Digital Task Design to Support Students' Developing Graphing Meanings

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The ability to construct and interpret graphs is critical for students as learners across STEM fields (e.g., Paoletti et al., 2020; Potgieter et al., 2008). However, from algebra through calculus, there are well documented difficulties that students experience with graphs and graphing (e.g., Glazer, 2011; Thompson et al., 2017). A small but growing body of literature addresses ways to support students' development of graphing meanings by utilizing digital tasks (Ellis et al., 2015; Liang & Moore, 2021; Paoletti & Moore, 2017). We describe an emerging digital task design principle: That enabling interaction with theoretically salient aspects of a representation directs students' attention there. We use Paoletti et al.'s (2023) framework to determine which aspects of the representation are theoretically salient and to analyze students' activity.

Building on the research on students' covariational reasoning and graphing meanings (Carlson et al., 2002; Moore, 2021; Moore & Thompson, 2015; Thompson, 2011), Paoletti et al. (2023) argue that providing students with repeated opportunities to bridge their meanings for quantities in situations and the graphical representations of those quantities is productive for developing graphing meanings. This entails constructing quantities from a situation and representing the "amount-ness" (Stevens & Moore, 2017) of the quantity as a magnitude bar. A different part of the process entails conceiving of a coordinate point as a multiplicative object which simultaneously represents the length of magnitude bars along orthogonal axes.

In this poster, we address the question: *How does Paoletti et al.'s (2023) framework inform the iterative design of digital tasks intended to support students' developing graphing meanings?* We use data collected during a two-year design experiment (Cobb et al., 2003) consisting of multiple teaching experiments (Steffe & Thompson, 2000). We describe how Paoletti et al.'s (2023) framework supported iterative cycles of task design, fine-grained analysis of students' activity, and task revision. We draw on examples in which our task design resulted in students' activity that we did not intend. We describe 1) how we use the framework to determine where students' attention could be directed to better elicit the intended activity, and 2) how we applied this emerging digital task design principle to direct students' attention in those ways.

For example, we designed a task to support students' conceiving of a coordinate point as a multiplicative object. We first prompted students to drag dots along the vertical axis to represent increases in one quantity for equal increases in the other. Later in the task, we prompt them to do this again, but also show the related coordinate point in the plane. We intended for students to notice the coordinate points' movement as they manipulated the dots along the vertical axis, but their attention was not drawn to the coordinate point. When revising, we wanted students to attend to the relationship between the coordinate point and the length of the related magnitude bar along the vertical axis. We prompted them to accomplish a familiar goal (arrange dots along the vertical axis to represent a changing situational quantity) through a new type of interaction with the graphical representation (dragging a coordinate point in the plane). We provide implications for teachers and researchers as they adapt or design digital graphing tasks.

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References

Carlson, M. P., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education, 33*(5), 352–378. https://doi.org/10.2307/4149958

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, *32*(1), 9-13.

Ellis, A. B., Özgür, Z., Kulow, T., Williams, C. C., & Amidon, J. (2015). Quantifying exponential growth: Three conceptual shifts in coordinating multiplicative and additive growth. *The Journal of Mathematical Behavior, 39*, 135–155. https://doi.org/10.1016/j.jmathb.2015.06.004

Liang, B., & Moore, K. C. (2021). Figurative and operative partitioning activity: students' meanings for amounts of change in covarying quantities. *Mathematical Thinking and Learning*, 1–27. https://doi.org/10.1080/10986065.2020.1789930

Moore, K. C. (2021). Graphical shape thinking and transfer. In C. Hohensee & J. Lobato (Eds.), *Transfer of learning: Progressive perspectives for mathematics education and related fields* (pp. 145—172). Springer. https://doi.org/10.1007/978-3-030-65632-4_7

Moore, K. C., & Thompson, P. W. (2015). Shape thinking and students' graphing activity. In *Proceedings of the 18th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education* (pp. 782-789). RUME.

Paoletti, T., Gantt, A. L. & Corven, J. (2023) A local instructional theory for middle school students' emergent reasoning. *Journal for Research in Mathematics Education*, 54(*3*). https://doi.org/10.5951/jresematheduc-2021-0066

Paoletti, T., Lee, H. Y., Rahman, Z., Vishnubhotla, M., & Basu, D. (2020). Comparing graphical representations in mathematics, science, and engineering textbooks and practitioner journals. *International Journal of Mathematical Education in Science and Technology*, 1–20. https://doi.org/10.1080/0020739X.2020.1847336

Paoletti, T., & Moore, K. C. (2017). The parametric nature of two students' covariational reasoning. *The Journal of Mathematical Behavior, 48*, 137–151. https://doi.org/10.1016/j.jmathb.2017.08.003

Potgieter, M., Harding, A., & Engelbrecht, J. (2008). Transfer of algebraic and graphical thinking between mathematics and chemistry. *Journal of Research in Science Teaching 45*(2), 197–218. https://doi.org/10.1002/tea.20208

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. A. Lesh & A. E. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 267–307). Erlbaum.

