Representing Learning in Advanced Mathematics Courses for Secondary Mathematics Teachers

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It is essential to provide opportunities for prospective secondary mathematics teachers to connect advanced mathematics content to secondary mathematics teaching practice, if advanced mathematics courses are to be useful to these teachers. We argue that for teachers to frame teaching and learning in equitable ways, connections to teaching must represent learning in anti-deficit ways. We review a selection of curricular materials satisfying two criteria: first, they are written for use in advanced mathematics courses that prospective teachers may take; and second, they feature explicit connections to secondary teaching practice. We find that representations of learning in curricular materials tend to take binary views of mathematical legitimacy, and frame students as either being correct or incorrect. We conclude with implications for mathematics teacher educators in RUME.

Keywords: content knowledge for teaching, secondary mathematics teacher education, advanced mathematics courses

Many prospective secondary mathematics teachers (PSMTs) complete multiple courses in advanced mathematics (e.g., Ferrini-Mundy & Findell, 2004; Tatto & Bankov, 2018). As recent empirical work demonstrates, it is essential to provide opportunities for PSMTs to connect the content of these courses to the *practice of teaching*, if these courses are to be useful to prospective teachers (Álvarez et al., 2020a; Lai et al., 2023; Wasserman & McGuffey, 2021). Multiple scholars have followed this approach in creating curricular materials for secondary mathematics teacher education (e.g., Hauk et al., 2018; Heid et al., 2015; Lischka et al., 2020; Mathematical Education of Teachers as an Application of Undergraduate Mathematics [META Math], 2020; Wasserman et al., 2022). In this approach, scholars represent teaching through descriptions of student thinking and teaching scenarios. This approach to secondary mathematics teacher education raises the question of how secondary students' learning is represented in curricular materials. This is a critical issue, as instruction in teacher education, including in mathematics classes, is part of the socialization that shapes teachers' images of teaching practice (e.g., Lai et al., 2023).

Our purpose is to review how secondary students' learning is portrayed in curricular materials for advanced mathematics courses that enroll PSMTs. To do so, we reviewed a selection of curricular materials satisfying the following criteria: they were intended for use in advanced mathematics courses that PSMTs may take; and they featured explicit connections to secondary teaching practice. Our review was guided by the research questions: *(1) What approaches have been taken by curriculum writers to connect advanced mathematics* *coursework to secondary teaching practice? (2) How do these connections portray students' learning of mathematics?* We suggest that approaches taken are consistent with Grossman et al.'s (2009) pedagogies of practice. We then analyzed portrayal of students' learning of mathematics in line with Louie's (2017) exclusive and inclusive frames of mathematics and mathematical ability.

Background

Advanced mathematics knowledge can potentially strengthen secondary mathematics knowledge (Conference Board of the Mathematical Sciences [CBMS], 2012; Murray et al., 2017). A typical approach has been to follow Felix Klein's (1924/1932) prescription of considering "elementary mathematics from an advanced standpoint" (p. 1). The hope here is that with a more sophisticated understanding of secondary concepts, teachers will take more productive actions when teaching.

Yet secondary teachers lament their mathematical preparation was irrelevant to secondary teaching (e.g., Goulding et al., 2003; Wasserman & Galarza, 2018; Zazkis & Leikin, 2010). Even when prospective teachers develop a richer understanding of the secondary mathematical concepts that they will teach, they may still exit advanced mathematics believing that they may have become better mathematicians, but not better mathematics teachers (Wasserman & Ham, 2013).

Mathematics teacher educators have historically connected advanced mathematics to secondary teaching along two dimensions: *mathematics content at the secondary level* and *mathematical practice*. The concept of a capstone course for teachers, which is offered by many institutions in the U.S. (Cox et al., 2013), exemplifies the first dimension. In their review of then-present-day capstone coursework for secondary teachers, Murray and Star (2013) found that connections took the form of generalizations or abstractions of secondary mathematical ideas (e.g., geometric transformations and group theory, factoring polynomials and Galois theory), or specific uses of advanced mathematics to define concepts that secondary students may encounter (e.g., limits and irrational numbers). As for mathematical practice, scholars such as Cuoco et al. (1996) called attention to the importance of mathematical habits of mind in teaching and learning mathematics. CBMS (2001) theorized that mathematics teachers must experience mathematical practices in their own learning, so that they can cultivate experiences of mathematical practices when teaching secondary mathematics. This argument is compatible with research on the perspective of teachers. Some scholars have found that when teachers found coursework relevant, it was due to understanding the nature of mathematics practice, and how learning can feel (Baldinger, 2018; Even, 2011; Hoffman & Even, 2019).

