In Transition from Undergraduate to Postgraduate Mathematics: A Case of Re-learning to Learn

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The transition from undergraduate to postgraduate studies has been underexplored in mathematics education research. Several studies showed that this transition is challenging as it requires countless changes in familiar practices. In this paper, we present the case of Jordan—an honors student who was developing mathematical background as a stepping stone toward his research project. With the commognitive framework, we analyzed the data from three interviews to reveal Jordan's mathematical learning network of routines. The findings detail the network's components and illuminate the role of agency in Jordan's network development. Agency appears as necessary to navigate learning routines common in undergraduate studies and enables Jordan to be less dependent on externally provided resources shaping his learning.

Keywords: mathematics learning, postgraduate studies, commognition, learning routines

Introduction and Background

The preparation of new researchers is vital for the flourishment of any discipline. Mathematics departments initiate students to research through a range of programs such as honors, master's and doctorates. Each of these postgraduate contexts is distinct, and their structures differ substantially from what undergraduate students are used to. Thus, a shift from undergraduate to postgraduate mathematics constitutes a critical transition, that is, one that involves "a noticeable change of point of view […and] a necessity for entering into a different type of discourse […] or more broadly […] changing 'lenses'" (Yerushalmi, 2005, p. 37). Yet, research into students' experiences of the undergraduate–postgraduate transition in mathematics is scarce.

Higher education research identified important obstacles in students' transition to postgraduate studies. For instance, O'Donnell and colleagues challenged the assumption that undergraduates are experts in disciplinary practices and elaborated on multiple changes postgraduate students must navigate simultaneously (O'Donnell et al., 2009; Tobbell & O'Donnell, 2013a; Tobbell & O'Donnell, 2013b). Adjusting to a higher level of independence has been reported to be particularly challenging for students (Tobbell et al., 2010). Lovitts (2005, 2008) detailed a range of factors underpinning the complexity of the shift from being expected to "[learn] what others know and how they know it" to "conducting original research and creating knowledge" (p. 140). Among other factors, the microenvironment composed of the department, advisors, and peers appeared to have a significant impact on the shift to independent research. Overall, the growth of independence constitutes a central theme in the literature on the transition to postgraduate studies. While the existing findings are relevant to many postgraduate programs, they may be difficult to appreciate without accounting for the discipline within which the transition occurs. Therefore, the transition from undergraduate to postgraduate is a topic of interest for mathematics education research.

A few mathematics education studies discussed the experience of postgraduate students (e.g., Herzig, 2002, 2004; Morton & Thornley, 2001) and even fewer focused on the undergraduate–postgraduate transition. Duffin and Simpson (2006) listed a few differences between undergraduate and postgraduate studies. Within the former, students' learning mainly occurs through courses, where lecturers determine syllabi, teach the material, set problems for students to solve, and suggest textbooks and references. The research component features in some undergraduate programs, but it is typically small and comes with comprehensive support (Dorff, Henrich & Pudwell, 2019). In turn, postgraduate students are expected to learn in an independent and self-directed manner. While research supervision is a typical component of many postgraduate programs, this role differs from that of a course lecturer. Accordingly, students' adaptation to the circumstances of postgraduate mathematics learning appears unavoidable.

An example of such adaptations comes from Geraniou (2010). She observed different stages in students' doctoral studies and explored factors contributing to students' success. Geraniou describes the first stage, which she calls *adjustment*, as "reading and gaining the background knowledge so as to be in a position to start their research" (p. 286). As this study was undertaken in the UK, coursework was not required, and students experienced learning without the structured environment of courses. Geraniou reports that in the absence of clear learning aims and imposed assessments, the students drew on their personal interest in the material and turned to their supervisors for motivation and guidance.

Some studies addressed the distinctions between how undergraduates and postgraduate students carry out specific learning-oriented practices. For instance, Shepherd and van de Sande (2014) explored students' and mathematicians' learning from mathematical texts. The researchers found that undergraduates limit their attention to the provided reference, relying only on the material presented inside the text for support. Postgraduate students and mathematicians both used the provided reference and external resources when seeking clarifications.

