSE (Dassa & Nichols, 2019; Hackett, 1985; Pajares & Miller, 1995). However, one issue that can arise is students' desire to give the "correct" answer, how they think they "should" feel. To account for this, in the interviews, we asked students to describe and elaborate on their MSE beliefs multiple times and then used member checking to ensure trustworthiness. However, it is always possible that students did not share the full truth with us as researchers.

Implications for Practice

The findings in this study indicate that instructors should be aware of students who are under or over confident and potentially take measures to help students calibrate their MSE beliefs with their performances. This is not to say that we need to go around lowering or raising students' MSE beliefs, but rather that we should try to help students become aware of their own abilities in realistic ways. Perhaps we can design instructional material that includes components of introspection alongside the content. Such introspection could increase the likelihood that students will adjust their MSE beliefs accordingly.

Another aspect of this study that is informative for instructors is that MSE beliefs can change over the course of the semester. What this means for instructors is the need to "keep a pulse" on their students and their MSE beliefs. This is especially true with repeated performances that are not what the student initially expected. Instructors could help students by encouraging them to have a personal meeting with the instructor after the first or each exam. This session would not have to be very long, but it would be a time to address the student's performance on the exam, any questions they missed, and the long-term effects of the exam and their current homework or quiz performances on their grade. This counseling session could help students to have a better understanding of the situation they are in and calibrate their MSE and performance better.

Overall, our results show that IMSE exists, is a result of specific reasons from the students' course (e.g., initially seeming easy), and is based on factors similar to MSE but with students perhaps interpreting these factors differently than instructors. Learning about IMSE serves as an opportunity for instructors to better understand their students and help their students better understand themselves and what leads to their success. If we can continue the pursuit of helping students achieve their goals and learn about themselves, we can increase student success in mathematics.

References

- Bandura, A. (1986). Self-Efficacy. In A. Bandura (Ed.) *Social foundations of thought and action* (pp. 290 – 453). Englewood Cliffs.
- Campbell, J. L., Quincy, C., Osserman, J., & Pedersen, O. K. (2013). Coding in-depth semistructured interviews: Problems of unitization and intercoder reliability and agreement. *Sociological Methods & Research*, 42(3), 294 – 320. DOI: 10.1177/0049124113500475
- Champion, J. (2010). *The mathematics self-efficacy and calibration of students in a secondary mathematics teacher preparation program*. [Doctoral Dissertation, University of Northern Colorado]. <https://www.proquest.com/docview/516275195?pq-origsite=gscholar&fromopenview=true>
- Chen, P. (2006). Relationship between students' self-assessment of their capabilities and their teachers judgements of students' capabilities in mathematics problem solving. *Psychological Reports*, 98, 765 – 778.
- Dassa, L., & Nichols, B. (2019). Self-efficacy or overconfidence? Comparing preservice teacher perceptions of their content knowledge and teaching abilities to the perceptions of their supervisors. *The New Educator*, 15(2), 156 – 174.
- Hackett, G. (1985). Role of mathematics self-efficacy in the choice of math-related majors of college women and men: a path analysis. *Journal of Counseling Psychology*, 32(1), 47 – 56.
- Hackett, G., & Betz, N. (1982). Mathematics self-efficacy expectations, math performance, and the consideration of math-related majors. *Proceedings of the Annual Meeting of the American Educational Research Association*. <https://files.eric.ed.gov/fulltext/ED218089.pdf>
- Hutchison, M. A., Follman, D. K., Sumpter, M., & Bodner, G. M. (2006). Factors influencing the self-efficacy beliefs of first-year engineering students. *Journal of Engineering Education*, 95(1), 39 – 47.
- Kelley, H. H., & Michela, J. L. (1980). Attribution theory and research. *Annual Review of Psychology*, 31, 457 – 501.
- Labuhn, A. S., Zimmerman, B. J., & Hasselhorn, M. (2010). Enhancing students' self-regulation and mathematics performance: the influence of feedback and self-evaluative standards. *Metacognition Learning*, 5, 173 – 194.
- Lichtman, M. (2012). *Qualitative research in education: a user's guide* (3rd ed.). Los Angeles, CA: Sage Publications.
- Lincoln, Y., & Guba, E. (1985). *Naturalistic inquiry*. Beverly Hills: SAGE Publications, Inc.
- Lopez, F. G., & Lent, R. W. (1992). Sources of mathematics self-efficacy in high school students. *The Career Development Quarterly*, 41, 3 – 12.
- MacPhee, D., Farro, S., & Canetto, S. S. (2013). Academic self-efficacy and performance of underrepresented STEM majors: gender, ethnic, and social class patterns. *Analyses of Social Issues and Public Policy*, 13(1), 347 – 369.
- Marra, R. M., Rodgers, K. A., Shen, D., & Bogue, B. (2009). Women engineering students and self-efficacy: a multi-year, multi-institution study of women engineering student self-efficacy. *Journal of Engineering Education*, 98(1), 27 – 38.
- Morony, S., Kleitman, S., Lee, Y. P., & Stankov, L. (2013). Predicting achievement: confidence vs self-efficacy, anxiety, and self-concept in Confucian and European countries. *International Journal of Educational Research*, 58, 79 – 96.
- Multon, K. D., Brown, S. D., & Lent, R. W. (1991). Relation of self-efficacy beliefs to academic outcomes: a meta-analytic investigation. *Journal of Counseling Psychology*, 38(1), 30 – 38.

- Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: a path analysis. *Journal of Educational Psychology*, 86(2), 195-203.
- Pajares, F., & Miller, M. D. (1995). Mathematics self-efficacy and mathematics performances: the need for specificity of assessment. *Journal of Counseling Psychology*, 42(2), 190 – 198.
- Russell, B., & Phelps-Gregory, C. M. (2022). Global and task-specific mathematics self-efficacy and performance: A qualitative study in intermediate college algebra. *Proceedings of the 44th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.
- Schraw, G. (1995). Measures of feeling-of-knowing accuracy: a new look at an old problem. *Applied Cognitive Psychology*, 9, 321 – 332.
- Sheldrake, R. (2016). Differential predictors of under-confidence and over-confidence for mathematics and science students in England. *Learning and Individual Differences*, 49, 305 – 313.
- Stolp, S., & Zabrocky, K. M. (2009). Contributions of metacognitive and self-regulated learning theories to investigations of calibration of comprehension. *International Electronic Journal of Elementary Education*, 2(1), 7 – 31.
- Tellhed, U., Bäckström, M., & Björklund, F. (2017). Will I fit in and do well? The importance of social belongingness and self-efficacy for explaining gender differences in interest in STEM and HEED majors. *Sex Roles*, 77, 86 – 96.
- Wyatt, M. (2014). Using qualitative research methods to assess the degree of fit between teachers' reported self-efficacy beliefs and their practical knowledge during teacher education. *Australian Journal of Teacher Education*, 40(1), 117 – 145.
- Zeldin, A. L., Britner, S. L., & Pajares, F. (2008). A comparative study of the self-efficacy beliefs of successful men and women in mathematics, science, and technology careers. *Journal of Research in Science Teaching*, 45(9), 1036 – 1058.