A “Different Vibe”: The Effect of Discrete Mathematics on Undergraduate Students’ Mathematics Beliefs

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This paper presents emerging results from an investigation of undergraduate students’ beliefs about mathematics (and themselves as learners) within the context of a discrete mathematics course. Our data sources include findings from a survey study with 11 participants and a follow-up interview study with 5 participants. Results suggest that students perceived discrete mathematics as being significantly different from their previous mathematics coursework, offering additional options for creativity and a shift in focus from computation to proof and logic. The impact of this shift on students’ mathematical self-efficacy was mixed, with some students reporting unchanged self-efficacy but multiple students reporting decreased self-efficacy (either temporary or permanent). These results have implications for the teaching of discrete mathematics courses and shed additional light on our existing knowledge of students’ belief change during the transition from K-12 school to undergraduate mathematics courses.

Keywords: Beliefs, Discrete mathematics, Mathematics self-efficacy

We present preliminary results of two studies examining undergraduate discrete mathematics students’ beliefs about themselves as learners (their mathematics self-efficacy or confidence) and their beliefs about discrete mathematics. Discrete mathematics is a valuable site for study because it may allow for changes in students’ beliefs. In particular, discrete mathematics courses typically include a variety of accessible topics that allow for deep mathematical thinking (Goldin, 2018; Sandefur et al., 2022). Particularly given recent increases in the number of undergraduate students majoring in the computer and information sciences, which typically require a course in discrete mathematics (Berg et al., 2023), it is timely to consider the impact of this course. However, little previous research has examined the effects of discrete mathematics on students’ beliefs.

Conceptual Framework and Literature Review

In this study, we examine two types of beliefs held by undergraduate students enrolled in discrete mathematics courses. First, we examine students’ mathematics self-efficacy (MSE), that is, their beliefs about their ability to do and learn mathematics (Bandura, 1986). The construct of MSE is related to mathematical confidence, although confidence is normally a broad measure whereas self-efficacy is task- or subject-specific (Morony et al., 2013). Self-efficacy beliefs are important because high MSE has been linked to higher achievement and improved learning, as well as non-academic skills such as perseverance and self-regulation (Hackett & Betz, 1989; Muenks et al., 2018; Usher et al., 2019). Generally, high MSE is seen as more beneficial than low MSE, though some researchers suggest that self-efficacy needs to align with actual ability (called calibration) to be fully beneficial (Russell & Phelps-Gregory, 2022). Thus, a goal of researchers and instructors should be, at the very least, to help students develop calibrated but confident self-efficacy.

Secondly, we examine students' beliefs about mathematics, including some of their generalized beliefs about the nature of mathematics (such as if math is creative) as well as specific beliefs about discrete mathematics. Unproductive beliefs (like the belief that mathematics is memorizing rules and is not creative) are commonplace among US students (Boaler, 1998; Kloosterman & Stage, 1992; Mann, 2006; Phelps-Gregory et al., 2020) and may harm students' learning because these beliefs correlate with higher anxiety and lower achievement (Geist, 2010; House, 2006).

We specifically examine students' beliefs (and possible beliefs changes) in discrete mathematics, a collegiate mathematics course typically taken by aspiring mathematicians, teachers, computer scientists and engineers, and information technology/information science majors. Discrete mathematics courses generally cover topics related to discrete objects and structures, though the exact topics covered are not always agreed upon. Sandefur and colleagues (2022) identified core topics and key themes of all discrete mathematics classes, including topics such as combinatorics and recursion. Previous research on discrete mathematics has examined student work (e.g., Greefrath et al., 2022) and teachers' pedagogical decisions and techniques in building student thinking (e.g., Alsardary & Blumberg, 2009; Love et al., 2006; Soto et al., 2022). However, one under-researched area is students' beliefs during and after discrete mathematics courses.

Discrete mathematics may be a good site to examine students' beliefs because researchers have argued that discrete students can engage in deep mathematical thinking regardless of their mathematical backgrounds and experiences (Colipan & Liendo, 2022; Goldin, 2018). This engagement might provide a site for changes in students' beliefs (Sandefur et al., 2022). In addition, discrete mathematics may also be a source of changes to MSE because mathematical transitions such as those required in discrete mathematics may be a site for disruption and change (Bandura, 1986; Gill, 2019). In particular, discrete mathematics courses often (but not always) include a focus on mathematical proof, thus prompting a transition to the so-called "axiomatic-formal" mode of mathematics (see Tall, 2008, for a fuller discussion of this transition). However, as noted by Stylianou and colleagues (2015), little research has examined undergraduate students' beliefs about proof, including their self-efficacy for learning proof. In general, few previous studies have examined students' beliefs in discrete mathematics. Our study sought to begin to address this. We examined the research questions:

1. How does discrete mathematics affect students' beliefs about their mathematics ability (their mathematics self-efficacy)?
2. What are students' beliefs about the nature of discrete mathematics?