The existing literature indicates that the undergraduate–postgraduate transition requires new ways to learn mathematics. This study was initiated to enrich the existing literature by exploring how these ways of learning come about. Specifically, we aim *to characterize the processes of students' adjustment to the learning of mathematics at the postgraduate level.* We pursue this aim in the case of an honors student who was developing mathematical background outside of courses as a preparation for his research project.

Theoretical Framework

Research has shown the usefulness of the commognitive framework (Sfard, 2008) in exploring mathematics learning and teaching in the university context (e.g., Nardi et al., 2014; Karavi et al., 2022; Kontorovich & Ovadiya, 2022). In this study, we rely on the framework to cast light on students' learning of mathematics at the postgraduate level.

Commognition construes different mathematical areas as *discourses*, defined as "different types of communication, set apart by their objects, the kinds of mediators used, and the rules followed by participants, thus defining different communities of communicating actors" (Sfard, 2008, p. 93). Consistently, *learning* is associated with one "becoming a participant in certain distinct activities" (Sfard, 2008, p. 23). This may be evident in one's capability to operate with broadly accepted discourse-specific narratives, such as definitions, theorems, and proofs.

Postgraduate studies are expected to prepare students to contribute to a targeted mathematical discourse. This preparation imposes expectations on students, but students are "contributors to their life circumstances, not just products of them" (Bandera, 2006, p.1). In the context of postgraduate studies, students' contributions are captured through *agentive learning.* Brennan (2012) defines a learner's agency as an "ability to define and pursue learning" (p. 24). Knowles (1975) maintains that agentive learners take initiative, are sensitive to their learning needs, set relevant goals, determine helpful resources, and monitor their learning outcomes. Similarly, Lavie et al. (2019) specify that growth in agentivity, the commognitive version of agency, can be identified via the increasing number of self-determined decisions. Lavie et al. highlight the importance of one initiating activities without invitations from others and in the capability to set "the relevant [tasks] for herself, in response to her own needs" (p. 170). These activities can be captured in terms of one's *routines*.

Commognition defines routines as patterned courses of action. Initially, the construct referred to capture one's participation in a specific mathematical discourse (Sfard, 2008). Lavie et al. (2019) expanded this approach, arguing that "whatever we do involves routines. From the simplest, most mundane of our activities to the most abstract and sophisticated of them, we cope with the task by repeating something that we did or have seen done before" (p. 154). Sfard (2023) demonstrated that one's routines of teaching can be construed as "a tightly interconnected system" (p. 5) or a *network*, where one routine feeds into another or where several routines constitute part of a greater routine. Sfard argues that networks have a fractal-like structure due to the recursive nature of routines. In her words, "routines are built from parts that, in themselves, constitute routines" (Sfard, 2023, p. 5).

Following Sfard's (2023) steps, we propose that *mathematics learning*, that is an activity targeted as becoming an insider to a mathematical discourse, can be considered through the lens of routines. Indeed, students' progression through school and university mathematics can be seen as a journey across different discourses and joining these discourses requires the performance of various routines of learning. Thus, we propose that learning involves a repertoire of routines students employ to come to grips with new mathematics time and again. In undergraduate studies, this repertoire may include engaging with the course resources, working on the assigned exercises, and consulting with the lecturer during office hours. Identifying what routines students employ to learn mathematics at the postgraduate level is at the heart of our study.

Sfard (2008) distinguishes between the "how" and the "when" of a routine. The former refers to the *procedure* or the course of action, whereas the latter captures applicability conditions within which one *initiates* a routine and brings it to *closure*. Lavie et al. (2019) proposed that an analysis of one's routines should account for the *task*—an interpretation of a goal a routine performer sets for themselves in a specific *task situation*, that is, in circumstances in which they consider themselves bound to act.

