Resource Use and Student Engagement in Multivariable Calculus

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This article examines the relations between content, resource use, and the pedagogy of instructors in multivariable calculus. Content included topics that typically appear in multivariable calculus which were grouped into these general areas: introductory ideas, differential calculus, integral calculus, and vector calculus. The resources used included digital resources, 3D-printed models, and other physical resources. Pedagogy practices included both collaborative and individual in-class learning activities, instructor demonstrations, and homework. We also consider the instructor's assessment of the activities they use. Data were obtained from three instructors working at three different types of institutions in the United States: a community college, a small selective undergraduate college, and a large comprehensive research university. The results provide information about instructional needs, as instructors implement resources in multivariable calculus, and about patterns related to the content they cover, the technology they use, and their pedagogical practices.

Keywords: Multivariable calculus, digital resources, TPCK, 3D-printed models

Introduction

Learning multivariable calculus (MVC) is a challenge for students. After operating primarily in two-dimensional worlds to study algebra, pre-calculus, and single-variable functions in calculus, students move to MVC, which typically involves two-variable functions. This introduces a new situation for students, as they must now generalize from the two-dimensional context of their previous studies to the MVC context. Doing so requires students to model and visualize the mathematical objects and their relations in three-dimensional (3D) space. As an instructional aid in this regard, instructors can leverage resources to help students bridge the gap from two to three dimensions, providing learning opportunities that are closer (Parzysz, 1988) to MVC topics than drawing on a chalkboard. To clarify, by resources we mean digital technology, digitally generated technology (e.g., 3D-printed models), or other physical artifacts.

However, the use of resources is not ubiquitous in MVC instruction. This leads to questioning how to best support instructors' use of resources for teaching MVC. In order to provide this support, it is imperative to first try to understand instructors' choices of resources and how resource selection may depend on the topic at hand or a particular pedagogy. One way to initiate this quest is to investigate the relationship between MVC instructors' use of resources for topics in the MVC course and their pedagogical intentions when leveraging resources to engage students in certain types of activities. For this case study, we consider data obtained from three MVC instructors who reported using the digital resource CalcPlot3D, 3D-printed models, as well as other physical resources in their MVC instruction.

To understand the relation between content, resource use, pedagogical practices, and instructor self-assessment of activity quality, we use the following research questions.

- Which topic areas of MVC seem more amenable to resource adoption? What types of resources are adopted?
- Which topic areas of MVC allow for student-centered activities?
- Which topic areas of MVC have activities that instructors assess as being of good or excellent quality?
- Are any patterns noted above that are specific to certain instructors in the sample?

Literature Review

The survey study by Martínez-Planell and Trigueros (2021) shows there is an increasing number of studies dealing with the teaching and learning of multivariable calculus (MVC). However, there is not much research related to the use of digital resources in MVC (Trigueros et al., 2023). There are some studies that report improved learning with the use of physical manipulatives, including tangible surfaces and 3D-printed models (e.g., McGee et al., 2012, 2015; Paul, 2018; Sherer et al., 2013; Wangberg, 2020; Wangberg et al., 2013). A few articles consider the use of digital technology like GeoGebra, Maple, and Mathematica as an aid in visualization, to foment student discussion, and to help bridge the gap between single variable and multivariable calculus (Alves, 2012, Ingar 2014). Some of the studies using digital technologies focus on specific topics or areas of MVC, for example, Trigueros et al. (2023) for basic ideas of two-variable functions, Rojas et al. (2019) for directional derivative, Ingar and Silva (2019) for extrema of two-variable functions, Alves (2014) for using second-order Taylor polynomials for approximating two-variable functions near critical points and relating this to the second derivative test, Henriquez (2006) for multivariable integration, particularly for drawing complex regions and finding limits of integration, and VanDieren et al. (2020) for the cross product. There are also some articles discussing the effect of using digital resources for an entire MVC course (e.g., Carvalho & Pereira, 2011; Gemechu et al., 2018; Habre, 2001). Some more recent articles consider the use of virtual or augmented reality (Cheong, 2023, Jones et al., 2022; Kang et al., 2020; Karabina et al., 2023). Yet, overall, there is much to learn about how resources, especially digital resources, can support the teaching and learning of MVC. Further, all these studies primarily focus on student learning. There is only one published study we know about that considers the use of resources for teaching MVC from the point of view of the instructors' needs (Martínez-Planell & Moore-Russo, 2023). This research is an extension of that previous study, where more data were obtained about the relation between content, pedagogy, and resources for teaching MVC.

