Is There an Interrelation Between Graduate Students' Teaching of Undergraduate Courses and Their Own Learning of Advanced Mathematics?

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Keywords: mathematics graduate students, teaching, leaning, ROG, PCK

The academic role of mathematics graduate students is complex, as it entails being simultaneously a student, a teacher, and a researcher. The three roles are inseparable from the graduate school experience and shape graduate students' professional identity (Colbeck, 2008). Some studies (e.g., Lai et al., 2016; Speer & Hald, 2008; Schoenfeld, 2015) suggest a deeper consideration of the connection between teaching, research, and learning among graduate students. Furthermore, Beisiegel (2019) believes that some mathematics graduate students perceive teaching and research as contrasting and contradicting roles.

The purpose of this poster is to build and discuss a model of "Interrelation Between Teaching and Learning Mathematics of Graduate Students", based on Schoenfeld's (2015) "Resources, Orientations, and Goals" model along with Ball et al.'s (2008) "Pedagogical Content Knowledge" theoretical framework. As a teacher and researcher, being a student is an inseparable part of graduate students' professional identity (Colbeck, 2008); hence, constant learning is inevitably a part of graduate students' lives, resulting in the accumulation of mathematical knowledge. The framework of "Resources, Orientation, Goals, and Decision-Making" (ROGs) was developed to predict and map one's professional behavior (Schoenfeld, 2015). Schoenfeld's framework made knowledge to be the resource in focus, which, in the presented model, graduate students rely on the most. According to Schoenfeld, "an individual enters into a particular context with a specific body of resources, goals, and orientations" (p. 347), they observe and orient the situation, establish or re-establish goals, and make decisions as a result of this internal analysis (Schoenfeld, 2015). In teaching undergraduate courses, graduate students' decisions and choices are impacted by their previously acquired knowledge. The choice of lecture style, problems solved, examples, and explanations given are based on graduate students' mathematical knowledge acquired through their prior learning. For example, upon completing Real Analysis, graduate students' teaching of limits and integrals may be enhanced by their theory encountered in the class. In addition to mathematical knowledge, teachers have a pedagogical content knowledge acquired through teaching (Ball et al., 2008). As teachers, graduate students have a developed intuition about problems and concepts that are challenging for students and are "able to hear and interpret students' emerging and incomplete thinking" (p. 401) (Ball et al., 2008). The knowledge of the origin of common mistakes and misconceptions among students allows teachers to reliably identify students' thought processes behind solving math problems. The awareness of multiple ways to approach solving a problem, the knowledge behind made choices, and recognition of different strategies applied in the solution are a product of pedagogical content knowledge (PCK). Such gained skill set is expected to help graduate students with answering challenging homework problems in their classes by revealing alternative approaches. In light of the proposed model, our research questions to guide this study are: Does the pedagogical content knowledge gained from teaching undergraduate courses affect graduate students' learning of advanced mathematics? Does the knowledge acquired while studying for graduate courses impact the graduate students' teaching?

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Interrelation Between Teaching and Learning Mathematics of Graduate Students

Anna Mikulo & Sepideh Stewart

INTRODUCTION

The academic role of mathematics graduate students is complex, as it entails being simultaneously a student, a teacher, and a researcher. The three roles are inseparable from the graduate school experience and shape graduate students' professional identity (Colbeck, 2008). A study by Beisiegel (2019) reveal mathematics graduate students' view of the relation between teaching and research, where studied participants perceived two roles being "totally different" (p. 494). This perception could lead to research and teaching requiring separately invested time and energy, leaving graduate students with no choice but under-commit to one of the roles. The purpose of this poster is to build and discuss a model of "Interrelation Between Teaching and Learning Mathematics of Graduate Students", based on Schoenfeld's (2015) ROGs model along with Ball et al.'s (2008) PCK theoretical framework.

RESEARCH QUESTIONS

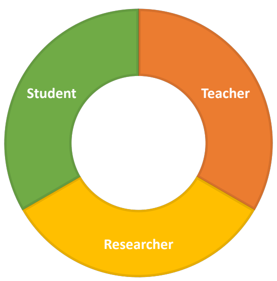
- Does the pedagogical content knowledge gained from teaching undergraduate courses affect graduate students' learning of advanced mathematics?
- Does the knowledge acquired while studying for graduate courses impact the graduate students' teaching?

DESCRIPTION OF THE MODEL

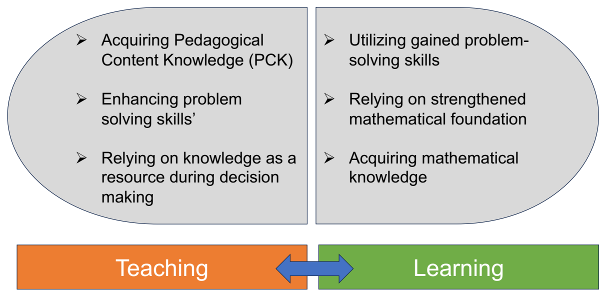
This model relies on two theoretical frameworks: PCK (Ball et al., 2008) and ROGs (Schoenfeld, 2015). During teaching, graduate students acquire pedagogical content knowledge, which enhances their understanding of the material taught and introduces them to different solution approaches undergraduate students take. With wider exposure to problem-solving techniques, we conjecture that producing proofs may become clearer for graduate students. On the other hand, while learning advanced mathematical content, graduate students expand their knowledge, which is the primary resource of their reliance. With knowledge as a resource, according to ROGs, graduate students' decisions in teaching change correspondingly to the resource. Hence, graduate courses students take influence their choices of material presentation, solution approaches, and variety of examples provided.