Lavie et al. (2019) maintain that in task situations, people replicate actions they either performed or observed others performing in circumstances they deem sufficiently similar to the current one. Common features of situations, called *precedents,* enable one to cope with unfamiliar task situations. As no two task situations are identical, drawing on precedents requires one to decide which aspects of past performance should be replicated as is and which elements need to be amended to new circumstances. Accordingly, the implementation of familiar routines still requires some degree of agency. It is especially needed when one realizes that existing routines are insufficient to navigate the current task situation. Routines must then be substantially revised or abandoned in favor of new habits. Given the systemic differences between undergraduate and postgraduate contexts, it seems reasonable to expect students to develop new learning routines deliberately and agentively. We attend to the agentive component when analyzing postgraduate students' routines when learning new mathematics.

Method

This study is part of the first author's doctoral research on mathematics students' transition from undergraduate studies to postgraduate research. In this paper, we report on the case of Jordan (pseudonym), a mathematics major enrolled in a one-year honors program. The study unfolded in a small department of mathematics at a research-intensive university in New Zealand. Jordan's program consisted of six semester-long graduate courses and a research project that he carried out in parallel under the guidance of two supervisors. Jordan's project revolved around group theory and topology. Group theory was part of Jordan's undergraduate and graduate coursework, but topology was not. Nonetheless, Jordan had to develop the mathematical background needed for his research project in the first few months of his program.

Our data came from three interviews, around 50 minutes each. The interviews took place when Jordan was in the third, fifth and sixth months of his program. The first interview focused on Jordan's work with the supervisors and his general impression of research so far. Through his responses, Jordan elaborated on his self-guided learning of topology, and we used the opportunity to delve into this process. In the following interview, Jordan elaborated on his typical practices of learning new content, now including group theory, and demonstrated some of them in the third interview.

We engaged with the interview transcripts with the commognitive apparatus, searching for routines Jordan employed to join new mathematical discourses. Our data mainly consisted of Jordan's reflections, and thus, we operated with *routines-narratives*, i.e. self-descriptions of patterned actions. Different routines-narratives varied in the level of detail, but we attempted to delineate Jordan's procedures and tasks where possible. In the last stage of the analysis, we structured the identified routines into a network, with particular attention to how Jordan's postgraduate learning routines compare to typical actions students employ in undergraduate mathematics courses.

Findings

To report our results on Jordan's adjustments to the learning of mathematics at the postgraduate level, we report on one network of learning routines central to Jordan's joining mathematical discourses. Building on the fractal nature of networks, we then zoom in on one component of this network, which is a network in its own right.

Agentive Navigation of Network of Learning Routines

As mentioned, Jordan had to develop the background needed for his research in topology and group theory. Thus, the overarching task of the learning network we identified was to become an insider of these discourses or, in Jordan's words, to "get the background knowledge." Here is an example of how Jordan described this endeavor in the case of topology:

> I spent the first couple of weeks learning some topology and I read [some] lecture notes from a graduate course. It's a sort of introduction to topological groups, and it had like exercises. So, yesterday, I met with my adviser, and we just went over the exercises that I did and checked if they were roughly correct, and tried to approach the ones that I didn't get correct or just didn't know how to do. This was all building up to this theorem called Van Dantzig theorem. The goal of the first couple weeks was to understand that because it's apparently a foundation theorem in this area of topological groups.

The synthesis of similar self-reflections and observations of Jordan's work showed that Jordan's learning activity included finding references online; engaging with them with a special attention to definitions, examples, and exercises; discussing selected exercises and their solutions with his supervisor; and writing up selected solutions in LaTeX. The activities had a patterned nature and fed into each other, giving rise to a network of learning routines.

Some elements of the network are not foreign to routines undergraduates employ to join mathematical discourses in their coursework. Indeed, it is not rare for students to engage with provided references, solve exercises, and attend office hours to clarify issues with the course lecturer. At some point in the first interview, Jordan reflected on this similarity explicitly:

I remember when I was taking algebra and calculus theory, and you're doing line integrals or decomposition of matrices, I could just do exercises, and there were loads of exercises, like hundreds of exercises. I could just sort of go, do them quite quickly, and I would get a good understanding.