Methods

We present here the related results of two studies of student beliefs. We first conducted a qualitative survey study examining undergraduate students enrolled in discrete mathematics at two regional universities. The survey used open-ended prompts to ask participants about their MSE and their beliefs about mathematics and discrete mathematics in particular. Eleven participants completed the survey, answering in their own words in sentences and phrases. We analyzed this data using an open coding procedure where codes were developed emergently from the data, following Campbell and colleagues (2013).

Once analysis of this data was complete, we conducted a second qualitative interview study to again examine undergraduate students' beliefs in discrete mathematics but with a new group of participants, enrolled in a different semester and with a different instructor. The goal of the interview was to explore the same questions as the survey, but in further depth and with the opportunity for follow-up questions. Questions included asking participants to describe their mathematics experiences before discrete mathematics, asking students to describe their discrete mathematics experiences, asking about the effect of discrete mathematics on their confidence, and asking belief questions (e.g., about the relevance and usefulness of mathematics). There were also questions asking participants to examine specific discrete mathematics questions and their confidence (MSE) in solving them. Five students completed interviews, lasting on average 29 minutes. Further interviews will be conducted in future semesters, but we present the results of the survey study and initial round of interviews here.

To analyze the interview data, we transcribed the interviews and then built cases for each participant by examining their data and highlighting important themes related to the research questions. Analysis then focused on categorizing participants by the effect of discrete mathematics on their MSE. We then compared and contrasted the groups of participants.

Results

Our preliminary results from both studies show that discrete mathematics may raise some students' MSE, temporarily or permanently, lower other students' MSE, and have no effect on the MSE of some students. This result is tied to students' beliefs about discrete mathematics, with multiple students mentioning their feeling that the course was qualitatively different compared to other mathematics classes. We will examine these findings in more detail below.

Findings of Survey Study

Of the eleven students who completed the survey, three participants reported that discrete mathematics did not affect their self-efficacy, three reported that it increased their self-efficacy and one reported that it temporarily lowered their MSE. Students' reasons for a change in self-efficacy included their course performance (i.e., grades), particularly given the reputation of the course as difficult and the nature of the mathematics in the course. For example, Noah[1] said, "I feel a little more confident because it isn't as scary as my dad made it out to be." Caleb reported that discrete mathematics temporarily lowered his MSE because the material was "different." When asked if discrete mathematics effected his confidence, Caleb said "Yes and no, it was harder for me to pick up the different topics because it's so different from, for example, a calculus class, but at the same time I just had to work harder to understand it and when I did my confidence was back."

Students' view of discrete mathematics as "different" could be seen in their other survey responses as well. Seven participants reported that discrete mathematics required more creativity than other mathematics subjects. Ava said, "The proofs in discrete math allow for slightly more creativity than in other math classes, because there is no one 'right' answer." However, not every participant agreed, with Noah saying, "Less creativity, [because] physics and calculus requires a lot of thinking outside of the box." Overall, the results of the survey study suggest that discrete mathematics could be a source of MSE change for some students due to the nature of the mathematics in the course.

Preliminary Findings of Interview Study

We are currently conducting an interview study to follow-up on the survey results. Based on preliminary analysis of data from five participants, one reported that their MSE was permanently lowered; one reported that, like Caleb, their MSE was temporarily lowered; and three reported no

[1] All names are pseudonyms to protect participants' identities.

change to their MSE. In contrast to the survey study, no students in the interview study reported that their MSE was raised as a result of discrete mathematics.

Jacob reported discrete mathematics permanently lowered his MSE, and Kaitlyn reported that discrete mathematics temporarily lowered her MSE. The major source of this lowered MSE for both students seemed to be their performance in discrete mathematics; both reported struggling some with the class material. Jacob said he was doing “not so hot [laughs]” in his discrete class. Kaitlyn said, “Usually math would come to me easy. Like I can pretty pick up pretty fast on it. But I just like for whatever reason cannot like wrap my head around some of the stuff that we're doing.” Both attributed their performance and lowered confidence to the nature of the mathematics in the class being different from previous classes. Jacob said, “the material is like… logic based. So, it's really hard to like wrap your mind around sometimes.” He also said:

> I guess there's not so many … like set formulas that I have to memorize. It's kind of just like applying all of the math that I've ever learned into one, without like formulas. And it's just kind of hard for me to … grasp my head around.