Theoretical Framework

We use the Technological Pedagogical Content Knowledge (TPCK) model (Mishra & Koehler, 2006; Koehler & Mishra, 2009) as a theoretical framework for this study. This framework of instructor knowledge builds on the Pedagogical Content Knowledge model initially introduced by Shulman (1986) while also considering the role of technology in instructional decisions and actions. Koehler and Mishra (2009) situate the flexible knowledge needed for teaching as a "complex interaction among three bodies of knowledge: content, pedagogy, and technology" (p. 60). We adopt Mishra and Koehler's (2006) terms "pedagogy" and "content" to mean respectively the "processes and practices or methods of teaching and learning and how [they] encompass educational purposes, values, and aims (p. 1026)" and "the actual subject matter that is to be learned or taught (p. 1025)" respectively. We also adopt their TPCK stance on technologies as being both more traditional items (e.g., pencils and chalkboards) and what they refer to as newer technologies, including digital resources that include computers, tablets, and mobile devices as well as the digital applications they use.

Mishar and Koehler (2006) do not treat technology, pedagogy, and content as mutually exclusive domains. Instead, there is an interplay between the three that is noted in Figure 1 by the overlapping intersections in the TPCK Venn Diagram. In fact, certain resources are more likely to be employed for instructional tasks due to the affordances and capabilities of the resources (Koehler & Mishra, 2008), the fit between the characteristics of the resources and the characteristics of the task at hand (Goodhue & Thompson, 1995), and the instructor's perceptions of how useful and easy-to-use the resources are (Davis, 1989).

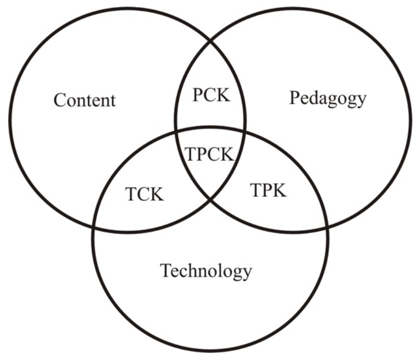

Figure 1. TPCK: Technological pedagogical content knowledge.

Methodology

We used a convenience sample of three MVC professors who employ the CalcPlot3D digital platform, digitally generated 3D-printed models, and other physical artifacts as aids in their MVC instruction. The professors taught at three different universities in three different states in the USA and had taught MVC anywhere from 12 to almost 30 times over their respective academic careers. Professor A teaches at a public community college with over 10,000 students in the Northeast. Professor H teaches at a small selective undergraduate college with about 4,000 students in the Midwest. Professor T teaches at a large public comprehensive research university with over 35,000 students in the East. The three were participating for a second year in a project involving CalcPlot3D and 3D-printed models. The data obtained came directly from the professors, and informal interviews were used to clarify their responses when needed.

The data were instructors' responses to a list of 35 common MVC topics, which was grouped into the following general *content areas* during analysis: introductory information, differential calculus, integral calculus, and vector calculus. The professors were given this list and asked to respond to questions about the types of resources they used in teaching each topic, describe the activities in which the resources were used, and rate the quality of each activity. An activity was the unit of analysis. The professors were asked to self-analyze their instruction using the following codes under each italicized coding category: *resource type* (digital resources, 3D-printed models, other), *pedagogical practice* (instructor demonstration, individual in-class student activity, collaborative in-class student activity, student homework), and *instructor rating* (i.e., evaluation of each activity as excellent, pretty good, needs work, no longer used).