This excerpt suggests that engaging with exercises provided in lecture notes and textbooks was a routinized and valuable way for Jordan to learn in at least some of his undergraduate courses. Hence, we propose that features of undergraduate studies and the learning routines they shape constitute precedents informing Jordan's routines as he joins the topological discourse.

Notwithstanding, it is hard to ignore the differences between typical ways of learning in undergraduate courses and Jordan's learning network. During the first months of his program, Jordan studied topology and group theory in an independent, self-directed manner, without a teacher providing carefully selected readings, exercises, and assessments. This new situation did not enable the replication of routinized habits and necessitated Jordan to be agentive and make multiple decisions. Let us elaborate on some of them regarding his engagement with multiple references.

Engaging with multiple references The supervisors directed Jordan to the main concepts and theorems in the area, which focused on Jordan's search for references. Consider an example of Jordan reflecting on his engagement with one reference:

Jordan: Right now I'm reading through this particular document [lecture notes]. I'll have like, 20 tabs opened on my computer, and it's a couple of different textbooks and this paper and I'm also looking up Wikipedia articles, looking up math stack exchange articles, or posts to explain certain things.

Rox-Anne: And you use all those resources to understand better the notes that you have, right?

Jordan: Yeah. It explains sparsely the notes [my supervisors] want me to go through. I have a couple of textbook PDFs that I have downloaded, and also a friend of mine in a course. He's doing a master's, and he has some notes on topology, so I'm also using his notes…just referencing all this stuff. Sometimes, I want a particular definition, so I have to go to another reference.

In an undergraduate course, students are typically offered single references to learn from. These resources cover the relevant material and provide explanations that lecturers deem satisfactory and appropriate to support students' learning. However, in the case of Jordan, he has to be the one to search for relevant references. The process occurs online, requiring particular search procedures (e.g., choosing keywords). Jordan needs to estimate the usefulness of each reference to decide whether to engage with it more deeply or search for an alternative. No single reference appears to totally fulfil Jordan's needs, which yields to search-engagement cycles. Furthermore, Jordan also has to be the one to decide when the reference has fulfilled its purpose. For instance, once "a particular definition" has been found, he can return to his main notes. In this way, we argue that Jordan's performance of search-for-references and engagement-with-references routines are more agentive than those typically expected from students in undergraduate courses. Agency pertains to all components of these routines, including their initiation, procedures, and closure.

Detachment from Externally Proposed Exercises in Favor of Self-Posed Ones

In the first interview, Jordan shared that working on exercises he finds in textbooks and lecture notes plays a major role in his learning. He also recognized the limitations of learning through exercises. In his words:

I guess I am not really sure what I'm going to do when the exercises run out because that's kind of the main way I've always learned. So if there are no exercises, I don't really know, I'll have to ask my advisors to, like, create exercises for me or something, or maybe I'll just try to come up with it myself.

Working on externally assigned exercises constitutes a central component of Jordan's learning network. By engaging with them, Jordan puts himself in task situations he encountered at the undergraduate level: an authoritative source, such as a lecturer or a textbook, ensures that the exercise is well-formulated, relevant, and potentially contributes to students' learning. The relevant material that is sufficient to solve the exercise is typically presented in the preceding text, whereas some learning resources may even provide final answers, hints, or complete solutions. A self-generated solution is often interpreted as a marker of understanding and progress, whereas difficulties can be clarified with a course lecturer or a supervisor in Jordan's case. In this way, exercises contribute to Jordan's becoming a participant in his target mathematical discourse.

However, exercises also confine Jordan to a particular type of resource. Indeed, Jordan was cognizant that exercises are not characteristic of the literature found in postgraduate mathematics, or at least not in the way Jordan is used to. This may potentially impede his engagement with that literature since he is "not really sure [what] to do when the exercises run out." As in undergraduate courses, he considers the supervisors as potential setters of exercises in these cases. Jordan acknowledges the possibility of coming up with his own exercises, but this possibility was mentioned in passing.

Later interviews revealed a number of routines Jordan developed to reduce his dependency on externally set exercises. He labelled one of these routines as *filling in the gaps* and gave the following example:

Jordan: For the Van Dantzig theorem, they say that the translation map is a continuous map and they just make a little remark about it, but the reason it works is because it comes from the fact that the product map is continuous as well.

