Analyzing Calculus Textbooks' Introduction of the Dynamic Derivative Using a Framework of Shape-Thinking and (Co)Variational Reasoning

Allison Olshefke
University of Delaware

Keywords: calculus, content analysis, covariational reasoning, shape thinking, textbook analysis

Research over the past few decades demonstrates that calculus students have difficulties making sense of the derivative conceptually, even if they can calculate derivatives fluently (e.g., Orton, 1983; Carlson et al., 2002; Thompson & Carlson, 2017; Epstein, 2013). In a review of literature on ideas foundational to calculus learning, Thompson and Harel (2021) argue that that the underdevelopment of covariational reasoning skills may be the missing link in students' ability to engage deeply with major calculus concepts including the derivative. Covariational reasoning, (i.e., reasoning about how quantities vary simultaneously and in relation to one another) is evident in the MAA's recommended conception of the derivative as dynamic (Bressoud, 2015). Recent work emphasizes the need to promote "derivative as instantaneous rate of change or as a measure of sensitivity of one variable to change in another," rather than the static conception of "derivative as slopes of tangent lines," which does not make explicit the relationship between quantities (Bressoud et al., 2015, p. 18).

A few recent studies suggest that calculus textbooks do not frequently provide opportunities for students to develop covariational reasoning skills with quantities related to the derivative. These studies used Carlson et al.'s (2002) five levels of (co)variational reasoning to compare the opportunities for covariational reasoning across international calculus textbooks (Chen, 2023) and between a regular and applied Calculus textbook (Mkhatshwa, 2022).

This study adds a different perspective on opportunities for reasoning covariationally in the context of the derivative by analyzing introductory material about the derivative in a two widely used calculus textbooks, one traditional (Larson & Edwards, 2018) and one reform (Hughes-Hallet et al., 2013) by applying a novel framework introduced by Tasova et al., 2018. Their framework for analyzing written curriculum, which they used to analyze precalculus content in calculus textbooks, merges Thompson & Carlson's (2017) (co)variational reasoning framework with Moore and Thompson's (2015) framework for static and emergent graphical shape thinking. This multi-layered approach allows for a more nuanced description of the opportunities for students to understand the derivative from a covariational perspective and for exploration of the alignment with a dynamic or static conception of the derivative.

In this poster, I present the results of my directed content analysis (Hsieh et al., 2005) to answer the following questions: *What opportunities does each textbook provide in its narrative and worked example sections for students to reason about quantities and variables related to the derivative as dynamic?* and *What are the similarities/differences between the textbooks in the nature and frequency of these opportunities?* Results are supplemented with an expanded presentation of Tasova et al.'s (2018) framework that includes examples illustrating the range of opportunities textbooks provide. Finally, I present suggestions for both research and practice that focus on supplementing textbook opportunities to foster a dynamic conception and the implications of these results for equity and inclusion in STEM.

Acknowledgments

This material is based upon work supported by NSF under Grant No. DRL-2142000.

References

Bressoud, D. M., Johnston, E., Murphy, C., Rhea, K., Williams, J., & Zorn, P. (2015). The Calculus Sequence. In P. Zorn, C. S. Schumacher, & M. J. Siegel (Eds.), *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences* (pp. 17–25). Mathematics Association of America. https://maa.org/sites/default/files/CUPM%20Guide.pdf

Carlson, M. P., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 33(5), 352–378

Chen, Y. (2023). *An Analysis of Covariational Reasoning Pedagogy for the Introduction of Derivative in Selected Calculus Textbooks*. Teachers College Columbia.

Epstein, J. (2013). The Calculus Concept Inventory–Measurement of the Effect of Teaching Methodology in Mathematics. *Notices of the American Mathematical Society*, *60*(08), 1018. https://doi.org/10.1090/noti1033

Hsieh, H.-F., & Shannon, S. E. (2005). Three Approaches to Qualitative Content Analysis. *Qualitative Health Research*, *15*(9), 1277–1288. https://doi.org/10.1177/1049732305276687

Hughes-Hallett, D., McCallum, W. G., & Marks, E. J. (2013). Calculus: Single and Multivariable (6th ed.). Wiley.

Larson, R., & Edwards, B. H. (2018). Calculus of a single variable (11th ed). Brooks/Cole.

Mkhatshwa, T. P. (2022). Quantitative and covariational reasoning opportunities provided by calculus textbooks: The case of the derivative. *International Journal of Mathematical Education in Science and Technology*, 1–28. https://doi.org/10.1080/0020739X.2022.2129497

Moore, K. C., & Thompson, P. W. (2015). Shape Thinking and Students' Graphing Activity. In T. Fukawa-Connelly, N. Infante, & M. Zandieh (Eds.), *Proceedings of the Eighteenth Annual Conference on Research in Undergraduate Mathematics Education* (pp. 782–789).

Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics*, 14(3), 235–250

Tasova, H. I., Stevens, I. E., & Moore, K. C. (2018). A Framework for Analyzing Written Curriculum from a Shape-Thinking and (Co)Variational Reasoning Perspective. In A. Weinberg, J. Rasmussen, M. Rabin, M. Wawro, & S. Brown (Eds.), *Proceedings of the Twenty-First Annual Conference on Research in Undergraduate Mathematics Education* (pp. 1527–1533). http://sigmaa.maa.org/rume/RUME21.pdf

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), Compendium for research in mathematics education. (pp. 421–456). National Council of Teachers of Mathematics.

