An Exploratory Study of Students' Understanding of Graphical Optimization in a Cost Minimization Context

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This study reports on first-semester calculus students' reasoning about a univariate optimization problem that involves finding the production level at which the cost per yard is minimized when given the graph of a function that represents the relationship between the cost per yard and the number of yards produced by a factory. Analysis of verbal responses and work written by four students when solving the problem revealed that determining the production level at which the cost per yard is minimized was straightforward for all the students. However, explaining how this production level is related to the first derivative of the given function was problematic for most of the students.

Key words: Optimization problems, graphical optimization, problem solving, calculus education

Unlike algebraic optimization that uses algebraic methods (that may sometimes be sophisticated, especially when working with complex objective functions) to solve univariate optimization problems or numerical optimization (that requires some level of technical skills such as proficiency in MATLAB programing) to solve univariate optimization problems, graphical optimization (Bhatti, 2000) is the simplest method for solving univariate optimization problems (UOPs) in that it only requires making sense of graphs of objective functions. A number of studies have reported on several difficulties typically exhibited by students when solving optimization problems algebraically, including formulating the objective function, finding and interpreting critical values or extrema of the objective function, and determining if a critical value(s) results in a minimum/maximum value of the objective function (cf. Borgen & Manu, 2002; Dominguez, 2010; LaRue & Infante, 2015; Mkhatshwa, 2019; Swanagan, 2012). We are not aware of any research that has examined students' thinking in the context of graphical optimization, which is the motivation for this study. To address this knowledge gap, task-based interviews (Goldin 2000) were conducted with four calculus students.

Contrary to findings of studies that have reported on students' thinking about algebraic optimization (cf. Borgen & Manu; Swanagan, 2012), nearly all the students in the present study were successful in finding the critical value or extremum as well as justifying extremum while working with a UOP where the graph of the objective function was provided. To some extent, this may suggest that while students may struggle with solving UOPs algebraically, partly due to the lack of facility with some algebraic techniques such as calculating derivatives of complex objective functions, students have better success with solving UOPs graphically not only because they can visualize the objective function, but also because having access to the graph of the objective function supports their quantitative reasoning (Thompson, 1993; 2011) such as the ease of identifying critical values and extrema. Additionally, three of the four students made remarks that suggested that they had difficulty understanding that the derivative of the objective function ought to be zero at the critical value (i.e., the cost minimizing quantity), something that generally comes easy for students when solving UOPs algebraically.

References

Bhatti M.A. (2000) Graphical optimization. *Practical Optimization Methods, 18*(2), 47-73.

Borgen, K. L., & Manu, S. S. (2002). What do students really understand? *The Journal of Mathematical Behavior*, *21*(2), 151-165.

Dominguez, A. (2010). Single Solution, Multiple Perspectives. In Lesh, R., Galbraith, P. L., Haines, C. R., & Hurford, A. (Eds.), *Modeling Students' Mathematical Modeling Competencies: ICTMA 13* (pp. 223-233). New York: Springer.

Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517-545). Mahwah, NJ: Laurence Erlbaum Associates.

LaRue, R., & Infante, N. E. (2015). Optimization in first semester calculus: a look at a classic problem. *International Journal of Mathematical Education in Science and Technology*, *46*(7), 1021-1031.

Mkhatshwa, T. (2019). Students' quantitative reasoning about an absolute extrema optimization problem in a profit maximization context. *International Journal of Mathematical Education in Science and Technology, 50*(8), 1105-1127.

Swanagan, B. S. (2012). *The impact of students' understanding of derivatives on their performance while solving optimization problems* (Doctoral dissertation). University of Georgia, Georgia.

Thompson, P. W. (1993). Quantitative reasoning, complexity, and additive structures. *Educational Studies in Mathematics, 25*(3), 165–208.

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education.* WISDOMe Mongraphs (Vol. 1, pp. 33- 57). Laramie, WY: University of Wyoming.