Rox-Anne: But it's not that obvious, right?

Jordan: I mean, it kind of is, I don't know, I feel like I'll write it out for myself eventually. Just write out a little proof in full like we show that the translation is just a product of [continuous maps].

Filling in the gaps emerged as a network of routines where Jordan sets new task situations for himself. Within this network, Jordan engaged with a reference with attention to "gaps", such as statements that were not justified in full or inferences that were unclear to him. Then, he turned the generation of a justification or proof into a task for him to pursue. This routine was discussed more deeply as Jordan's research project became more oriented on group theory. To develop the background for his research, Jordan engaged with various scientific papers, and the new routine of filling in the gaps gained importance.

Jordan specified that he initially filled in gaps that the references' authors often indicated with phrases such as "we omitted the proof because it's not difficult." He interpreted these indicators as a "signal that ok, I should probably fill that in, for the details, to put in my research [thesis]". At some point, Jordan started to create gaps on his own by pausing his reading and attempting to prove or justify mathematical statements before engaging with how this was done in the reference. In his words, I "create exercises by looking at the statement, then [trying] to prove it, and then reading what their proof is." Gaps of this sort can be opened in any mathematical paper, turning it into a prolific source of learning-inducing task situations. Whether noticed or created, the routine of filling in the gaps enabled Jordan to distance his learning network from the dependence on externally set exercises.

Summary and Discussion

Mathematics departments invest considerable efforts and resources to involve their majoring students in mathematics research through postgraduate programs. However, only a few studies in mathematics education explored the transition from undergraduate to postgraduate studies. With all the research attention that transitions received in mathematics education (Gueudet, 2016), the undergraduate-postgraduate transition appears to be overlooked.

In this paper, we focused on Jordan—an honors student who was developing mathematical background on his own as a stepping stone toward his research project. Higher education research emphasizes the shift in independence in students' transition from undergraduate studies to postgraduate research (Lovitts, 2005, 2008). Jordan's case illustrates that undergraduate studies do not always prepare students to embark on research right away, and students may need to develop the necessary background related to their research area. Geraniou (2010) referred to such situations as the *adjustment* stage and also identified this stage among doctoral studies. Our study shows that an adjustment stage may also be necessary in transitioning to other types of postgraduate research.

Using the commognitive framework, we identified a network of learning routines Jordan employed and developed in his adjustment stage. We saw similarities between some network components and ways of learning students often employ in undergraduate courses. Instances of these similar network components are the engagement with resources and the discussions with the advisors who provided clarifications about the material. These correspondences are somewhat expected considering that Jordan majored only a few months ago and was offered learning aims similar to those found in undergraduate courses. Additionally, we note that the main reference used by Jordan when joining the topological discourse was lecture notes, which may have facilitated the replication of routines common in undergraduate courses. Nonetheless, a distinctive feature of Jordan's network pertained to agency. In particular, agency was needed to navigate the network routines since each required him to make decisions regarding the initiation of a routine, the course of action, and a decision that the targeted outcome was achieved. Initially, the network depended on Jordan having access to exercises from textbooks and lecture notes, which confined his learning opportunities to specific references. Then, he demonstrated agency when introducing the routine of filling in the gaps that enabled him to become a poser of his own exercises. This new routine had a major impact on the whole network, broadening its applicability and enabling Jordan to learn from research papers. Accordingly, we propose that in mathematics, the shift to independence mentioned by Lovitts (2005, 2008) may be characterized by a growing distance from externally provided resources supporting learning.

The network development we observed fully exemplifies what has been called agentive learning (Brenan, 2012; Knowles, 1975). Our findings show that in the case of Jordan, the shift to this kind of learning, which was required at the postgraduate level, was a gradual and time-consuming process and a part of his adjustment stage. This makes us wonder about the role of agentive learning in the transition from undergraduate to postgraduate mathematics, specifically considering students' expected contribution to research.

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