Novice Statistics Students' Forms of Reasoning When Reasoning About and With Sampling Distributions

Claire Miller
University of Georgia

Keywords: statistics education, sampling distribution, deductive reasoning, inductive reasoning, abductive reasoning

Understanding ideas of sampling, sampling distributions, and statistical inference are important for improving students' statistical literacy and productive citizenship, particularly given calls for students to become "critical consumers of statistically-based results reported in populator media" (GAISE College Report ASA Revision Committee, 2016, p. 8). Despite the importance of understanding statistical inference and sampling distributions, research suggests that students struggle with these ideas (Sotos et al., 2007). One reason for this difficulty stems from the complex and abstract concept of sampling distribution, which requires the coordination of multiple statistical ideas, including distribution, the relationship between sample and population, sampling variability, randomness, and probability (Chance et al., 2004; Saldanha & Thompson, 2002; Noll & Shaughnessy, 2012). I hypothesize that another reason students experience difficulty understanding sampling distributions and statistical inference is because these ideas require students to reason with uncertainty, a kind of reasoning that differs greatly from the forms of reasoning that are expected and highlighted in mathematics, such as deductive and inductive reasoning.

Deduction, induction, and abduction are three classic forms of reasoning that can be modeled with a triadic structure involving a case, rule, and result. (Peirce, 1878; Reid & Knipping, 2010). A case is a specific observation that a condition holds. A condition describes an attribute of something, or a relation between things. A rule is a general proposition that states that if one condition occurs then another one will also occur. A result is a specific observation, similar to a case, but referring to a condition that depends on another one linked to it by a rule. The order in which one links a case, rule, and result determines the kind of inference—deduction, induction, or abduction—needed to gain more knowledge about the situation.

Using this triadic structure, I examined the forms of reasoning—deductive, inductive, and abductive—that novice statistics employed when reasoning about and with sampling distributions. The data come from two clinical interviews, each 60-75 minutes in length, with undergraduate students who recently completed an introductory statistics course. Participants were asked to work through a series of statistical tasks related to sampling distributions. I identified reasoning excerpts—instances in which participants provided justification or an explanation for a claim—from the interview transcripts. Within each reasoning excerpt, I identified case, rule, and result, then examined how the participant linked them, which provided evidence for the form of reasoning they employed.

Preliminary results indicate that participants used all three forms of reasoning when reasoning about and with sampling distributions, though not all three forms were productive ways of reasoning. However, abductive reasoning was powerful for some participants when making inferences from sample data to an unknown population of interest. In this poster presentation, I will provide examples of reasoning excerpts from a small subset of the participants and discuss possible implications for this work.

References

Chance, B., delMas, R., & Garfield, J. B. (2004). Reasoning about sampling distributions In D. Ben-Zvi & J. B. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 295-323). Springer Dordrecht. https://doi.org/10.1007/1-4020-2278-6_13

GAISE College Report ASA Revision Committee. (2016). *Guidelines for assessment and instruction in statistics education (GAISE) college report 2016*. American Statistical Association. https://www.amstat.org/docs/default-source/amstat-documents/gaisecollege_full.pdf

Noll, J., & Shaughnessy, M. (2012). Aspects of students' reasoning about variation in empirical sampling distributions. *Journal for Research in Mathematics Education*, *43*(5), 509-556. https://doi.org/10.5951/jresematheduc.43.5.0509

Peirce, C. S. (1878). Illustrations of the Logic of Science VI: Deduction, Induction, and Hypothesis. *The Popular Science Monthly*, *13*(August), 470-482.

Reid, D. A., & Knipping, C. (2010). *Proof in mathematics education: Research, learning and teaching*. Sense.

Saldanha, L. A., & Thompson, P. W. (2002). Conceptions of sample and their relationship to statistical inference. *Educational Studies in Mathematics*, *51*(3), 257-270. https://doi.org/10.1023/A:1023692604014

Sotos, A. E. C., Vanhoof, S., Van den Noortgate, W., & Onghena, P. (2007). Students' misconceptions of statistical inference: A review of the empirical evidence from research on statistics education. *Educational Research Review*, *2*(2), 98-113. https://doi.org/10.1016/j.edurev.2007.04.001

Claire Miller
UNIVERSITY OF GEORGIA

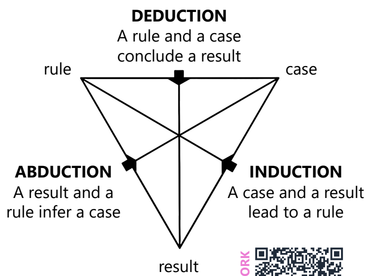
PAPER

Abductive reasoning is powerful when making inferences from sample data.

Background

Data are everywhere. Being able to make sense of data collected from samples is important for improving students' statistical literacy and their ability to critically evaluate data-based claims.

Forms of Reasoning



FRAMEWORK

Methods

PARTICIPANTS
Undergraduate students, with varying majors, who recently completed an introductory statistics course

CLINICAL INTERVIEWS
Two task-based clinical interviews designed to examine participants' existing ways of reasoning about sampling distributions

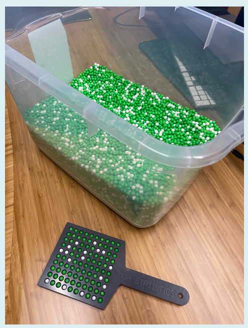
PEIRCE'S (1878) FORMS OF INFERENTIAL REASONING
Identified case, rule, and result in each reasoning excerpt and examined how the participant linked them to infer the form of reasoning (deductive, inductive, or abductive) they employed

Results

Inductive reasoning is useful when estimating sampling variability, and abductive reasoning is powerful when estimating an unknown population parameter.

Department of Mathematics, Science, and Social Studies Education
Mary Frances Early College of Education
UNIVERSITY OF GEORGIA

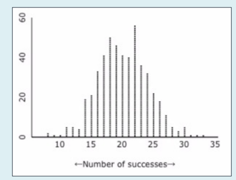
Lyla **induced** a range for a single sample outcome from observing a pattern in multiple sample outcomes.

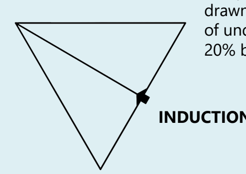


"I would say between 10 and 30. And really just from the samples we've taken so far, we've taken a lot, we haven't seen many go above 30. I don't know if we've seen one. I've seen we've reached 30, but I don't think we've gone above 30 and we haven't really gone below 10 besides these two in 500 [samples]."

I expect that any one single random sample of 100 undergraduates drawn from this population will produce between 10 and 30 business majors

Many random samples drawn from a population of undergraduates with 20% business majors





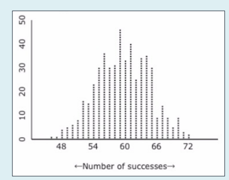
Very few samples produced a number of business majors that was less than 10 or greater than 30

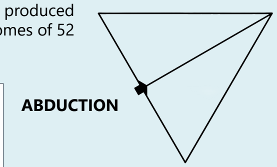
Lorraine **abduced** that her sample could have come from a population with a parameter of 60%.

"It looks like [my sample outcome of] 52 would be reasonable. There's a pretty significant number of results that are 52. And then past that is where [the distribution] tends to kind of taper out. So maybe if the true proportion were 60, the lower bound would be like 48 or so."

A population with 60% business majors produced several sample outcomes of 52

The sample I drew could have come from a population with 60% business majors





A random sample drawn from a population with an unknown proportion of business majors produced a sample outcome of 52 business majors