The model aims to track and establish the existence of a relation between teaching and learning, which would help to understand graduate students' experience as teachers and students and help them witness the influence of this interrelation.

PROFESSIONAL IDENTITY



THE MODEL: INTERRELATION BETWEEN TEACHING AND LEARNING MATHEMATICS OF GRADUATE STUDENTS



THEORETICAL BACKGROUND

The framework of ROGs was developed to predict and map one's professional behavior (Schoenfeld, 2015). Schoenfeld's framework made knowledge to be the resource in focus, which, in the presented model, graduate students rely on the most. According to Schoenfeld, "an individual enters into a particular context" (p. 347), they observe the situation, establish goals, and make decisions as a result of this internal analysis (Schoenfeld, 2015). In teaching undergraduate courses, graduate students' decisions and choices are impacted by their previously acquired knowledge.

As teachers, graduate students have a developed intuition about problems and concepts that are challenging for students and are "able to hear and interpret students' emerging and incomplete thinking" (p. 401) (Ball et al., 2008). The awareness of multiple ways to approach solving a problem, the knowledge behind made choices, and recognition of different strategies applied in the solution are a product of PCK. Such gained skill set is expected to help graduate students with answering challenging homework problems in their classes by revealing alternative approaches.

Graduate students observe norms of social interactions in their professional environment, interpret them based on their personal experience, and adapt such norms according to how they see themselves fit in that professional environment. Adapting Colbeck's definition (2008), the identity is "'what it means to be who one is'" (p. 10) and "individuals' identities are often associated with labels for social positions or roles" (p. 10) (Colbeck, 2008). So, having a professional identity as a *graduate student* entails having teacher, student, researcher, mathematician, tutor, grader, and TA as part of their professional identities.

FUTURE STUDY

To test the developing model, the future research may be a case study using a qualitative research approach and it may be carried out in two phases. The fist phase will investigate the relation between teaching and graduate students' learning where the participants will be University of Oklahoma mathematics graduate students two have taken Real Analysis sequence, have no prior experience teaching Calculus, and are assigned to teach Calculus I or Calculus II. Before the semester of teaching, graduate students will be asked to answer a list of Real Analysis questions that relate to Calculus topics other advanced Real Analysis questions. At the end of the semester, graduate students will be asked a set of similar questions to gage any enhancement of understanding of Real Analysis concepts.

REAL ANALYSIS TEST

1. Find an increasing function f such that $f' = 0$ a.e. but f is not constant on any open interval (p. 129) (Bass, 2013).
2. If f is real-valued and differentiable at each point of $[0,1]$, is f necessarily absolutely continuous on $[0,1]$? If not, find a counterexample (p. 129) (Bass, 2013).
3. Prove or disprove: If $F \subseteq \mathbb{R}$ is such that every bounded continuous function from F to $\mathbb{R}$ can be extended to a continuous function from $\mathbb{R}$ to $\mathbb{R}$, then F is a closed subset of $\mathbb{R}$ (p. 72) (Axler, 2020).
4. Prove or disprove: If $A \subseteq \mathbb{R}$ a d $f_1, f_2, ...$ is a sequence of uniformly continuous functions from A to $\mathbb{R}$ that converges uniformly to a function $f : A \to \mathbb{R}$, then f is uniformly continuous on A. (p. 71) (Axler, 2020).
5. Suppose $f, g : [a,b] \to \mathbb{R}$ are Riemann integrable. Prove that $f + g$ is Riemann integrable on $[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$ (p. 7) (Axler, 2020).
6. Suppose $f : [a,b] \to \mathbb{R}$ is a bounded function. Prove that f is a Riemann integrable if and only if for each $\epsilon > 0$, there exists a partition P on $[a,b]$ such that $U(f, P, [a,b]) - \mathcal{L}(f, P, [a,b]) < \epsilon$ (p. 7) (Axler, 2020).
7. Suppose μ is a signed measure on $(\mathcal{X}, \mathcal{A})$. Prove that if $A \in \mathcal{A}$, then $\mu^+ = sup\{\mu(B) \mid B \in B, B \subset A\}$ and $\mu^- = -inf\{\mu(B) \mid B \in B, B \subset A\}$. (p. 98) (Bass, 2013).

The second phase will consist of exploring ways studying advanced mathematics may influence teaching undergraduate courses. The participants will be mathematics graduate students who have not taken Real Analysis sequence and they will be interviewed before and after the sequence. The interview questions will target graduate students' approach to teaching Calculus I or II related topics. The Interviews will be conducted and after before taking Real Analysis sequence.

INTERVIEW QUESTIONS RELATED TO TEACHING

1. Describe your teaching approach of derivatives.
2. Give three examples of continuous functions and justify your choice.
3. Please explain the relation between continuity and differentiability.

REFERENCES AND CONTACT