Results

Table 1 displays the counts for several coding categories. However, we will primarily discuss the more qualitative aspects of the data in this section based on the research questions.

Table 1. Activity counts by pedagogical practice, instructor rating, and instructor for each content area and resource type.

Content Area	*Resource Type* Digital Resource	 3D-Printed Model	 Other Physical Resource
Introductory Information	**39 activities** *pedagogical practice* 5 ind act, 37 demos, 10 hwk* *instructor rating* 9 excellent, 30 good	**3 activities** *pedagogical practice* 3 collab *instructor rating* 2 excellent, 1 good	**11 activities** *pedagogical practice* 3 collab, 4 demos *instructor rating* 7 good, 4 not used
Differential Calculus	**19 activities** *pedagogical practice* 2 collab, 1 ind, 18 demos, 4 hwk *instructor rating* 9 excellent, 9 good, 1 needs work	**10 activities** *pedagogical practice* 10 collab *instructor rating* 7 excellent, 2 good, 1 needs work	**4 activities** *pedagogical practice* 1 collab, 3 hwk *instructor rating* 1 excellent, 3 good
Integral Calculus	**21 activities** *pedagogical practice* 21 demos, 1 hwk *instructor rating* 3 excellent, 14 good, 4 needs work	**6 activities** *pedagogical practice* 6 collab *instructor rating* 3 good, 3 not used	**0 activities**
Vector Calculus	**21 activities** *pedagogical practice* 21 demos, 4 hwk *instructor rating* 3 excellent, 15 good 3 needs work	**4 activities** *pedagogical practice* 3 collab, 1 demo *instructor rating* 2 excellent, 2 not used	**0 activities**

* Instructor H allows students to use CalcPlot3D for all homework, but she did not detail how this resource is leveraged. For this reason, we did not count her homework assignments.

We start with the first research question, dealing with TCK, by considering the activities as well as the *content areas* that were more amenable to resource adoption. There were 138 activities reported by the three instructors. Of that total, there are 53 activities dealing with introductory ideas (38%), 33 with differential calculus (24%), 27 with integral calculus (20%), and 25 with vector calculus (18%). Of the 138 activities, 100 (72%) use digital technology (CalcPlot3D), 23 (17%) use 3D-printed models, and 15 (11%) use other resources. This shows there was a clear preference among the participating instructors for using digital technology.

As for *resource type*, the 100 activities that used digital resources were spread across all four *content areas*. The most activities (39%) were employed for introductory ideas with a near even split for differential (19%), integral (21%), and vector (21%) calculus covering every individual topic area except limits and continuity and the chain rule.

Ten of the 23 activities (43%) used with 3D-printed models were for differential calculus. This suggests that instructors find differential calculus more natural or amenable to the design of activities based on 3D-printed models. There are two topics for which the three instructors used 3D-printed models (functions of several variables, partial derivatives), and five topics for which two of the three instructors used 3D-printed models (maxima and minima, Lagrange multipliers, triple integrals in rectangular, cylindrical, and spherical coordinates). There were six other topics for which a single instructor used 3D-printed models. This suggests that the use of 3D-printed models and their implementation should be further explored.

Eleven of the 15 activities were used with resources other than digital or 3D-printed addressed introductory information. The other resources include K'nex rods, a desktop coordinate system, large vector props made from wooden dowels, extendable vectors, a hill outside class, and static images on paper. For the most part, the use of these resources predated the adoption of digital and digitally based resources and attest to instructors' initial TPCK and motivations for engaging in a project to work on using digital and digitally generated resources.

For the second research question, we consider *pedagogical practices* to determine which class activities (by *content area*) allow for more student-centeredness (i.e. individual and collaborative in-class activities). Eleven of the 53 activities for introductory ideas (21%), 14 of the 33 differential calculus activities (42%), 6 of the 27 integral calculus activities (22%), and 3 of the 25 vector calculus activities (12%) were student-centered. All three instructors reported valuing student engagement, but their TPCK allowed them to design more student-centered activities for differential calculus than other areas, especially for vector calculus.