The potential of these approaches is unmet. Begle (1979) found that secondary students' performance was associated with neither the number of tertiary mathematics courses taken by teachers nor the average grade received by teachers in these courses. Goulding et al.'s (2003) and Zazkis and Leikin's (2010) surveys found that many teachers report their mathematical preparation is disconnected from teaching. In a study of an abstract algebra course, Ticknor (2012) found that even when secondary teachers wanted to do well in the course, and the instructor saw connections between abstract algebra and secondary mathematics, the teachers still perceived the course as irrelevant to teaching. Wasserman et al. (2018) articulated concrete reasons, given by prospective and in-service teachers, for why their knowledge of real analysis did not inform their teaching, even when they understood the material.

There is a promise of advanced mathematics to shape teachers' practice and an observed inefficacy of advanced mathematics courses to do so. In response, secondary mathematics teacher educators have pushed for explicit connections from advanced mathematics to secondary teaching practice and enacted such connections in written curriculum materials (Álvarez et al., 2020a; Hauk et al., 2018; Heid et al., 2015; Lai, 2019; Lischka et al., 2020; Wasserman et al., 2022). Many of these authors are in the RUME community; indeed members of the above cited teams behind curricula and research have all presented at RUME in the past 5 years.

Another factor in the ecology of secondary mathematics teacher education, particularly when it comes to courses taught by mathematics faculty for PSMTs, is a shift in the teacher education community's focus. For a long time, the sentiment that 'you can't teach math and have students learn without knowing the math yourself' drove funding, research, and course design; now, there is the urgent problem of increasing equitable access to mathematics. A corollary of this observation is that when assessing connections of advanced mathematics to secondary teaching practice, one must not only assess the mathematics, but also the images of equity and inclusiveness in teaching in connections.

To our knowledge, there has been no systematic review of materials taking the approach of connecting advanced mathematics to teaching, including the ways that these curricular materials represent student learning and teaching. In view of the posited connection to teaching practice, and the abundance of materials aligned with this approach, now is the right moment to analyze how exactly teaching and learning are portrayed. The results of this analysis, particularly in terms of equitable and inclusive educational principles, is essential to future work by the RUME community in improving teacher education.

Conceptual Perspective

Mathematics *teaching practice* at any level entails relational work with students, routines for disciplinary discourse, and norms for establishing the legitimacy of solutions. In this sense, the term *practice* is about what practitioners do rather than think or know (Lampert, 2010). Moving up a level from teaching to the teaching of teaching, Grossman et al. (2009) proposed the concept of *pedagogies of (teaching) practice*. By this term, Grossman and colleagues refer to the intentional design of experiences for future teachers that develop their future teaching practice.

Representations and approximations of practice

To articulate potential pedagogies of practice, Grossman et al. (2009) analyzed the education of future teachers, pastors, and therapists. These professions were chosen as comparisons due to their commonality of requiring relational work in combination with the expectation of particular routines and harnessing technical knowledge. Through this work, Grossman and colleagues identified pedagogies of practice that spanned the fields of teacher education, liturgical education, and clinical education. These pedagogies of practice included representations and approximations of practice. In the descriptions below, we follow Grossman et al. (2009).

Representations of practice allow novices to observe aspects of practice. When teachers engage with representations, they can develop ways of noticing and understanding teaching practice. Representations can vary in comprehensiveness and authenticity, and can take a variety of forms, including short narratives of teaching scenarios, videos, animations, and case studies. We consider curriculum materials (for advanced mathematics courses for PSMTs) to feature a representation of practice if they feature a description of a teaching scenario that for instance engages PSMTs in evaluating, describing, or reflecting upon features of the presented scenario.

Approximations of practice allow novices to simulate practice so they can attend to particular aspects of practice, rather than all aspects of the complex, relational practice that is teaching. Approximations may vary in authenticity and complexity. We consider curriculum materials to feature an approximation of practice if teachers are asked to simulate responding to students' mathematical work, evaluate students' work, explain content, or teach a full lesson. Note that approximations of practice may embed representations of practice, as the prompt for an approximation of practice may describe a teaching scenario.

Exclusionary and inclusive frames for the nature of mathematics and mathematical ability

We posit that inherent in any representation of practice of mathematics teaching is an at least implicit frame for mathematics and mathematical ability. Louie (2017) conceptualized exclusionary and inclusive frames for the nature of mathematical activity and the nature of mathematical ability. Within the exclusionary frame are the "rote practice frame", where "mathematics is a fixed body of knowledge" involving closed questions and a focus on answers; and the "hierarchical ability frame", where speed and correctness are valorized, and some students are positioned as helpers and others as in need of help. Within the inclusive frame are the "sense-making frame", where "mathematics is about making sense of ideas and understanding connections" and "the multidimensional math frame"; and the "multidimensional ability frame", where a variety of students are named as resources for peers' learning and skills outside of speed and correctness are valorized (p. 496).