Thompson, P. W., & Harel, G. (2021). Ideas foundational to calculus learning and their links to students' difficulties. *ZDM – Mathematics Education*, *53*(3), 507–519. https://doi.org/10.1007/s11858-021-01270-1

Static and Dynamic:
How Two Textbooks Introduce the Derivative

PRESENTER:

Allison Olshefke

MOTIVATION:
- Even when they can work with derivatives procedurally, students still struggle understanding the derivative conceptually.
- Covariational reasoning may help bridge this gap.
- How do textbooks promote conceptions of the derivative that are productive?

METHODS:
- I compared the intro chapter on derivatives from two commonly used calculus textbooks:
  - Larson's *Calculus*
  - Hughes Hallett et al.'s *Calculus: Single and Multivariable*
- I coded narrative sections of the textbooks using a framework that distinguishes between static and dynamic conceptions of the derivative.

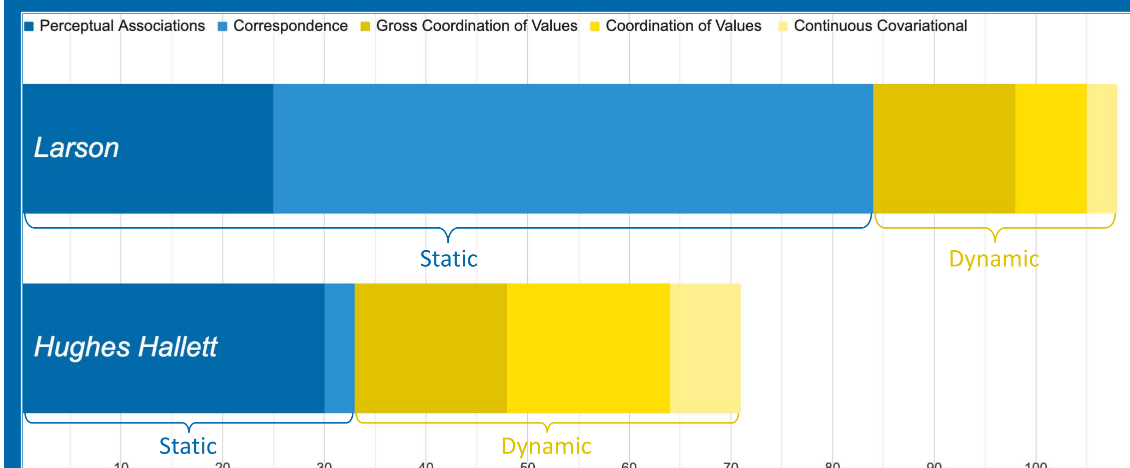
MAIN RESULTS:

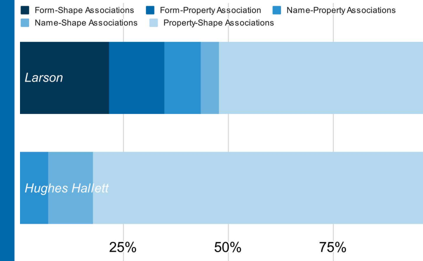
Frequency

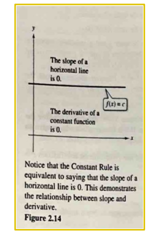
	Static	Dynamic	Total
Larson	76.4% (n = 84)	23.6% (n = 26)	110
Hughes Hallett	46.5% (n = 33)	53.5% (n = 38)	71

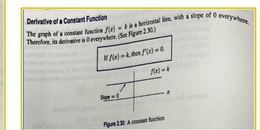
Scan for references & more info!

Calculus textbooks differ in the ways that they promote the **derivative** as static and dynamic.

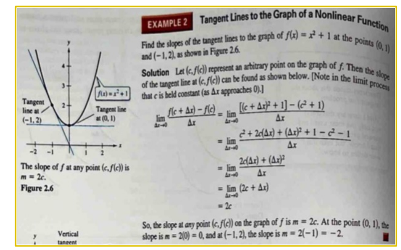




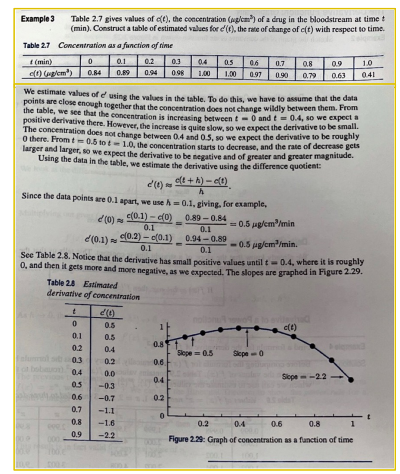




Property-Shape Association worked examples (*Larson* p. 110 & *Hughes Hallett* p. 94)



Larson Correspondence worked example (p. 102)



Hughes Hallett Continuous Covariation Worked Example (p. 92-93)