There were only two topics for which all three instructors designed in-class (individual or collaborative) student-centered activities (functions of several variables, partial derivatives) and five topics for which two of the three instructors designed student-centered activities (maxima and minima, Lagrange multipliers, and triple integrals in rectangular, cylindrical, and spherical coordinates). These topics suggest commonalities in instructors' PCK. At the same time, there were 11 topics for which a single instructor designed student-centered activities.

As for homework, which is also student-centered, but not completed in class, Instructor H shared that she allowed students to use CalcPlot3D on all homework, but she did not provide explicit details. There were 6 topics where the other two instructors expected students to leverage digital resources to complete their homework (equations of lines and planes; vector functions and space curves; functions of several variables; tangent plane and linear approximations; maxima and minima; Lagrange multipliers). There were also 13 other topics in which one of the instructors explicitly stated how he expected students to leverage digital resources. Note that none of the six common topics fall under the integral or vector calculus general topic areas.

We now consider the activities that involve instructor demonstrations, which are less student-centered than the other in-class activities just reported, and which also stem from instructors' TPCK. Demonstrations include 41 of the 53 activities for introductory ideas (77%), 18 of the 33 for differential calculus (55%), 21 of the 27 for integral calculus (78%), and 22 of the 25 for vector calculus (88%). So, the activities designed for introductory ideas, integral calculus, and particularly, vector calculus are mainly instructor demonstrations.

There were 13 topics where the three instructors used classroom demonstrations (vectors; equations of lines and planes; cylinders and quadric surfaces; vector functions and space curves; motion in space-velocity and acceleration; functions of several variables; partial derivatives; tangent planes and linear approximations; directional derivatives and the gradient vector; maxima and minima; Lagrange multipliers; vector fields; fundamental theorem of line integrals). There were also 14 other topics in which exactly two of the three instructors used classroom demonstrations. So, there is an ample collection of available demonstrations, as the most common pedagogical practice used with digital technology implementation, which might speak to instructors' TPK.

For the third research question, we consider the topics for which *instructor ratings* were that their activities were of good or excellent quality. Not many activities (only 20%) were self-rated as excellent. Of these, nine dealt with introductory topics (33%), 12 with differential calculus (44%), 3 with integral calculus (11%), and 3 with vector calculus (11%). So, we see that a higher proportion of differential calculus activities rated as "excellent", and there is a lack of highly rated activities in integral and vector calculus.

In examining instructors' PCK, there was only one topic for which all three instructors self-rated their activity as excellent (partial derivatives), and five topics for which two of the three instructors self-rated their activity as excellent (motion in space-velocity and acceleration; functions of several variables; directional derivatives and the gradient vector; maxima and minima; Lagrange multipliers). At the same time, there were 11 other topics for which an activity was self-rated as excellent by a single instructor.

We now move to the fourth research question which considers if any patterns were noted specific to certain instructors in the sample. First, *pedagogical practices*, as determined by student-centeredness (collaborative and individual in-class activities), vary not only by *content area* but also by instructor. For example, in introductory information, only one of the 27 activities of instructor H (4%) was student-centered, while in differential calculus, five of the 13 activities (38%) reported by the same instructor were student-centered. So, the *pedagogical practice* seems to depend on the *content area*. Similarly, in the areas of integral and vector calculus, instructor T designed six of his 13 activities (46%) to be student-centered (collaborative) activities, while none of the nine activities of Instructor A in these areas were student-centered. So, *pedagogical practices* also seem to depend on the instructor.

The choice of *resource type* also seems to be instructor dependent. For example, Instructor H used CalcPlot3D 23 times when discussing introductory ideas, while Instructor T used it only four times. Similarly, Instructor T, when teaching smaller classes, used 3D-printed models in 12 activities, while Instructor A used them for three activities.