Data & Method

We sought to review curricular materials satisfying two criteria: (1) they were intended for use in advanced mathematics courses that secondary teachers may take, and (2) they sought to make explicit connections to secondary mathematics teaching practice. We also restricted this review to materials developed in the US. In searching for curricular materials, we looked for both the materials themselves as well as reports of their enactment. We considered textbooks commonly used in capstone courses (as reviewed by Cox et al., 2013), those published by professional organizations of mathematicians, and the literature reporting the use of these materials and textbooks in advanced mathematics courses. As well, we conducted a Fastlane search for materials created with the support of the U.S. National Science Foundation.

The authors of this manuscript are developers of curricular materials, namely the MODULE(S^2) materials (Lai, Strayer, Casey, Lischka) and ULTRA materials (Wasserman, Weber, Fukawa-Connelly). We contacted all curriculum writers and investigators that came up in our search (other than ourselves) for copies of materials. Not all authors and investigators responded to requests. Table 1 (on the next page) displays materials and reports we obtained. Note that in this table, "algebra" refers to the study of number systems, functions, or relations, whereas "abstract algebra" refers to the study of mathematical groups, rings, fields, or related constructs.

We analyzed descriptions of enactments and the text of curricular materials in three stages. First, we identified instances of presenting secondary teaching context to PSMTs. By context we mean the contextual elements of teaching practice such as hypothetical or actual secondary student talk or pieces of hypothetical or actual secondary curriculum materials (Lai & Jacobson, 2018). Second, we coded these instances for whether they satisfied conceptualizations of representation of practice and approximation of practice (consistent with the descriptions given in the section on Conceptual Perspective) and then inductively coded categories of representations of practice and approximations of practice. Third, we analyzed these instances holistically for how their prompts and descriptions of students and teachers in the teaching context aligned or not with various aspects of Louie's (2017) framework. For instance, if student work was described as "correct" or "incorrect", this is indicative of an exclusionary frame for mathematical activity. Or, if teachers were asked to name strengths of student work beyond correctness, this would align with an inclusive frame for mathematical ability.

Table 1. Reviewed materials and reports.

Citations reviewed, grouped by project	Topics addressed
1. Álvarez et al. (2021)	Algebra
2. Bremigan, Bremigan & Lorch (2011)	Algebra
3. Buchbinder & McCrone (2018, 2020); Buchbinder (2018)	Number Theory, Geometry, Use of conditionals
4. Hauk, Hsu, & Speer (2017, 2018)	Algebra, Abstract Algebra, Geometry
5. Heid, Wilson, & Blume (2015)	Algebra, Abstract Algebra, Geometry, Statistics, Proof by induction
6. Lai & Hart (2021); Hart & Lai (2021), Aubrey et al. (2021); Casey et al. (2021a, 2021b, 2021c); Alibegović & Lischka (2021a, 2021b, 2021c); Anhalt et al. (2021a, 2021b, 2021c)	Abstract Algebra, Algebra, Geometry, Mathematical Modeling, Statistics
7. MAA META Math (2020a, 2020b, 2020c, 2020d, 2020e, 2020f, 2020g, 2020h, 2020i)	Abstract Algebra, Calculus, Discrete Mathematics, Proof, Statistics
8. Sultan & Artzt (2011); Artzt et al. (2011)	Algebra, Geometry, Statistics
9. Usiskin et al. (2003); implementation of using this textbook in Winsor (2009)	Algebra, Geometry, Abstract algebra
10. Wasserman & McGuffey (2021); Weber et al. (2020); Fukawa-Connelly et al. (2020), Wasserman et al. (2022); McGuffey et al. (2019); Wasserman et al. (2019)	Real analysis

Results

In all 10 projects, we found evidence of representations of practice and approximations of practice. Within representations of practice, we found 5 aspects of teaching context constituting the representation: student contributions (that is, depictions of student talk, work, or thinking, where teachers are not explicitly described); teacher-student interactions; written curriculum and assessments (such as actual or hypothetical textbook or lesson plans); personal experience with secondary-level tasks (where teachers are asked to notice features of their own experiences solving a secondary-level task); and an educator's modeling of a teaching practice (where the experience of the task becomes an object to notice). Within approximations of practice, we found 4 teaching practices that PSMTs were asked to simulate: responding to students' mathematical contributions; evaluating students' work; explaining content; and teaching a full lesson (where PSMTs were either teaching a full lesson of the advanced mathematics course in which teachers were enrolled [Artzt et al., 2011; Winsor, 2009], or teaching a lesson to secondary students [Buchbinder & McCrone, 2020]).