For Jacob, this lowered MSE seemed to be permanent; that is, he said overall now he believed he just was not as good at mathematics and would struggle in future classes. In contrast, for Kaitlyn, this change was temporary, and she assumed that she would be successful again in the future. The main difference between the two was that Jacob assumed that future classes would be like discrete mathematics. He said, “So I know a lot of like math from now on … basically builds off of discrete, like proofs and stuff like that.” When asked to elaborate he said:

> Because I feel like the math before, … like calculus and stuff like that, it's all like here's the question and then there's an answer. But I feel like discrete and everything moving past this is more going to be like, why is this an answer? Or are there other answers? Or is there no answer? And like then explain why there isn't that and then prove it.

In contrast, Kaitlyn assumed that future mathematics classes would be similar to her previous classes and discrete mathematics was just an anomaly. She said, “I know it's just … one class. I know … it's like I'm good at math. I know I am, but it's just this one class. So, I guess short term [it affected my confidence], yes, but long term not really.”

It is important to note that all five students reported that discrete mathematics was different from their previous classes, even students who reported that their MSE did not change. Four of the five said discrete mathematics required more creativity, with the fifth student reporting it was about the same. Makayla said:

> I would say it's more, more creative. I think there's definitely different ways you can go about getting the answer for some things. Which I know is true of like other math classes, but I think it's more, more true of this.

And Madison said it was more creative saying, “Probably more [strongly emphasized] because I would think the other classes are more just numbers and equations, whereas this one is thinking more outside the box with how they're related, I guess.”

In addition, all participants implied that discrete mathematics had a “different vibe” from previous mathematics classes. Makayla said:

> I feel like it is just like a different overall like vibe, but I don't really know how to explain that. Like it just, it does kind of feel different in terms of math. It is more like logic and circuits and, and just kind of like… Um, I don't know. It's just … kind of like different.

Madison said, “How I described it to my friends is it's low-level math but use the most confusing words possible. [She laughs here.]” Tyler reported that discrete mathematics did not affect his MSE but did say the math was different and perhaps he was less confident in it. He said:

> I will say that I'm less confident about this class than most other math classes because with discrete math it's a bit different from other math. It's a lot less about computation and more about proofs and how the different concepts work together. And so I'm less confident in that than, say, solving equations. But … I would say it hasn't really affected my total confidence in math.

Overall, as with the survey study, the results of the interview study suggest that discrete mathematics could be a source of MSE change for some students due to the nature of the mathematics in the course.

Discussion

The results of this study add nuance to the field's understanding of how undergraduate mathematics courses such as discrete mathematics might influence students' beliefs. Previous research (e.g., Sandefur et al., 2022) suggests that the conceptual nature of discrete mathematics courses could be a source of increased MSE, because students might be able to engage more deeply with mathematical ideas compared with more computationally-focused undergraduate coursework which relies heavily on prerequisite knowledge and skills. In some ways, our results align with this result, in that nearly all our participants in both the interview and survey study described discrete math as being different from the math they have experienced in other coursework. Students described a "different vibe" including mathematics that seemed more creativity, had more options for completing problems, and had a focus on proof, logic, and concepts rather than computation. This aligns with existing research about students' perceptions of the transition to more formal mathematics at the undergraduate level (e.g., Tall, 2008).

However, in contrast with previous research and particularly within the interview study, we found that students reported that these perceived differences between their previous coursework and discrete math actually lowered their MSE (either permanently or temporarily). We might conjecture that this difference from previous research could be caused by the fact that the students in the interview study described themselves as having been successful in their prior mathematics coursework. Thus, rather than serving as a site where their insufficient prerequisite knowledge was not a barrier to achievement, for these students, discrete mathematics pushed them to think about math in unfamiliar and potentially uncomfortable ways. These results suggest that instructors of discrete mathematics courses should be aware that their students might be experiencing lowered self-efficacy, and this could cause students to disengage from the course. These students might need additional support to continue persevering despite the new challenges.

Questions for Further Discussion

We anticipate collecting further data for an additional two semesters, as well as finishing our analysis of existing data. Given this research is ongoing, we pose the following questions for the reader and the audience. First, the nature of interview work is that the questions you ask guide the interview. *What further questions should we ask during interviews to better explore students' beliefs?* In addition, as our analysis is ongoing, we are open to further interpretations of our data. *What analysis techniques and questions could help us better understand our data?* Finally, we pose a broader question to the field. *Given discrete mathematics, and possibly other advanced mathematics classes, may impact students' beliefs, what should be instructors' goals in these classes? How can instructors support students in positive belief change while also mitigating any effects of negative belief change (like permanently lowered self-efficacy)?*

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