Discussion and Summary

Martínez-Planell and Moore-Russo (2023) found that concern for students' learning and the conviction that geometrical understanding is crucial in multivariable calculus motivated their sample of instructors to consider a variety of educational resources. It was found that the instructors were sensitive to students' needs and that over time their teaching shifted from being instructor-centered (demonstrations presented in class) to being more student-centered (activities to engage students often in collaborative group work). The present study examines instructors' TPCK by relating the resources instructors use, the pedagogy employed, and the content at hand.

To answer the first research question, we observed that the area of introductory ideas is more amenable to resource adoption in the sense that it was more frequently the target of activities, mainly for demonstrations using digital technology. This might relate to instructors' TCK. This area of content is also more likely to leverage resources other than digital or 3D-printed for in-class activities. In the case of 3D-printed models, we found that it seems to be more natural for the instructors in our study to use them for differential calculus. For the second research question, mainly focusing on PCK, we observed that all areas allow for student-centered activities through the use of resources. However, there were more student-centered activities for differential calculus followed by the area of introductory ideas. The areas of integral calculus, and particularly vector calculus, are comparatively lacking in the design of student-centered activities. Activities for 3D-printed models, although fewer, tend to be more student-centered than other activities. As regards the third research question, we found that differential calculus has more activities assessed as excellent, followed by the area of introductory ideas. The areas of integral and vector calculus do not show comparable development. For the fourth research question, we found that content addressed, pedagogy used, and resources employed vary by instructor. The general patterns that we found are that digital technology is preferred to other resource use; the use of in-class demonstrations by the instructor vastly outnumbers the use of student-centered activities; there are fewer activities for 3D-printed models, although they tend to be more student-centered; and the area of vector calculus is vastly overlooked in comparison with the others. These patterns suggest that MVC instructors' TPCK is not uniform across content area nor across resource type.

There were six topics for which only one instructor used 3D-printed models, 11 topics for which only one instructor designed student-centered activities, and 11 topics for which an activity was self-rated as excellent by only one instructor. This suggests that sharing materials might help instructors incorporate 3D-printed models into other topics, make the MVC course more student-centered, and disseminate activities that have been rated as excellent. Consistent with Martínez-Planell and Moore-Russo (2023), who argued that instructors could benefit from support on how to teach MVC to engage students by leveraging instructional resources, we found that establishing a means to facilitate instructors sharing student-centered activities, activities they self-rate as excellent, and activities that employ 3D-printed models, could help others and improve MVC teaching. This study makes a case for establishing a means to do so.

In the areas of integral calculus and vector calculus there are fewer activities, very few of which were student-centered or rated as excellent. Correspondingly, most of the relatively few integral and vector calculus activities are instructor demonstrations. This perceived inadequacy or imbalance in these two areas of MVC in comparison with the areas of introductory ideas and differential calculus could be a result of several factors: a consequence of the content itself; a consequence of these areas (integral and vector calculus) being taught towards the end of the course, after instructors have learned to use resources for the introductory and differential calculus areas (so they have yet to get there); or a consequence of a lack of digital capabilities or other tools to properly treat these two areas. More research is needed to better understand this issue. Documenting that these two areas need more work in terms of activities, especially those that are student-centered and of high quality is a contribution of our study.

In the MVC course, there were relatively many activities to address introductory ideas. However, not many of them were student-centered or rated excellent; in fact, classroom demonstrations constituted a large percentage of these activities. While this suggests the possibility of re-examining existing teaching activities in order to increase student engagement with the content, there might be more of a function of time constraint issues exerting pressure on instructional choices more than pedagogical beliefs playing a role. Further study is needed here.

In terms of content, this study suggests that more activities that use resources to improve the teaching of integral and vector calculus are needed. In regard to pedagogy, there is a preference for instructor demonstrations and not enough highly rated student-centered activities. For resources, we observe that more work is needed to explore the pedagogical potential of 3D-printed models. Overall, this study contributes to a better understanding of resource use in MVC, while also contributing to a better understanding of MVC instructors' TPCK.

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