We now turn to frames within representations and approximations of practice. Evaluating student contributions was by far the most prevalent structure for engaging with representations of practice, and evaluating teacher contributions (such as appraising a described secondary teacher's teaching moves) was the second most common. A standard format for evaluating student contributions was showing teachers a sample of student work or dialogue, and then asking teachers to identify strengths and weaknesses of the mathematics shown (e.g., Bremigan et al., 2011; Casey et al., 2021a; Hauk et al., 2017, 2018; META Math 2020a; Sultan & Artzt, 2011; Wasserman et al., 2022). See Figure 1 for an example. Across the multiple secondary education projects reviewed, the majority of approximations featured the practices of responding to students' mathematical contributions, evaluating students' work, and explaining content (e.g., Álvarez et al., 2020b; Alibegović & Lischka, 2021b; Aubrey et al., 2021; Bremigan et al., 2011; Buchbinder & McCrone, 2020; Hauk et al., 2017; MAA META Math, 2020b; Sultan and Artzt, 2011; Wasserman et al., 2022). For instance, the Capstone Math materials provided teachers with a set of student work samples and asked teachers to work in groups to "sort the work into categories that represent different ways of thinking and/or difficulties" (Hauk et al., 2017, p. 9).

Two parts of [a student's] work on a task are shown.
(i) Name things [that this student] has done well when analyzing the data.
(ii) Name things [that this student] needs to work towards understanding about analyzing univariate quantitative data.

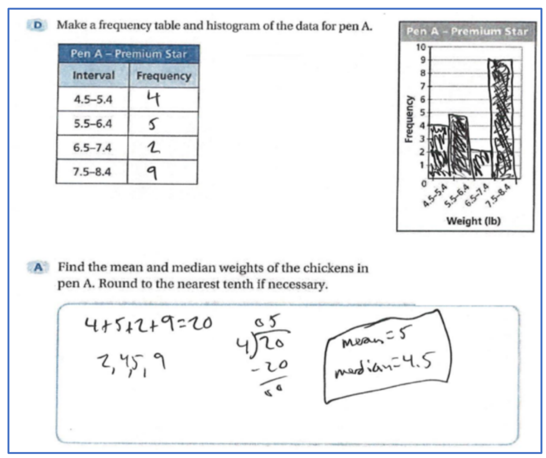

Figure 1. Example approximation of practice with embedded teaching context (Casey et al., 2021a, p. 175).

Across the reviewed materials, we saw a dominance of the inclusive sense-making frame, where mathematics is about making connections across ideas. However, within representations and approximations of practice, the secondary-level tasks tended to be closed rather than open. While this may a practical consideration for the curriculum developers, it also means that when PSMTs are asked to simulate teaching practice, they engage with an exclusionary frame. In multiple approximations of practice, an incorrect response was juxtaposed with a correct response. This juxtaposition may feed into an exclusionary hierarchical ability frame where some students are positioned as helpers and others as in need of help.

For brevity, we discuss only one example here in depth, that shown in Figure 1. The approximation of practice shown potentially supports an inclusive multidimensional frame of mathematical ability mathematics. In this example, PSMTs are asked to name strengths of student work. However, PSMTs may also gravitate toward correctness or incorrectness of the student work, which aligns with the exclusionary frame of mathematics as a fixed body of knowledge. On the other hand, within the materials reviewed, we saw multiple instances of representations and approximations which explicitly asked PSMTs to evaluate student work in terms of selecting which was the best work, or being presented with work that was incorrect and being asked to articulate what is incorrect (without asking PSMTs to articulate what may be worth building on). One set of materials contained prompts that asked PSMTs to rate solutions as "high, medium, or low". While assessing mathematical work for mathematical correctness and incorrectness is a skill that teachers need to have, only turning to this skill in engaging with student work leads to an exclusionary frame of mathematics and mathematical ability.

Conclusion

How secondary teaching is represented impacts the opportunities that PSMTs have to engage with inclusive frames of mathematics and mathematical ability. Representations and approximations may also reinforce exclusionary frames of mathematics and mathematics ability within PSMTs. In our review, we found examples of representations and approximations of practice that may foster an inclusive frame of mathematics. At the same time, these representations and approximations of practice did not explicitly support reframing away from exclusionary frames. Supporting PSMTs in inclusive framing of mathematics and mathematical ability is ongoing work that will require potential revision of existing materials (including our own) and broadening constructions of what counts as representations and approximations of secondary teaching practice in advanced mathematics courses that PSMTs experience.

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