Stevens, I. E., & Moore, K. C. (2017). The Intersection between Quantification and an All-Encompassing Meaning for a Graph. In E. Galindo & J. Newton, (Eds.), *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 709-716). Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In S. Chamberlin, L. L. Hatfield, & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education: Papers from a planning conference for WISDOM^e* (pp. 33–57).

EMERGING PRINCIPLES OF DIGITAL TASK DESIGN

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Study Goal: Develop an instructional sequence that supports 6th graders (11-12 years old) in developing emergent graphical shape thinking (Moore & Thompson, 2015).

Methodological Context: Design-based research study (Cobb et al., 2003) with multiple rounds of small-group teaching experiments (Steffe & Thompson, 2000).

Framework: Provide repeated opportunities to develop meanings for situations (MS), develop meanings for graphical representations (MR), and bridge those meanings (MS ⟷ MR).

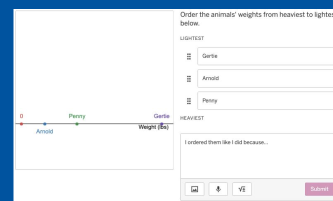
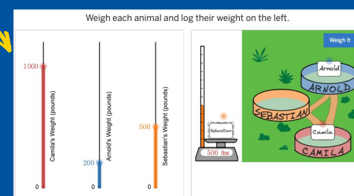
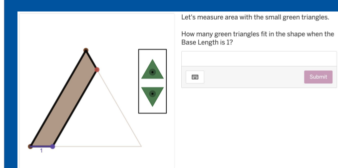
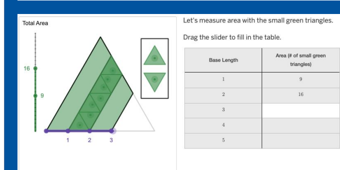
Partial LIT for EGST (Paoletti et al., 2023)

M.S. - Situational quantitative and covariational reasoning	**M.R. - Reasoning with graphical representations of covarying quantities**
M.S.1 - Construct quantities in a contextualized or decontextualized situation.	M.R.1 - Consider a varying segment length as representing a quantity's magnitude.
M.S.2 - Coordinate how two quantities change in relation to each other.	M.R.2 - Consider variations in two orthogonal segment lengths on axes in a coordinate system in relation to two covarying quantities.
M.S.3 - Develop an operative image of covariation that entails a multiplicative object.	M.R.3 - Conceive of or anticipate a point as a multiplicative object in the coordinate system simultaneously representing the two segments' magnitudes.

Direct students' attention to theoretically salient aspects of a representation by making it interactable.

Goal: MS1

New Interactions:
- Measure weight of mystery animals
- Construct magnitude bar representations by dragging

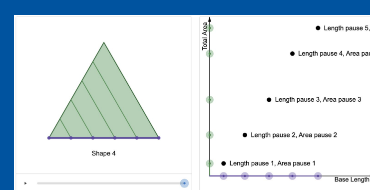
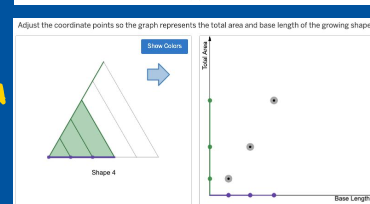





Goal: MS2

New Interactions:
- Drag Base Length
- Construct a table to coordinate values

Goal: MS3⟵⟶MR3

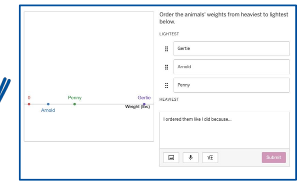
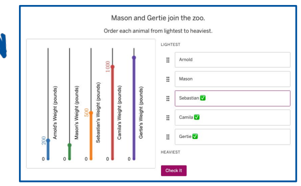
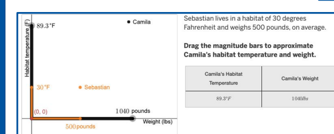
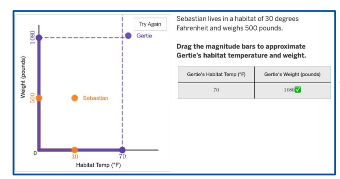
New Interaction:
- Drag coordinate point to arrange orthogonal magnitude bars

Use visual feedback to represent intended ways of thinking.

Visual Feedback: visual information provided automatically in response to student actions with potential for mathematical meaning (Margolis & Boyce, in press).

Feedback Design: Click "Check It." Magnitude bars reorder to reflect the student's selection.

Feedback Design: Drag mag. bars to estimate amts. Press "Check It" and see dotted lines extend toward point.

Feedback Design: Drag coordinate point to create the graph via dynamic trace. See mag. bars on the axes follow the motion of the point.